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Reg. No. :

Question Paper Code : 90808

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

First Semester

Civil Engineering

MA 8151 — ENGINEERING MATHEMATICS – I

(Common to : All branches (Except Marine Engineering))

(Regulations 2017)

Time : Three hours Maximum : 100 marks

(Codes/Tables/Charts to be permitted, if any may be indicated)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Evaluate $\lim_{x\to 0} \frac{\log x}{\cot x}$.

2. Find the points of inflexion of $y = 4x^3 + 3x^2 - 2x$.

3. If $f(x, y) = x\cos y + ye^x$ then find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$.

4. Compute the Jacobian where $u = \frac{y^2}{x}$ and $v = x^2 + y^2$.

5. Determine $\int (2x-5)^7\, dx$.

6. Evaluate $\int_0^2 \frac{1}{(4+x^2)}\, dx$.

7. Sketch the region of integration for the integral $\int_0^2 \int_{x^2}^{2x} (4x+2)\, dy\, dx$ and write an equivalent integral with the order of integration reversed. (No need to evaluate)

8. Change $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) dy dx$ into polar coordinates. (No need to evaluate)

9. Write the superposition principle for homogeneous linear ODE.

10. Determine the Wronskian determinant value for $y_1 = \cos wx$ and $y_2 = \sin wx$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A rectangular sheet of metal of length 6 meters and width 2 meters is given. Four equal squares are removed from the corners. The sides of this sheet are now turned up to form on open rectangular box. Find the height of the box, approximately, such that the volume is maximum. (8)

(ii) Show that the function $y = |x|$ is differentiable on $(-\infty, 0)$ and on $(0, \infty)$ but has no derivative at $x = 0$. (8)

Or

(b) (i) Evaluate (1) $\lim_{x \to 0} \frac{\sqrt{x^2 + 100} - 10}{x^2}$ (2) $\lim_{x \to \infty} \frac{5x + 2}{2x^2 + 5}$. (8)

(ii) Find the maximum and minimum values of $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$ in the interval (0,2). (8)

12. (a) (i) Find the local extreme values of $f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy$. (8)

(ii) Use Taylor's series formula to find the quadratic approximation to $f(x, y) = \sin x \sin y$ near the origin. (8)

Or

(b) (i) Find the maximum and minimum values of the function $f(x, y) = 3x + 4y$ on the circle $x^2 + y^2 = 1$. (10)

(ii) If $u(x, y) = \frac{x^2 + y^2}{\sqrt{x + y}}$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$. (6)

13. (a) (i) Determine (1) $\int \frac{1}{\sqrt{(a^2 - x^2)}} dx$ (2) $\int \sin^2 t \cos^4 t dt$. (8)

(ii) Find (1) $\int_2^3 \frac{2x^2 - 9x - 35}{(x-2)(x+1)(x+3)} dx$ (2) $\int_0^2 \frac{3x}{\sqrt{2x^2 + 1}} dx$. (8)

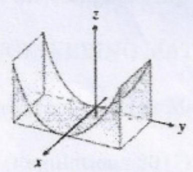
Or

(b) (i) Evaluate (1) $\int_1^9 \sqrt{x} \ln x \, dx$ (2) $\int_2^3 \frac{x^3 - 2x^2 - 4x - 4}{x^2 + x - 2} dx$. (8)

(ii) Determine (1) $\int x^2 \sin x \, dx$ (2) $\int_0^{\frac{\pi}{2}} 2\theta \sin \theta d\theta$. (8)

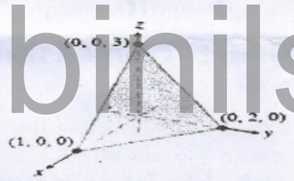
14. (a) (i) Find the area of the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$. (8)

(ii) Find the volumes of the region between the cylinder $z = y^2$ and the xy-plane that is bounded by the planes $x = 0$, $x = 1$, $y = -1$, $y = 1$. (8)



Or

(b) (i) Find the volumes of the tetrahedron in the first octant bounded by the coordinate planes and the plane passing through (1,0, 0),(0, 2,0), and (0,0, 3). (10)



(ii) Sketch the region bounded by the coordinate axes and the line $x + y = 2$. Then express the region's area as an iterated double integral and evaluate the integral. (6)

15. (a) (i) Solve the IVP $y'' + 25y = 0$, $y(0) = 4.6$, $y'(0) = -12$. (8)

(ii) Solve Euler Cauchy equation $x^2y'' + 3xy' + 0.75y = 0$, $y(1) = 1$, $y'(1) = -1.5$. (8)

Or

(b) (i) Solve $y'' + 5y' + 4y = 10e^{-3x}$. (8)

(ii) Solve $(D^2 - 2D + I)y = 35x^{\frac{3}{2}}e^x$ by variation of parameter. (8)

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