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Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 90810

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Second Semester

Civil Engineering

MA 8251 — ENGINEERING MATHEMATICS – II

(Common to : All Branches (Expect Marine Engineering/Artificial Intelligence and Data Science/Computer Science and Business Systems)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Given that $A = \begin{pmatrix} 5 & 4 \\ 1 & 2 \end{pmatrix}$ find the eigen values of A^2.

2. Write down the matrix of the quadratic form $3x_1^2 + 5x_2^2 + 5x_3^2 - 2x_1x_2 + 2x_2x_3 + 6x_3x_1$.

3. Find the unit normal to the surface $x^3 - xyz + z^3 = 1$ at the point (1,1,1).

4. Give the physical meaning of $\nabla \times \bar{F}$.

5. Does $f(z) = \bar{z}$ analytic? Justify.

6. Write any two properties of analytic function.

7. State simply and multiply connected regions.

8. Define removable singular point with an example.

9. State unit step function.

10. Find Laplace transform of $t \cos t$ t.

PART B — (5 × 16 = 80 marks)

11. (a) Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix}1&2&7\\4&2&3\\1&2&1\end{pmatrix}$ and also use it to find A^{-1}.

Or

(b) Diagonalise the matrix $A=\begin{pmatrix}2&1&-1\\1&1&-2\\-1&-2&1\end{pmatrix}$ by means of an orthogonal transformation.

12. (a) (i) Show that $\overline{F}=(y^2+2xz^2)\hat{i}+(2xy-z)\hat{j}+(2x^2z-y+2z)\hat{k}$ is irrotational and hence find its scalar potential. (8)

(ii) Find work done by the force $\overline{F}=z\hat{i}+x\hat{j}+y\hat{k}$, when it moves a particle along the arc of the curve $\bar{r}=\cos\hat{i}+\sin t\hat{j}+t\hat{k}$, from $t=0$ to $t=2\pi$. (8)

Or

(b) Verify Stock's theorem for $\overline{F}=y^2z\hat{i}+z^2x\hat{j}+x^2y\hat{k}$, where S is the open surface of the cube formed by the planes $x=\pm a, y=\pm a$ and $z=\pm a$ in which the plane $z=-a$ is cut.

13. (a) (i) If $u=x^2-y^2$ and $v=\frac{-y}{x^2+y^2}$, prove that both u and v satisfy Laplace equations, but that $(u+iv)$ is not a regular function of z. (8)

(ii) Find the bilinear transformation that maps the points $1+i,-i,2-i$ of the z-plane into the points 0,1,i of the w-plane. (8)

Or

(b) (i) Find the image in the w-plane of the region of the z-plane bounded by the straight lines $x=1$, $y=1$ and $x+y=1$ under the transformation $w=z^2$. (8)

(ii) Find the image of the half-plane $x>c$, when $c>0$ under the transformation $w=\frac{1}{z}$. (8)

14. (a) (i) Use Cauchy's integral formula to evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-3)} dz$, where C is the circle $|z| = 4$. (8)

(ii) Find the Laurent's series of $f(z) = \frac{1}{z(1-z)}$ valid in the region $1 < |z+1| < 2$. (8)

Or

(b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$ using contour integration, where $a > b > 0$.

15. (a) (i) Find the Laplace transform of the square wave function $f(t)$ defined by $f(t) = \begin{cases} k & in\ 0 \le t \le a \\ -k & in\ a \le t \le 2a \end{cases}$ and $f(t+2a) = f(t)$ for all t. (8)

(ii) Find the inverse Laplace transform of the function $\frac{s}{(s^2+a^2)^2}$ by using convolution theorem. (8)

Or

(b) Solve $y''-4y'+8y = e^{2t}$, $y(0) = 2$ and $y'(0) = -2$ by using Laplace transform.