Reg. No. :

**Question Paper Code : 12196**

M.E./M.Tech. DEGREE EXAMINATIONS, JANUARY 2022.

First Semester

Aeronautical Engineering

MA 4153 — ADVANCED MATHEMATICAL METHODS

(Common to M.E. Aerospace Technology / M.E. Soil Mechanics and Foundation Engineering / M.E. Structural Engineering)

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State the conditions for the existence of the Laplace Transform of $f(t)$.

2. Find the Laplace transform $t \sin t$.

3. Define Dirac delta function.

4. Write the possible solution of two dimensional Laplace equation.

5. State the necessary condition for the extremum of the Euler's equation.

6. Find the extremals of the functional $\int_{x_0}^{x_1} \frac{y'^2}{x^3}\, dx$.

7. Find the map of the circle $|z| = k$ by the transformation $w = (1+i)z$.

8. Find the fixed points of the transformation $w = \frac{6z-9}{z}$.

9. Write the Christoffel symbol of first kind and second kind.

10. What is the divergence of the contravariant vector of $A^i$?

PART B — (5 × 13 = 65 marks)

11. (a) (i) Apply convolution theorem to find $L^{-1}\left(\frac{s}{(s^2+a^2)^2}\right)$. (6)

(ii) Find the Laplace transform of $f(t)=\begin{cases} t-1 (1<t<2) \\ 3-t (2<t<3) \end{cases}$. (7)

Or

(b) Solve by Lapalce transform $\frac{\partial w}{\partial x}+x\frac{\partial w}{\partial t}=x, w(x,0)=1, w(0,t)=1$.

12. (a) (i) Find the Fourier transform of $f(x)=\begin{cases} x \text{ if } 0<x<a \\ 0 \text{ otherwise} \end{cases}$. (6)

(ii) A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially in a position given by $y(x,0)=y_0\sin^3\frac{\pi x}{l}$. If it is released from rest from this position find the displacement $y$ at any distance $x$ from one end at any time $t$. (7)

Or

(b) A bar AB of length 10 cm has its ends A and B kept at 30°C and 100°C respectively, until steady state condition is reached. Then the temperature at A and B suddenly reduced to 0°C. Find the temperature distribution of the bar. (13)

13. (a) (i) Solve the boundary value problem $y''-y+x=0$ $(0\le x\le 1)$, $y(0)=y(1)=0$ by Ritz method. (6)

(ii) Find the plane curve of fixed perimeter and maximum area. (7)

Or

(b) (i) Find the path on which a particle in the absence of friction, will slide from one point to another in the shortest time under the action of gravity. (6)

(ii) Find the extremals of the functional

$$v(y,z)=\int_{x_0}^{x_1}\left(2yz-2y^2+y'^2-z'^2\right)dx.$$ (7)

14. (a) (i) Find the bilinear transformation that maps the points $z_1 = 1$, $z_2 = i, z_3 = -1$ into the points $w_1 = 2$, $w_2 = i$ and $w_3 = -2$. (6)

(ii) Find the transformation, which will map the interior of the infinite strip bounded by the lines $v = 0$, $v = \pi$ onto the upper half of the $z$-plane. (7)

Or

(b) (i) Find the temperature field around a long thin wire of radius $r_1 = 1$ mm that is electrically heated to $T_1 = 500°F$ and is surrounded by a circular cylinder of radius $r_2 = 100$ mm, which is kept at temperature $T_2 = 60°F$ by cooling it with air. (The wire is at the origin of the coordinate system). (6)

(ii) Show that $F(Z) = iZ^2$ also models a flow around a corner. Sketch streamlines and equipotential lines. Find $V$. (7)

15. (a) (i) Show that any inner product of the tensors $A_r^p$ and $B_t^{qS}$ is a tensor of rank 3. (6)

(ii) If $(ds)^2 = (dr)^2 + r^2(d\theta)^2 + r^2\sin^2\theta(d\phi)^2$, find the values of

(1) [22, 1] and [13, 3]

(2) $\begin{Bmatrix}1\\22\end{Bmatrix}$ and $\begin{Bmatrix}3\\13\end{Bmatrix}$. (7)

Or

(b) (i) Prove that $\frac{\partial g_{ij}}{\partial x^k} = [ik, j] + [jk, i]$. (6)

(ii) Prove that the covariant derivative of $g^{ij}$ is zero. (7)



PART C — (1 × 15 = 15 marks)

16. (a) (i) Show that $L[tJ_0(at)] = \frac{s}{(s^2 + a^2)^{\frac{3}{2}}}$. (8)

(ii) Evaluate $\int_0^\infty \frac{x^2}{(a^2 + x^2)(b^2 + x^2)} dx$ and hence find $\int_0^\infty \left(\frac{x}{x^2 + 1}\right)^2 dz$. (7)

Or

(b) (i) Prove that the shortest distance between two points in a plane is a straight line. (8)

(ii) Prove that $w = \frac{z}{1 - z}$ maps the upper half of the $z$-plane onto the upper half of the $w$-plane. What is the image of the circle $|z| = 1$ under this transformation? (7)