Reg. No. :

Question Paper Code : 90813

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Third Semester

Computer Science and Engineering

MA 8351 – DISCRETE MATHEMATICS

Common to: Artificial Intelligence and Data Science/ Computer Science and Business Systems/ Information Technology

(Regulations 2017)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Show that $\neg(A \leftrightarrow B) \Leftrightarrow (A \vee B) \wedge \neg(A \wedge B)$.

2. Find the inverse and converse of the statement "If you win the lottery, then you can buy a house"

3. Find the minimum number of students needs to guarantee that five of them belong to the same subject, if there are five different major subjects.

4. In how many ways can 5 Mathematics books, 4 History books, 3 Chemistry books and 7 Engineering books arranged on a shelf so that all books of the same subjects are together?

5. Give an example for graph which is Eulerian but not Hamiltonian.

6. Is it possible to have a 3-regular graph with 11 vertices? Justify your answer.

7. Prove that every subgroup of a cyclic group is a normal subgroup.

8. Find the generator(s) in the multiplicative group $\langle 1, -1, i, -i \rangle$.

9. Justify the statement: Every modular lattice is distributive.

10. Give an example for a distributive complemented lattice.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that $((P \vee Q) \wedge (\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology by using equivalences. (6)

(ii) Show that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$ by using proof by contradiction. (10)

Or

(b) (i) Obtain the principal disjunctive normal form of $(Q \to \neg R) \wedge (Q \leftrightarrow P)$ by using equivalences. (8)

(ii) Show that "It rained" is a conclusion obtained from the statements.

"If it does not rain or if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on". "If the sports day is held, the trophy will be awarded" and "the trophy was not awarded". (8)

12. (a) (i) Find the number of integers between 1 and 1000 that are not divisible by any of the integers 2,3,5 or 6. (8)

(ii) Solve the recurrence relation $S(n) = 8S(n-1) + (10)^{n-1}$ *with* $S(2) = 82$ by using generating function. (8)

Or

(b) (i) Using mathematical induction, show that $\sum_{r=0}^{n} 3^r = \frac{3^{n+1}-1}{2}$. (6)

(ii) Using characteristics equation method, solve the recurrence relation for $n \geq 3, S(n) + 3S(n-1) + 3S(n-2) + S(n-3) = 0$, with $S(0) = 5$, $S(1) = -9$, *and* $S(2) = 15$. (10)

13. (a) (i) Prove that a simple graph with n vertices must be connected if it has more than $\frac{(n-1)(n-2)}{2}$ edges. (6)

(ii) If G is a connected simple graph with n vertices with $n \geq 3$, such that the degree of every vertex in G is at least $\frac{n}{2}$, then prove that G has Hamilton cycle. (10)

Or

22

(b) (i) Prove that in a graph G, $\sum_{v \in V(G)} d(v) = 2 \times |E(G)|$. Hence prove that the number of odd degree vertices in any graph is even. (6)

(ii) Prove that a connected graph G is bipartite if and only if all its cycles are of even length. (10)

14\. (a) (i) Show that the group $\langle G, * \rangle$ is abelian if and only if $(a * b)^2 = a^2 * b^2$, for all $a, b \in G$. (6)

(ii) State and prove Lagrange's theorem on group. (10)

Or

(b) (i) Prove that a finite group of order n is isomorphic to a permutation group of order n. (8)

(ii) Let $f : G \to H$ be a homomorphism from the group $\langle G, * \rangle$ to the group $\langle H, \Delta \rangle$. Prove that the kernel of the homomorphism is a normal subgroup of G. (8)

15\. (a) (i) Show that every chain is a distributive lattice. Justify the converse. (8)

(ii) Prove that the De Morgan laws are valid in a distributive complemented lattice. (8)

Or

(b) (i) Prove that the Cartesian product of two lattices is also a lattice. (8)

(ii) In a distributive complemented lattice. Show that the following are equivalent. (8)

(a) $a \leq b$

(b) $a \wedge \bar{b} = 0$

(c) $\bar{a} \vee b = 1$

(d) $\bar{b} \leq \bar{a}$