Reg. No. : ___________

Question Paper Code : 50831

B.E./B.Tech DEGREE EXAMINATIONS, APRIL/MAY 2023.

Third Semester

Biomedical Engineering

MA 8352 – LINEAR ALGEBRA AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to: Computer and Communication Engineering/Electronics and Communication Engineering/Electronics and Telecommunication Engineering/Medical Electronics)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define subspace.

2. Check whether the set $\left\{\begin{pmatrix} 1 & -3 & 2 \\ -4 & 0 & 5 \end{pmatrix}, \begin{pmatrix} -3 & 7 & 4 \\ 6 & -2 & -7 \end{pmatrix}, \begin{pmatrix} -2 & 3 & 11 \\ -1 & -3 & 2 \end{pmatrix}\right\}$ is linearly dependent or not.

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation defined by $T(a_1, a_2, a_3) = (a_1 - a_2, 2a_3)$. Find $N(T)$.

4. Find the eigenvalues of $\begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.

5. Is $\{(1, 1, 0), (1, -1, 1), (-1, 1, 2)\}$ an orthonormal set?

6. Define adjoint' of a linear transformation.

7. Form the partial differential equation by eliminating the arbitrary functions from $z = f(x + at) + g(x - at)$, where a is a constant.

8. Solve $p - q = 1$.

9. State Dirichlet conditions for the expansion of $f(x)$ in Fourier series.

10. Write down the equation of the vibrating string.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Let V be a vector space over a field F and W be a subset of V. Prove that W is a subspace of V if the following three conditions hold for the operations defined in V (8)

(1) $0 \in W$

(2) $\forall x, y \in W$, we have $x + y \in W$

(3) $\forall x \in W, c \in F$, we have $cx \in W$

(ii) Prove that the span of any subset S of a vector space V is a subspace of V. (8)

Or

(b) (i) Let S be a linearly independent subset of a vector space V, and let v be a vector in V that is not in S. Then prove that $S \cup \{v\}$ is linearly dependent if and only if $v \in span(S)$. (8)

(ii) Let V be a vector space and $\beta = \{u_1, u_2, ..., u_n\}$ be a subset of V. Then prove that if β is a basis of V then each $v \in V$ can be uniquely expressed as a linear combination of vectors of β. (8)

12. (a) State and prove Dimension Theorem. (16)

Or

(b) (i) Find eigen values of $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$. (6)

(ii) Reduce the matrix $A = \begin{pmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$ to the diagonal form. (10)

13. (a) (i) Let V be an inner product space over F. Let $x, y \in V$. Prove that $|<x, y>| \le \|x\| \cdot \|y\|$. (8)

(ii) Let V be an inner product space over F. Let $x, y \in V$. Prove that $\|x + y\| \le \|x\| + \|y\|$. (8)

Or

(b) (i) In $\mathbb{R}^4$, the set $\{(1,0,1,0),(1,1,1,1),(0,1,2,1)\}$ is linear independent. Using Gram-Schmidt process, obtain an orthonormal set. (8)

(ii) Fit the least squares line for the data (1,2), (2,3), (3,5) and (4.7). (8)

14. (a) (i) Find the differential equation of all spheres whose centres lie on the z–axis. (8)

(ii) Solve $xzp + yzq = xy$. (8)

Or

(b) (i) Solve $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = 0$. (4)

(ii) Solve $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} = 2e^{2x} + 3x^2 y$. (12)

15. (a) (i) Find the Fourier series of $f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ x^2, & 0 \le x \le \pi \end{cases}$ which is assumed to be periodic with period 2π. (10)

(ii) Express $f(x)$ as a half-range sine series in $0 < x < 2$. (6)

Or

(b) (i) Using method of separation of variables, solve $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial z}{\partial z} + \frac{\partial z}{\partial y} = 0$ (8)

(ii) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is vibrating by giving to each of its points a velocity $\lambda x(l - x)$, find the displacement of the string at any distance x from one end at any time t. (8)