Reg. No. :

Question Paper Code : 50836

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

Fourth Semester

Computer and Communication Engineering

MA 8451 — PROBABILITY AND RANDOM PROCESSES

(Common to Electronics and Communication Engineering/Electronics and Telecommunication Engineering)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Suppose that two events A and B are mutually exclusive and $P[B] > 0$. Under what conditions will A and B be independent?

2. Balls are tossed at random into 50 boxes. Find the expected number of tosses required to get the first ball in the fourth box.

3. X and Y are two continuous random variables whose joint PDF is given by
$$f_{XY}(x, y) = \begin{cases} e^{-(x+y)}, & 0 \le x < \infty, 0 \le y < \infty \\ 0 & otherwise. \end{cases}$$
Are X and Y independent?

4. The joint PMF of two random variables X and Y is given by
$$p_{XY}(x, y) = \begin{cases} k(2x + y) & x = 1, 2; y = 1, 2 \\ 0 & otherwise \end{cases}$$
where k is a constant. What is the value of k?

5. Define irreducible Markov Chain.

6. Does difference of two independent Poisson process a Poisson process? Justify.

7. If $X(t)$ is periodic, then prove that its auto correlation function is also periodic.

8. Define power spectral density function.

9. Give an example of linear time invariant system.

10. What is time-invariant system with example?

PART B — (5 × 16 = 80 marks)

11. (a) (i) A student buys 1000 integrated circuits (ICs) from supplier A, 2000 ICs from supplier B, and 3000 ICs from supplier C. He tested the ICs and found that the conditional probability of an IC being defective depends on the supplier from whom it was bought. Specifically, given that an IC came from supplier A, the probability that it is defective is 0.05; given that an IC came from supplier B, the probability that it is defective is 0.10; and given that an IC came from supplier C, the probability that it is defective is 0.10. If the ICs from the three suppliers are mixed together and one is selected at random, what is the probability that it is defective? (8)

(ii) A test engineer discovered that the CDF of the lifetime of an equipment in years is given by $F_X(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-\frac{x}{5}}, & 0 \le x < \infty \end{cases}$

(1) What is the expected lifetime of the equipment?

(2) What is the variance of the lifetime of the equipment? (8)

Or

(b) (i) Messages arrive at a switchboard in a Poisson manner at an average rate of 6 per hour. Find the probability for each of the following events:

(1) Exactly two messages arrive within one hour.

(2) No message arrives within one hour.

(3) At least three messages arrive within one hour.

(4) Also, calculate the mean and variance. (8)

(ii) The weights in pounds of parcels arriving at a package delivery company's warehouse can be modeled by an $N(5; 16)$ normal random variable, X.

(1) What is the probability that a randomly selected parcel weighs between 1 and 10 pounds?

(2) What is the probability that a randomly selected parcel weighs more than 9 pounds? (8)

12. (a) (i) A fair coin is tossed three times. Let X be a random variable that takes the value 0 if the first toss is a tail and the value 1 if the first toss is a head. Also, let Y be a random variable that defines the total number of heads in the three tosses.

(1) Determine the joint PMF of X and Y.

(2) Are X and Y independent? (8)

(ii) The joint PMF of two random variables X and Y is given by

$$p_{XY}(x, y) = \begin{cases} k(2x + y), & x-1, 2;\ y = 1, 2 \\ 0, & otherwise \end{cases}$$

where k is a constant.

(1) What is the value of K?

(2) Find the marginal PMFs of X and Y.

(3) Are X and Y independent? (8)

Or

(b) (i) The joint PDF of the random variables X and Y is defined as follows:

$$f_{XY}(x, y) = \begin{cases} 25\, e^{-5y}, & 0 < x < 0.2,\ y > 0 \\ 0, & otherwise \end{cases}$$

(1) Find the marginal PDFs of X and Y.

(2) What is the covariance of X and Y? (8)

(ii) Assume that the random variable S_n, is the sum of 48 independent experimental values of the random variable X whose PDF is given by

$$f_X(x) = \begin{cases} \frac{1}{3}, & 1 \le x \le 4 \\ 0, & otherwise \end{cases}$$

Find the probability that S_n lies in the range $108 \le S_n \le 126$.

13. (a) (i) Consider the process $\{X(t)\} = \cos(\omega_0 t + \theta)$, where θ is uniformly distributed in the interval $-\pi$ to π. Check whether $X(t)$ is stationary or not? (8)

(ii) The transition probability matrix of the Markov chain $X(n)$, $n = 1,2,3$ ... having 3 states 1, 2, and 3 is

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix} \end{array}$$

and the initial distribution is (0.7 0.2 0.1). Find $P(X_2 = 3)$ and $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$ (8)

Or

(b) (i) Find the mean and covariance of the random telegraph signal (8)

(ii) A machine goes out of order, whenever a component fails. The failure of this part follows a Poisson process with a mean rate of 1 per week. Find the probability that 2 weeks have elapsed since last failure. If there are 5 spare parts of this component in an inventory and that the next supply is not due in 10 weeks, find the probability that the machine will not be out of order in the next 10 weeks. (8)

14. (a) A random process is defined by $X(t) = K \cos wt \geq 0$, where w is a constant and K is uniformly distributed between 0 and 2. Determine the following:

(i) $E[X(t)]$ (6)

(ii) The autocorrelation function of $X(t)$ (5)

(iii) The autocovariance function of $X(t)$ (5)

Or

(b) (i) The auto correlation of the random binary transmission is given by $R_{XX}(\tau) = \begin{cases} 1 - \frac{|\tau|}{T}, & for\ |\tau| \leq T \\ 0, & for\ |\tau| > T \end{cases}$. Find the power spectrum. (8)

(ii) The cross-power spectrum of real random process $X(t)$ and $Y(t)$ is given by $S_{XY}(w) = \begin{cases} a + jbw, & for\ |w| \leq 1 \\ 0, & elsewhere. \end{cases}$ Find the cross correlation function. (8)

15. (a) Let $X(t)$ be a zero-mean WSS process with $R_X(\tau) = \exp(-|\tau|)$. $X(t)$ is input to an LTI with

$$|H(f)| = \begin{cases} \sqrt{1 + 4\pi^2 f^2} & if\ |f| < 2 \\ 0 & otherwise \end{cases}$$

Let $Y(t)$ be the output. Find $E[Y(t)]$, $R_Y(t)$ and $E[Y(t)^2]$. (16)

Or

(b) Let $X(t)$ be a random process with mean function $\mu_X(t)$ and autocorrelation function $R_X(s, t)$ ($X(t)$ is not necessarily a WSS process). Let $Y(t)$ be given by

$$Y(t) = h(t) * X(t)$$

where $h(t)$ is the impulse response of the system. Show that

(i) $\mu_Y(t) = \mu_X(t) * h(t)$ (8)

(ii) $R_{XY}(t_1, t_2) = h(t_2) * R_X(t_1, t_2) = \int_{-\infty}^{\infty} h(\alpha) R_X(t_1, t_2 - \alpha) d\alpha$. (8)

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