Reg. No. :

Question Paper Code : 20816

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2022.

Fourth Semester

Computer and Communication Engineering

MA 8451 — PROBABILITY AND RANDOM PROCESSES

(Common to : Electronics and Communication Engineering/Electronics and Telecommunication Engineering)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Suppose that two events A and B are mutually exclusive and $P[B] > 0$. Under what conditions will A and B be independent?

2. Balls are tossed at random into 50 boxes. Find the expected number of tosses required to get the first ball in the fourth box.

3. X and Y are two continuous random variables whose joint PDF is given by

$$f_{XY}(x, y) = \begin{cases} e^{-(x+y)}, & 0 \le x < \infty,\ 0 \le y < \infty \\ 0 & otherwise \end{cases}$$

Are X and Y independent? Justify.

4. The joint PMF of two random variables X and Y is given by

$$P_{XY}(x, y) = \begin{cases} k(2x + y) & x = 1, 2;\ y = 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant What is the value of k?

5. Distinguish random variable and random process.

6. Draw the state-transition diagram for the Markov chain with the following transition probability matrix

$$P=\begin{bmatrix}1/2 & 0 & 0 & 1/2\\ 1/2 & 1/2 & 0 & 0\\ 1/4 & 0 & 1/2 & 1/4\\ 0 & 1/2 & 1/4 & 1/4\end{bmatrix}$$

7. State any two properties of the power spectral density.

8. The random process $X(t)$ is given by

$$X(t)=Y\cos(2\pi t),\ t>=0$$

where Y is a random variable that is uniformly distributed between 0 and 2. Find the expected value.

9. Define system transfer function.

10. When is a system called a linear system?

PART B — (5 × 16 = 80 marks)

11. (a) (i) An aircraft maintenance company bought equipment for detecting structural defects in aircrafts. Tests indicate that 95% of the time the equipment detects defects when they actually exist, and 1% of the time it gives a false alarm that indicates the presence of a structural defect when in fact there is none. If 2% of the aircrafts actually have structural defects, what is the probability that an aircraft actually has a structural defect given that the equipment indicates that it has a structural defect? (8)

(ii) Assume the random variable X has the PDF $f_X(x)=\lambda e^{-\lambda x},\ x\geq 0$. Find the third moment of X, $E[X^3]$. (8)

Or

(b) (i) Messages arrive at a switchboard in a Poisson manner at an average rate of six per hour. Find the probability for each of the following events: (8)

(1) Exactly two messages arrive within one hour.

(2) No message arrives within one hour.

(3) At least three messages arrive within one hour.

(ii) The mean weight of 200 students in a certain college is 140 lbs, and the standard deviation is 10 lbs. If we assume that the weights are normally distributed, evaluate the following: (8)

(1) The expected number of students that weigh between 110 and 145 lbs.

(2) The expected number of students that weigh less than 120 lbs.

(3) The expected number of students that weigh more than 170 lbs.

12. (a) (i) A fair coin is tossed three times. Let X be a random variable that takes the value 0 if the first toss is a tail and the value 1 if the first toss is a head. Also, let Y be a random variable that defines the total number of heads in the three tosses. (8)

(1) Determine the joint PMF of X and Y.

(2) Are X and Y independent?

(ii) Assume that the random variables X and Y have the joint PDF

$$f_{XY}(x,y) = \begin{cases} \frac{1}{2} x^3 y, & 0 \le x \le 2, 0 \le y \le 1 \end{cases}$$

Determine if X and Y are independent. (8)

Or

(b) (i) The joint PDF of the random variables X and Y is defined as follows : (8)

$$f_{XY}(x,y) = \begin{cases} 25\,e^{-5y}, & 0 < x < 0.2, \quad y > 0 \\ 0, & \text{otherwise} \end{cases}$$

(1) Find the marginal PDFs of X and Y.

(2) What is the covariance of X and Y?

(ii) Assume that the random variable S_n is the sum of 48 independent experimental values of the random variable X whose PDF is given by

$$f_X(x) = \begin{cases} \frac{1}{3}, & 1 \le x \le 4 \\ 0, & \text{otherwise} \end{cases}$$

Find the probability that S_n lies in the range $108 \le S_n \le 126$. (8)

13. (a) Alan is conducting an experiment to test the mean lifetimes of two sets of electric bulbs labeled A and B. The manufacturer claims that the mean lifetime of bulbs in set A is 200 hours, while the mean lifetime of the bulbs in set B is 400 hours. The lifetimes for both sets are exponentially distributed. Alan's experimental procedure is as follows: He started with one bulb from each set. As soon as a bulb from a given set fails (or burns out), he immediately replaces it with a new bulb from the same set and writes down the lifetime of the burnt-out bulb. Thus, at any point in time he has two bulbs on, one from each set. If at the end of the week Alan tells you that 8 bulbs have failed, determine the following:

(i) The probability that exactly 5 of those 8 bulbs are from set B.

(ii) The probability that no bulb will fail in the first 100 hours.

(iii) The mean time between two consecutive bulb failures.

Or

(b) A taxi driver conducts his business in three different towns 1, 2, and 3. On any given day, when he is in town 1, the probability that the next passenger he picks up is going to town 1 is 0.3, the probability that the next passenger he picks up is going to town 2 is 0.2, and the probability that the next passenger he picks up is going to town 3 is 05. When he is in town 2, the probability that the next passenger he picks up is going to town 1 is 0.1, the probability that the next passenger he picks up is going to town 2 is 0.8, and the probability that the next passenger he picks up is going to town 3 is 0.1. When he is in town 3, the probability that the next passenger he picks up is going to town 1 is 0.4, the probability that the next passenger he picks up is going to town 2 is 0.4, and the probability that the next passenger he picks up is going to town 3 is 0.2.

(i) Determine the state-transition diagram for the process.

(ii) Give the transition probability matrix for the process.

(iii) What are the limiting-state probabilities?

(iv) Given that the taxi driver is currently in town 2 and is waiting to pick up his first customer for the day, what is the probability that the first time he picks up a passenger to town 2 is when he picks up his third passenger for the day?

14. (a) (i) A random process is defined by $X(t) = K\cos wt, t \geq 0$, where w is a constant and K is uniformly distributed between 0 and 2. Determine the auto-covariance function of $X(t)$

(ii) The sample function $X(t)$ of a stationary, random process Y(t) is given by X(t) = Y(t)sin(wt + Θ)

where w is a constant, Y(t) and Θ are statistically independent, and Θ is uniformly distributed between 0 and 2π. Find the autocorrelation function of X(t) in terms of $R_{YY}(T)$.

Or

(b) Two jointly stationary random processes $X(t)$ and $Y(t)$ have the cross-correlation function given by:

$R_{XY}(T) = 2e^{-2T}, T \geq 0$

Determine the following:

(i) The cross-power spectral density $S_{XY}(W)$

(ii) The cross-power spectral density $S_{YX}(W)$.

15. (a) A random process X(t) is the input to a linear system whose impulse response is $h(t) = e^{-t}, t \geq 0$. If the autocorrelation function of the process is $R_{XX}(T) = 2e^{-2|T|}$, determine the following:

(i) The cross correlation function $R_{XY}(T)$ between the input process X(t) and the output process Y(t).

(ii) The cross correlation function $R_{YX}(T)$ between the output process Y(t) and the output process X(t).

Or

(b) (i) For a linear system with random input X(t) the impulse response h(t) and output Y(t). Obtain the cross correlation function and cross power spectral density functions. (8)

(ii) Find the cross-correlation function corresponding to the cross-power spectrum $S_{XY}(w) = \dfrac{6}{(9 + w^2)(3 + jw)^2}$. (8)

———————