Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 50838

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

Fourth/Fifth/Sixth Semester

Civil Engineering

MA 8491 – NUMERICAL METHODS

(Common to : Aeronautical Engineering/Aerospace Engineering/ Agriculture Engineering/Electrical and Electronics Engineering/Electronics and Instrumentation Engineering/Instrumentation and Control Engineering/Manufacturing Engineering/Mechanical Engineering (Sandwich)/Mechanical and Automation Engineering/Biotechnology and Biochemical Engineering/Chemical Engineering/Chemical and Electrochemical Engineering/Plastic Technology/Polymer Technology/Textile technology)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What is the First iteration approximate solution of the equation $4x + y = 8$ and $2x + 3y = 7$ solved by Gauss Jacobi Method?

2. Find all eigen values of the matrix A by Jacobi's method where $A = \begin{pmatrix} 2 & -3 \\ -3 & 2 \end{pmatrix}$.

3. Form the divided difference table for the following data :

x :	4	5	7	10
y :	48	100	294	900

4. Find the Lagrange's interpolating polynomial passing through the points $(0,0), (1,1), (2,20)$.

5. Evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ by using two-point Gaussian quadrature formula.

6. Find $\frac{dy}{dx}$ of $x = 50$ by using the following Forward difference

x	y	Δy	$\Delta^2 y$
50	3.6840	0.0244	–0.0003
51	3.7084	0.0241	
52	3.7325		

7. Using Euler's method, find y at $x = 0.1$ if $\frac{dy}{dx} = 1 + xy$, $y(0) = 2$.

8. State the Milne's predictor and corrector formula for solving differential equation numerically.

9. Write the finite difference scheme for $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} = 8x^2y^2$ for a square region with mesh size $\Delta x = \Delta y = 1$.

10. Write the explicit formula for one-dimensional wave equation if $1 - \lambda^2\alpha^2 = 0$ and $\lambda = \frac{k}{h}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find a real root of the equation $\cos x = 3x - 1$ correct to three decimal places by iteration method. (6)

(ii) Find the largest Eigen value and its corresponding Eigen vector of the matrix $A = \begin{pmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ by power method Take $X_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. (10)

Or

(b) Solve the following system of equations by Gauss-Seidal Method (16)

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

12. (a) (i) Use Newton's backward difference formula to construct an interpolating polynomial of degree 3 for the data : (10)

x :	–0.75	–0.5	–0.25	0
$f(x)$:	–0.07181250	0.024750	0.33493750	1.10100

(ii) In the following table, the values of y are consecutive terms of a series of which 23.6 is the 6th term. Find the first term of the series. (6)

x :	3	4	5	6	7	8	9
y :	4.8	8.4	14.5	23.6	36.2	52.8	73.9

Or

(b) Using the following table values, find the natural cubic spline approximation are hence evaluate the value of y at $x = 2.5$. (16)

x :	0	1	2	3
y :	1	2	33	244

13. (a) (i) Using the approximate Newton's Interpolation formula to find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 2.2$ from the following data: (10)

x :	1.4	1.6	1.8	2.0	2.2
y :	4.0552	4.9530	6.0496	7.3891	9.0250

(ii) Use Gaussian quadrature three points formula to evaluate the integral $\int_1^2 \frac{dx}{x}$. (6)

Or

(b) Consider the following data:

x :	0	0.125	0.250	0.375	0.50	0.675	0.750	0.875	1
$y = \frac{1}{1+x^2}$:	1	0.9846	0.9412	0.8767	0.8	0.7191	0.64	0.5664	0.5

with $h = 0.5, 0.25, 0.125$ and use Romberg's method to compute $\int_0^1 \frac{1}{1+x^2} dx$. Hence deduce an approximate value of π. (16)

14. (a) (i) Given $\frac{dy}{dx}=\left(\frac{1}{2}\right)(1+x^2)y^2$ and $y(0)=1$, $y(0.1)=1.06$, $y(0.2)=1.12$, $y(0.3)=1.21$, evaluate $y(0.4)$ by Milne's predictor-Corrector Method correct to 4 decimal places. (10)

(ii) Solve the equation $\frac{dy}{dx}=1-y$, $y(0)=0$ using modified Euler's method and tabulate the solutions at $x=0.1$ and 0.2 correct to 4 decimal places. (6)

Or

(b) Given $\frac{dy}{dx}=y-x^2+1$, $y(0)=0.5$. (16)

(i) Using the modified Euler's method, find $y(0.2)$

(ii) Using the 4[th] order Runge-Kutta method, find $y(0.4)$ and $y(0.6)$.

(iii) Using Adams-Bash forth Predictor-Corrector Method, find $y(0.8)$.

15. (a) Solve $2u_t=u_{xx}$, $u(0,t)=0$, $u(4,t)=0$, $u(x,0)=x(4-x)$ with $h=1$. Find the values of u upto $t=5$. (16)

Or

(b) Find the model values of the wave equation $\frac{\partial^2 u}{\partial t^2}=16\frac{\partial^2 u}{\partial x^2}$ given that $u(0,t)=u(5,t)=0$, $u(x,0)=x^2(5-x)$ and $u_t(x,0)=0$ taking $h=1$ and upto one half of the period of vibration.

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