Reg. No. :

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B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022

Fourth Semester

Computer and Communication Engineering

MA 8451 – PROBABILITY AND RANDOM PROCESSES

(Common to : Electronics and Communication Engineering/Electronics and Telecommunication Engineering)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If two dice are thrown, find the probability that the sum is either 6 or 10.

2. The time required to repair a machine is exponentially distributed with parameter $\frac{1}{2}$. What is the probability that the repair time exceeds 2 hours?

3. Let X and Y be any two random variables and a, b are constants. Prove that $Cov\,[aX, bY] = ab Cov[X, Y]$.

4. The joint PMF of two random variables X and Y is given by $p(x, y) = k(2x + y)$, $x = 1, 2$; $y = 1, 2$ where k is a constant. What is the value of k?

5. Define Wide sense stationary process.

6. If the transition probability matrix of a Markov Chain is $P = \begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$, find the limiting distribution of the Chain.

7. If $R(\tau)$ is the autocorrelation function of a stationary process $\{x(t)\}$, prove that $|R(\tau)| \le R(0)$.

8. State any two properties of the power spectral density.

9. Define the power transfer function of the system.

10. A random process $x(t)$ in the input to a linear system whose impulse response is $h(t) = 2e^{-t}$, $t \geq 0$. Find the transfer function of the linear system.

PART B — $(5 \times 16 = 80$ marks)

11. (a) (i) Obtain the mean, variance and moment generating function of a Poisson random variable.

(ii) The contents of urns I, II and III are as follows:

I : 2 white, 3 black and 4 red balls

II : 2 black, 3 white and 2 red balls

III : 4 white, 1 black and 3 red balls.

An urn is chosen at random and two balls are drawn. They happen to be white and red. What is the probability that they come from urn I? (10)

Or

(b) (i) Consider the function $f(x) = \begin{cases} c, & a \leq x \leq b \\ 0, & otherwise \end{cases}$

(1) For what value of c is $f(x)$ a legitimate probability density function?

(2) Find the CDF of the random variable X with the above PDF. (6)

(ii) The number of accidents in a year to taxi drivers in a city follows a Poisson distribution with mean equal to 3. Out of 1,000 taxi drivers, find approximately the number of drivers with

(1) No accidents in a year

(2) More than 3 accidents in a year. (10)

12. (a) (i) The joint probability density function of a two dimensional random variable (X,Y) in given by $f(x,y) = e^{-(x+y)}$, $x \geq 0$, $y \geq 0$. Find the conditional densities of X given Y and Y given X. (8)

(ii) Determine if random variables X and Y are independent when their joint PDF is given by $f(x,y) = \frac{x^3 y}{2}$, $\begin{matrix} 0 \leq x \leq 2 \\ 0 \leq y \leq 1 \end{matrix}$. (8)

Or

29

(b) (i) If X and Y are independent random variables having density function $f(x)=\begin{cases}2e^{-2x}, & x\ge 0\\ 0, & x<0\end{cases}$ and $f(y)=\begin{cases}3e^{-3y}, & y\ge 0\\ 0, & otherwise\end{cases}$

Find the density function of their sum $X+Y$. (8)

(ii) Assume that the random variable S_n is the sum of 48 independent experimental values of the random variable X whose PDF is given by

$$f(x)=\frac{1}{3},\ 1\le x\le 4$$

Find the probability that S_n lies in the range $108\le S_n\le 126$. (8)

13. (a) (i) State the properties of Poisson Process. (6)

(ii) Show that the random process $x(t)=A\cos(wt+\theta)$ is a wide sense stationary if A and w are constants and θ is a uniformly distributed random variable in $(0, 2\pi)$. (10)

Or

(b) (i) Let $\{x_n = n = 0,1,2...\}$ be a Markov Chain having state space $S=\{1,2,3\}$ with one step transition probability matrix $\begin{bmatrix}1 & 0 & 0\\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4}\\ 0 & 0 & 1\end{bmatrix}$

and the initial distribution $P[X_0=i]=\frac{1}{3},\ i=1,2,3$.

Find

(1) $P[X_3=1, X_2=1, X_1=1, X_0=2]$

(2) $P[X_2=2, X_1=1/X_0=1]$

(3) $P[X_3=3/X_2=2, X_1=1, X_0=3]$

(4) $P[X_2=3, X_0=3]$. (10)

(ii) If $p_{ij}(2)$ is the conditional probability that the system will be in state j after exactly 2 transition, given that it is presently in state i, then prove that $p_{ij}(2)=\sum_k p_{ik}p_{kj}$. (6)

14. (a) (i) Compute the variance of the random process $X(t)$ whose autocorrelation function is given by $R(\tau) = 25 + \frac{4}{1+6\tau^2}$. (6)

(ii) Determine the autocorrelation function of the random process with the power spectral density given by $S(w) = \begin{cases} S_0, & |w| < w_0 \\ 0, & otherwise \end{cases}$. (10)

Or

(b) (i) A random process $Y(t)$ consists of the sum of the random process $X(t)$ and a statiscally independent noise process $N(t)$. Find the cross correlation function of $X(t)$ and $Y(t)$. (6)

(ii) If $R(\tau) = e^{-2\lambda|\tau|}$ is the autocorrelation function of a random process $X(t)$, obtain the spectral density of $X(t)$. (10)

15. (a) A random process $X(t)$ is the input to a linear system whose impulse response is $h(t) = 2e^{-t}$, $t \geq 0$. If the auto correlation function of the process is $R(\tau) = e^{-2|\tau|}$, determine the cross correlation function $R_{XY}(\tau)$ between the input process $X(t)$ and the output process $Y(t)$. (16)

Or

(b) Given that $X(t)$ is the input to a linear time-invariant system with impulse response $h(t)$ and $Y(t)$ is the corresponding output of the system. Determine the mean and autocorrelation function of $Y(t)$ if $X(t)$ is a wide sense stationary process and also prove that $|H(w)|^2 = \frac{S_{YY}(w)}{S_{XX}(w)}$. (16)