Reg. No. :

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B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Third Semester

Electronics and Telecommunication Engineering

MA 3355 – RANDOM PROCESSES AND LINEAR ALGEBRA

(Common to: B.E. Electronics and Communication Engineering)

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Using the axioms of probability, prove $P(A^c) = 1 - P(A)$.

2. Consider a random experiment of tossing a fair coin three times. If X denotes the number heads obtained find, $P(X < 2)$.

3. For a bi-variate random variable (XY), prove that if X and Y are independent, then every event $a < X \le b$ is independent of the other event $c < X \le d$.

4. Let the joint probability mass function of (X,Y) be given by $P_{xy}(x,y) = \begin{cases} k(x+y) & x = 1,2,3;\ y = 1.2 \\ 0, & otherwise \end{cases}$. Find the value of k.

5. Let $X_1, X_2 \ldots$ be independent Bernoulli random variables with $P(X_n = 1) = p$ and $P(X_n = 0) = q$ for all n. Describe the Bernoulli process.

6. Consider a Markov chain with two states and transition probability matrix $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Find the stationary distribution of the chain.

7. Determine whether the vectors $u = (1,1,2), v = (1,0,1)$, and $w = (2,1,3)$ span the vector space R^3.

8. Is a set of all vectors of the form $(a,1,1)$, where a is real, a subspace of R^3? Justify.

9. Find the kernel and range of the identity operator.

10. Show that the vectors $u = (-2, 3, 1, 4)$ and $v = (1, 2, 0, -1)$ are orthogonal in R^4.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A lot of 100 semiconductor chips contain 20 that are defective. Two are selected randomly, without replacement, from the lot.

(1) What is the probability that the first one selected is defective?

(2) What is the probability that the second one selected is defective given that the first one was defective?

(3) What is the probability that both are defective? (8)

(ii) A company producing electric relays has three manufacturing plants producing 50, 30, and 20 percent respectively of its product. Suppose that the probabilities that a relay manufactured by these plants is defective are 0.02, 0.05 and 0.01 respectively. If a relay selected at random is found to be defective, what is the probability that it was manufactured by plant 2? (8)

Or

(b) (i) All manufactured devices and machines fail to work sooner or later. Suppose that the failure rate is constant and the time to failure (in hours) is an exponential random variable X with parameter λ. Measurements shows that the probability that the time to failure for computer memory chips in a given class exceeds 10^4 hours is e^{-1}. Find the value of λ and calculate the time X_0 such that the probability that the time to failure is less than X_0 is 0.05. (8)

(ii) A production line manufactures 1000 ohm resistors that have 10% tolerance. Let X denotes the resistance of a resistor. Assuming that X is a normal random variable with mean 1000 and variance 2500, find the probability that a resistor picked at random will be rejected. (8)

12. (a) Consider an experiment of drawing randomly three balls from an urn containing two red, three white, and four blue balls. Let (X, Y) be a bi-variate random variable where X and Y denote respectively the number of red and white balls chosen.

(i) Find the range of (X, Y).

(ii) Find the joint probability mass function of (X,Y).

(iii) Find the marginal probability function of X and Y.

(iv) Are X and Y independent? (16)

Or

(b) Test two integrated circuits one after the other. On each test, the possible outcomes are a (accept) and r (reject). Assume that all circuits are acceptable with probability 0.9 and that the outcomes of successive tests are independent. Count the number of acceptable circuits X and count the number of successful tests Y before you observe the first reject. (If both tests are successful, let $Y = 2$.)

(i) Find the joint probability mass function of X and Y.

(ii) Find the correlation between X and Y.

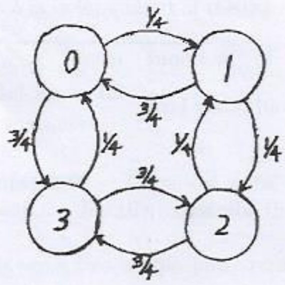
(iii) Find the covariance of X and Y. (16)

13. (a) (i) The input to a digital filter is an identical and independently distributed random sequence $\ldots, X_{-1}, X_0, X_1, \ldots$ with $E[X_i] = 0$ and $Var[X_i] = 1$. The output is a random sequence $\ldots, Y_{-1}, Y_0, Y_1, \ldots$ related to the input sequence by the formula $Y_n = X_n + X_{n-1}$ for all integers n. Find the expected value $E[Y_n]$ and auto-covariance function $C_Y[m,k]$. (8)

(ii) At the receiver of an AM radio, the received signal contains a cosine carrier signal at the carrier frequency fc with a random phase that is a sample value of the uniform $(0, 2\pi)$ random variable. The received carrier signal is $X(t) = A.\cos(2\pi f_c t + \theta)$. What are the expected value and autocorrelation of the process $X(t)$? (8)

Or

(b) Consider the Markov chain shown in the following figure.



(i) What is the period d of state 0?

(ii) What are the stationary probabilities π_0, π_1, π_2 and π_3?

(iii) Given the system is in state 0 at time 0, what is the probability the system is in state 0 at time nd in the limit as $n \to \infty$? (16)

14\. (a) Determine whether the set of all pairs of real numbers (x, y) with the operations $(x, y)+(p, q)=(x+p+1, y+q+1)$ and $k(x, y)=(kx, ky)$ is a vector space or not. If not, list all the axioms that fail to hold. (16)

Or

(b) Determine the basis and the dimension of the homogeneous system $2x_1+2x_2-x_3+x_5=0; -x_1-x_2+2x_3-3x_4+x_5=0;$ $x_1+x_2-2x_3-x_5=0 \quad x_3+x_4+x_5=0$. (16)

15\. (a) (i) State and prove the dimension theorem for linear transformation. (8)

(ii) Let $T: R^2 \to R^3$ be a linear transformation defined by $T\begin{pmatrix} x \\ y \end{pmatrix}=\begin{bmatrix} y \\ -5x+13y \\ -7x+16y \end{bmatrix}$. Find the matrix for the transformation T with respect to the bases $B=\{u_1, u_2\}$ for R^2 and $B_1=\{v_1, v_2, v_3\}$ for R^3 where $u_1=\begin{bmatrix} 3 \\ 1 \end{bmatrix}, u_2=\begin{bmatrix} 5 \\ 2 \end{bmatrix}, v_1=\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, v_2=\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, v_3=\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$. (8)

Or

(b) Find the orthogonal projection of the vector $u=(-3,-3,8,9)$ on the subspace of R^4 spanned by the vectors $v_1=(3,1,0,1), v_2=(1,2,1,1)$, $v_3=(-1,0,2,-1)$. (16)