Reg. No. :

Question Paper Code : 90814

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Third Semester

Biomedical Engineering

MA 8352 – LINEAR ALGEBRA AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to: Computer and Communication Engineering/Electronics and Communication Engineering/Electronics and Telecommunication Engineering/Medical Electronics)

(Regulations – 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Show that the vector $w = (9, 2, 7)$ is a linear combination of vectors $u = (1, 2, -1)$ and $v = (6, 4, 2)$.

2. Is the set {(1,2),(3,4)} in R^2 linearly independent? Justify.

3. Find the kernel of the matrix $A = \begin{bmatrix} 1 & 1 & 5 \\ 2 & -1 & 1 \end{bmatrix}$

4. State the dimension theorem for linear transformation.

5. Sketch the unit circle in an xy-coordinate system in R^2 using the weighted Euclidean inner product $< u,v >= \frac{1}{9}u_1v_1 + \frac{1}{4}u_2v_2$ where $u = (u_1, v_1)$ and $v = (v_1, v_2)$.

6. Prove that $\|u+v\| \le \|u\| + \|v\|$ if u and v are vectors in an inner product space V.

7. Form the partial differential equation by eliminating the arbitrary constants a and b from $(x-a)^2 + (y-b)^2 = z^2 \cot^2 \theta$

8. Classify the partial differential equation $f_{xx} - 2f_y = x^2 + y^2$.

9. What are the sufficient conditions for the existence of Fourier series of $f(x)$?

10. Find the Fourier coefficient a_0 if $f(x) = x^2 + x, -2 < x < 2$.

PART B — (5 × 16 = 80 marks)

11. (a) Determine whether the set of all pairs of real numbers (x,y) with the standard addition and scalar multiplication k(x,y) = (2kx, 2ky) is a vector space or not. If not, list all the axioms that fail to hold. (16)

Or

(b) Find a basis and dimension of the solution space of the homogeneous system $x_1 + 3x_2 - 2x_3 + 2x_5 = 0$; $2x_1 + 6x_2 - 5x_3 - 2x_4 + 4x_5 - 3x_6 = 0$; $5x_3 + 10x_4 + 15x_6 = 0$; $2x_1 + 6x_2 + 8x_4 + 4x_5 + 18x_6 = 0$ (16)

12. (a) Let $T : P_2 \to P_2$ be a linear operator defined by $T(p(x)) = p(3x - 5)$.

(i) Find $[T]_B$ with respect to the basis $B = \{1, x, x^2\}$.

(ii) Use the indirect procedure to compute $T(1 + 2x + 3x^2)$

(iii) Check the result in (ii) by computing $T(1 + 2x + 3x^2)$ directly. (16)

Or

(b) Find the matrix P that diagonalizes $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ (16)

13. (a) Find the least square solution of the system $Ax = b$ given by $x - y = 4$; $3x + 2y = 1$ and $-2x + 4y = 3$ and find the orthogonal projection b on the column space of A. (16)

Or

(b) Assume that the vector space R^3 has the Euclidean inner product. Apply the Gram-Schmidt process to transform the basis vectors u_1 = (1,1,1), u_2 = (0,1,1), and u_3 = (0,0,1) into an orthogonal basis and then normalize to obtain an orthonormal basis. (16)

14. (a) (i) Solve the partial differential equation $z = px + qy + p^2 + pq + q^2$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. (8)

(ii) Solve the partial differential equation $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$, where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. (8)

Or

(b) Solve the partial differential equation $(D^2 - 3DD' + 2D'^2 + 2D - 2D')z = x + y + \sin(2x + y)$. (16)

15. (a) A uniform string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form of the curve $y = kx(l - x)$ and then releasing it from this position at time t=0. Find the displacement of the string at a distance x from one end at time t. (16)

Or

(b) A square plate has its faces and its edge $y = 0$ insulated. Its edges $x = 0$ and $x = 10$ are kept at zero temperature and its edge $y = 10$ at temperature 100°C. Find the steady state temperature distribution in the plate. (16)