Reg. No. :

Question Paper Code : 30896

M.E./M.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

First Semester

Applied Electronics

MA 4101 – APPLIED MATHEMATICS FOR ELECTRONICS ENGINEERS

(Regulations 2021)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Mention the four types fuzzy propositions.

2. Write down the set T_n of truth values of an n–valued logic.

3. A typist types 2 letters erroneously for every 100 letters. What is the probability that the tenth letter typed is the first erroneous letter?

4. If a random variable X has the MGF $M(t)=\frac{3}{3-t}$, obtain the standard deviation of X.

5. Determine the value of c that makes the function $f(x,y)=c(x+y)$ a joint probability mass function over the nine points with $x=1,2,3$ and $y=1,2,3$.

6. If X and Y are independent continuous random variables, prove that $Cov\,(x,y)=0$.

7. If the customers arrive at a bank according to a Poisson process with mean rate of 2 per minute, find the probability that, during an 1 – min interval, no customer arrives

8. Find the variance of the stationary process $\{X(t)\}$, whose ACF is given by $R(\tau)=16+\frac{9}{1+6\tau^2}$.

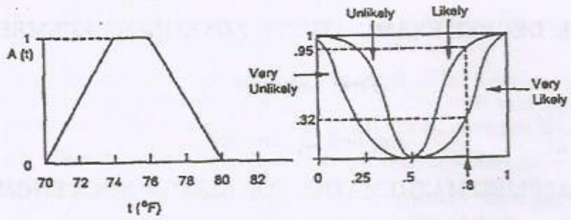
9. In a 3 server infinite capacity Poisson queue model if $\frac{\lambda}{s\mu}=\frac{2}{3}$, find P_0.

10. Write down the Little's formula.

PART B — (5 × 13 = 65 marks)

11. (a) Let V be the average daily temperature t in °F at some place on the earth during a certain month. Consider the probability – qualified proposition

p: Pro{temperature t (at given place and time)is around 75°F} is likely. The predicate $A(t)$ "around 75°F" and the quantifier likely" are defined in the following figures.



Given that the following probability distribution:

t	68	69	70	71	72	73	74	75	76
f(t)	.002	.005	.005	.01	.04	.11	.15	.21	.16
t	77	78	79	80	81	82	83		
f(t)	.14	.11	.04	.01	.005	.002	.001		

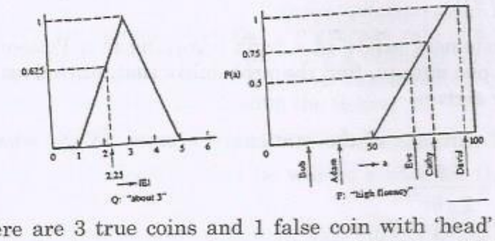
(i) Determine the truth value of the proposition *p*.

(ii) Discuss about what happen if we replace the qualification likely in the proposition p with very likely.

Or

(b) Consider the fuzzy proposition

p: there are about three students in I whose fluency in English V(i), is high. Where = *I ={Adam, Bob, Cathy David, Eve}* and *V* is a variable with values in the interval [0,100] that express the degrees of fluency in English. A fuzzy quantifier *Q* "about 3" and fuzzy set *F* "high fluency" on [0,100] are defined in the following figures. Determine the truth value of the proposition *p*.



12. (a) (i) There are 3 true coins and 1 false coin with 'head' on both sides. A coin is chosen at random and tossed 4 times. If 'head' occurs all the 4 times, what is the probability that the false coin has been chosen and used? (7)

(ii) A random variable X has the following probability distribution.

X	-2	-1	0	1	2	3
$p(x)$	0.1	K	0.2	$2K$	0.3	$3K$

Find (1) the value of K (2) evaluate $P(X < 2)$ and $p(-2 < X < 2)$ (3) find the CDF of X. (6)

Or

(b) (i) Out of 800 families with 4 children each, how many families would be expected to have (1) 2 boys and 2 girls, (2) at least 1 boy and (3) at most 2 girls. Assumes equal probabilities for boys and girls. (7)

(ii) The mean weight of 500 students is 150 lb, and the standard deviation is 15 lb. Assuming that the weights are normally distributed, find how many students weigh between 120 lb and 155 lb. (6)

13. (a) The following table represents the joint probability distribution of the discrete random variable (X,Y). Find all the marginal and conditional distributions.

Y \ X	1	2	3
1	1/12	1/6	0
2	0	1/9	1/5
3	1/18	1/4	2/15

Or

(b) Given $f_{XY}(x,y) = cx(x-y), 0 < x < 2, -x < y < x$, and 0 elsewhere,

(i) Evaluate c

(ii) Find $f_x(x)$

(iii) Find $f_{\frac{Y}{X}}\left(\frac{y}{x}\right)$

(iv) Find $f_y(y)$.

14. (a) (i) Show that the random process $X(t) = A\cos(w_0 t + \theta)$ wide – sense stationary, if A and W_0 are constants and θ is a uniformly distributed random variable in $(0, 2\pi)$. (7)

(ii) If customers arrive at a counter in accordance with a Poisson process with a mean rate of 2 per minute, find the probability that the interval between 2 consecutive arrivals is (1) more than 1 minute (2) between 1 mm and 2 mm and (3) 4 min or less. (6)

Or

(b) (i) The transition probability matrix of a markov chain $\{X_n\}$, n = 1,2,3, ... having 3 states 1, 2 and 3 is $p = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $p^{(0)} = (0.7, 0.2, 0.1)$.

Find (1) $P\{X_2 = 3\}$ and (2) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$. (8)

(ii) Prove that if a Gaussian process is wide-sense stationary, it is also strict-sense stationary. (5)

15. (a) A petrol pump station has 4 pumps. The service times follow the exponential distribution with a mean rate of 6 mm and cars arrive for service in a Poisson process at the rate of 30 cars per hour.

(i) What is the probability that an arrival would have to wait in line?

(ii) Find the average waiting time, average time spent in the system and the average number of cars in the system.

(iii) For what percentage of time would a pump be idle on an average?

Or

(b) At a railway station, only one train is handled at a time. The railway yard is sufficient only for 2 trains to wait, while the other is given signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average of 6 per hour. Assuming Poisson arrivals and exponential service distribution, find the probabilities for the number of trains in the system. Also find the average waiting time of a new train coming into the yard. If the handling rate is doubled, how will the above results get modified?

PART C — (1 × 15 = 15 marks)

16. (a) Obtain the equations of the regression lines from the following data, using the method of least squares. Hence find the coefficient of correlation between X and Y. Also estimate the value of (i) Y, when X = 38 and (ii) x, when Y= 18.

X :	22	26	29	30	31	31	34	35
Y :	20	20	21	29	27	24	27	31

Or

(b) Derive the values of P_0 and P_n for Poisson queue system. Also if people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 minutes before the picture starts and if its takes exactly 1.5 minutes to reach the correct seat after purchasing the ticket,

(i) Can he expect to be seated for the start of the picture?

(ii) What is the probability that he will be seated for the start of the picture?

(iii) How early he arrive in order to be 99% sure of being seated for the start of the picture?

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