Reg. No. :

Question Paper Code : 10808

M.E./M.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

First Semester

Communication Systems

MA 4156 — LINEAR ALGEBRA, PROBABILITY AND QUEUEING THEORY

(Common to M.E. Communication and Networking/M.E. Electronics and Communication Engineering/M.E. Electronics and Communication Engineering (Industry Integrated))

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Let $v = (1, -2, 2, 0)$. Find a unit vector u in the same direction as v.

2. Is $u = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$ an eigen vector of $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$? Justify.

3. State the axioms of probability.

4. If $f(x)$ is the density function of a random variable X given by

$$f(x) = \begin{cases} kx, 0 \le x \le 1 \\ k, 1 \le x \le 2, \\ 3k - kx,\ 2 \le x \le 3 \\ 0, elsewhere \end{cases}$$ Find the value of K.

5. What is the difference between the random variable and random process?

6. Given that the autocorrelation function $R_x(\tau) 25 + \frac{4}{1+6\tau^2}$, find the mean of the process.

7. What are the basic characteristics of a queueing system?

8. Find the average waiting time of a customer in the three server infinite capacity poisson queue if he happens to wait, given that $\lambda = 6$, $\mu = 4$ per hour.

9. What is an optimal solution?

10. When do we say a transportation problem is an unbalanced?

PART B — (5 × 13 = 65 marks)

11. (a) Construct QR-decomposition for $X = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$.

Or

(b) Find a matrix in Jordan canonical form that is similar to $A = \begin{bmatrix} 0 & 4 & 2 \\ -3 & 8 & 3 \\ 4 & -8 & -2 \end{bmatrix}$.

12. (a) The joint probability mass function of (X, Y) is given by $p(x, y) = k(2x + 3y)$, $x = 0,1,2;\ y = 1,2,3$. Find all the marginal and conditional probability distributions.

Or

(b) Compute the coefficient of correlation between X and Y using the data given below:

X	65	67	66	71	67	70	68	69
Y	67	68	68	70	64	67	72	70

13. (a) Show that the random process $X(t) = A\cos(\omega t + \theta)$ is wide sense stationary if A and ω are constants and θ is uniformly distributed random variable in $(0, 2\pi)$.

Or

(b) Statistically independent zero mean random processes X(t) and Y(t) have autocorrelation functions $R_{XX}(\tau) = e^{-|\tau|}$ and $R_{YY}(\tau) = \cos 2\pi\tau$ respectively. Find the autocorrelation function of the sum $W_1(t) = X(t) + Y(t)$, difference $W_2(t) = X(t) - Y(t)$ and the cross correlation of $W_1(t)$ and $W_2(t)$.

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14. (a) Customers arrive at a one-man barber shop according to a Poisson process with a mean inter-arrival time of 12 minutes- Customers spend on average of 10 minutes in the barbers chair.

(i) What is the expected number of customers in the barbershop?

(ii) Calculate the percentage of time for an arrival can walk straight into the barber's chair without having to wait.

(iii) How much time can a customer expect to spend in the barber's shop?

(iv) Management will provide another chair and hire another barber when a customer's waiting time in the shop exceeds 1.25 hours. How much must the average rate of arrivals increase to warrant a second barber?

(v) What is the average time that customers spend in the queue?

Or

(b) Patients arrive at a clinic having a single doctor according to Poisson distribution at a rate of 30 per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour. (i) Find the effective arrival rate at the clinic (ii) What is the probability that an arriving patient will not wait? (iii) What is the expected waiting time until a patient is discharged from the clinic?

15. (a) Use Big M method to solve:

Minimize $z = 4x_1 + 3x_2$

Subject to $2x_1 + x_2 \geq 0$

$-3x_1 + 2x_2 \leq 6$

$x_1 + x_2 \geq 6$

$x_1, x_2 \geq 0$

Or

(b) The assignment cost of assigning any one operator to any one machine is given below:

	I	II	III	IV
A	10	5	13	15
B	3	9	18	3
C	10	7	3	2
D	5	11	9	7

Find the optimal assignment.

PART C — (1 × 15 = 15 marks)

16\. (a) Find a singular value decomposition of $A = \begin{bmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{bmatrix}$

Or

(b) Fins the least squares solution to $x + 2y + z = 1$, $3x - y = 2$, $2x + y - z = 2$ and $x + 2y + 2z = 1$

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