Reg. No. :

Question Paper Code : 50839

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

Fifth Semester

Computer Science and Engineering

MA 8551 — ALGEBRA AND NUMBER THEORY

(Common to : Computer and Communication Engineering/Information Technology)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Give an example for a finite abelian group.

2. Find the inverse of 4 under the binary operation * defined in Z by $a * b = a + b - 2$.

3. What are the characteristics of the rings $(Z,+,.)$ and $(Q,+,.)$?

4. Give an example for an irreducible and reducible polynomial in $Z_2[x]$.

5. Find the number of positive integer's $\leq$ 1576 and not divisible by 11.

6. Obtain the gcd of (15, 28, 50).

7. Determine whether the LDE $5x + 20y + 30z = 44$ is solvable.

8. What is the remainder when 3^{31} is divided by 7.

9. State Wilson's theorem.

10. Compute $\phi(n)$ for $n = 146$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Determine whether $(Z,\oplus,\odot)$ is a ring with the binary operation $x \oplus y = x + y - 7$, $x \odot y = x + y - 3xy$ for all $x, y \in Z$. (8)

(ii) Prove that Z_n is a field if and only if n is a prime. (8)

Or

(b) (i) Prove that commutative properties is invariant under homomorphism. (8)

(ii) Find $[777]^{-1}$ in Z_{1009}. (8)

12. (a) (i) If R is a ring under usual addition and multiplication, show that $(R[x],+,x)$ is a ring of polynomials over R. (8)

(ii) Find all the roots of $f(x) = x^2 + 4x$ in $Z_{12}[x]$. (8)

Or

(b) (i) If $f(x) \in F[x]$ has degree $n \geq 1$, then prove that $f(x)$ has atmost n roots in F. (8)

(ii) If $f(x) = 3x^5 - 8x^4 + x^3 - x^2 + 4x - 7$, $g(x) = x + 9$ and $f(x), g(x) \in Z_{11}[x]$, find the remainder when $f(x)$ is divide by $g(x)$. (8)

13. (a) (i) Using the canonical decomposition of 1050 and 2574, find their lcm. (8)

(ii) Apply Euclidean algorithm to express the gcd of 3076 and 1976 as a linear combination of themselves. (8)

Or

(b) (i) Find the number of positive integers $\leq$ 999 that are divisible by 7 and 13. (8)

(ii) Prove that the product of gcd and lcm of any two positive integers a and b is equal to their products. (8)

14. (a) (i) Find the general solution of the linear Diophantine equation $6x+8y+12z=10$. (8)

(ii) Find the incongruent solutions of $5x \equiv 3 \pmod 6$. (8)

Or

(b) State Chinese Remainder Theorem. Using it solve $x \equiv 1 \pmod 3, x \equiv 2 \pmod 4$, and $x \equiv 3 \pmod 5$. (16)

15. (a) (i) Prove that the Euler's Phi function is multiplicative. (8)

(ii) Compute tau and sigma functions for $n = 2187$. (8)

Or

(b) State and prove Fermat's Little theorem. Hence, compute the remainder when 7^{1001} is divided by 17. (16)
