Reg. No. :

Question Paper Code : 20811

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2022.

Third Semester

Biomedical Engineering

MA 8352 — LINEAR ALGEBRA AND PARTIAL DIFFERENTIAL EQUATIONS

(Common to : Computer and Communication Engineering/ Electronics and Communication Engineering/Electronics and Telecommunication Engineering/Medical Electronics)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Let H be the set of all vectors of the form $(a-3b, b-a, a, b)$ where a and b are arbitrary scalars. Show that H is a subspace of R^4.

2. Let $v_1 = (1, -2, 3)$, $v_2 = (-2, 7, -9)$. Is $\{v_1, v_2\}$ a basis for R^2?

3. State Dimension Theorem.

4. If X is an eigen vector of A corresponding to λ, what is A^3X?

5. Use the following inner product to find $\|w\|$ where $(-1, 3)$. $\langle u,v\rangle = 3ac+2bd$ where $u=(a,b)$ and $v=(c,d)$.

6. Show that the following identify holds for vectors in any inner product space $\langle u,v\rangle = \frac{1}{4}\|u+v\|^2 - \frac{1}{4}\|u-v\|^2$.

7. Form the partial differential equation by eliminating the arbitrary constants a and b from $(x-a)^2+(y-b)^2 = z^2\cot^2\alpha$.

8. Classify the partial differential equation $xf_{xx} + yf_{yy} = 0$, $x>0$ and $y>0$.

9. Find the Fourier coefficient a_0 if $f(x)=x^2+x$, $-2<x<2$ and $f(x+4)=f(x)$.

10. State the sufficient condition for the existence of Fourier series of $f(x)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Determine whether the set of all 2 × 2 matrices of the form $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$ where a, b and d are real, with the standard matrix addition and scalar multiplication is a vector space or not. (10)

(ii) Express $-9-7x-15x^2$ as a linear combination of $2+x+4x^2$, $1-x+3x^2$ and $3+2x+5x^2$. (6)

Or

(b) (i) Determine the basis and the dimension of the homogeneous system
$2x_1+2x_2-x_3+x_5=0$; $-x_1-x_2+2x_3-3x_4+x_5=0$;
$x_1+x_2-2x_3-x_5=0$; $x_3+x_4+x_5=0$. (10)

(ii) Find whether the set of vectors $v_1=(2,-1,0,3)$, $v_2=(1,2,5,-1)$ and $v_3=(7,-1,5,8)$ form a linearly independent set. (6)

12. (a) (i) Diagonalize the matrix $A=\begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$. (10)

(ii) Show that the function $T: R^2 \to R^3$ given by $T(x_1, x_2)=(x_1+2x_2, 3x_1-x_2)$ is a linear operator. (6)

Or

(b) (i) Find the eigen values and eigen vectors of $A=\begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix}$. (10)

(ii) Consider the basis $\{v_1=(1,1,1),\ v_2=(1,1,0),\ v=(1,0,0)\}$ and let $T: R^3 \to R^2$ be the linear operator such that $T(v_1)=(1,0)$, $T(v_2)=(2,-1)$, $T(v_3)=(4,3)$. Find a formula $T(x_1, x_2, x_3)$ and use it to find $T(2,-3,5)$. (6)

13. (a) Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the basis $\{u_1 = (1, 1, 1),\ u_2 = (0, 1, 1),\ u_3 = (0, 0, 1)\}$ into an orthonormal basis. (16)

Or

(b) Find the least-square solution of $Ax = b$ for $A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 1 & 0 \\ 1 & 1 & 2 \\ 1 & 3 & 3 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ 5 \\ 7 \\ -3 \end{bmatrix}$. (16)

14. (a) (i) Solve the equation $z = px + qy + p^2 + pq + q^2$. (8)

(ii) Solve the equation $x(y^2 + z^2)p + q(x^2 + z^2)y = z(y^2 - x^2)$. (8)

Or

(b) Solve the partial differential equation $(D^2 - DD' + D' - 1)z = e^{2x+3y} + \cos^2(x + 2y)$ where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$. (16)

15. (a) Find the Fourier series of $f(x) = x^2,\ 0 < x < 2l$ and $f(x) = f(x + 2l)$. Hence deduce that (16)

(i) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots.. = \frac{\pi^2}{6}$

(ii) $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \ldots.. = \frac{\pi^2}{12}$

(iii) $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{5^2} + \ldots.. = \frac{\pi^2}{8}$

Or

(b) A uniform string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form of the curve $y = kx(l - x)$ and then releasing it from this position at time $t = 0$. Find the displacement of the point of the string at a distance x from one end at time t. (16)

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