Reg. No. :

Question Paper Code : 20805

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2022.

First Semester

Civil Engineering

MA 8151 — ENGINEERING MATHEMATICS — I

(Common to All Branches (Except Marine Engineering))

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find $\lim_{x\to 0}\left(x^3+\frac{\cos 5x}{10000}\right)$.

2. Represent the function $f(x)=x^2, x\in(-\infty,\infty)$ in Graphically.

3. If $z=u^2+v^2$ and $u=at^2, v=2at$ find $\frac{dz}{dt}$.

4. If $x=u(1-v), y=uv$, prove that $JJ'=1$.

5. Determine the value of $\int_0^{\pi}\sin^2 x\,dx$.

6. Check whether $\int_0^{\frac{\pi}{2}}\sec x\,dx$ converges or diverges.

7. Change the order of integration in $I=\int_0^1\int_{x^2}^{2-x} xy\,dx\,dy$.

8. Transform the integral $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy$ into polar coordinates.

9. Transform the differential equation $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x$ into linear differential equation with constant coefficients.

10. Check whether the functions $y_1=\sin 2x$, $y_2=\cos x$ are linearly independent or not by using Wronskian of y_1 and y_2.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the numbers at which f is discontinuous. At which of these numbers is f continuous from the right, from the left or neither? Sketch the graph of f

$$f(x)=\begin{cases}1+x^2, \text{ if } x\le 0\\ 2-x, \text{ if } 0<x\le 2\\ (x-2)^2, \text{ if } x>2\end{cases}$$

(ii) Find the absolute maximum and minimum values of the function $f(x)=x^3-3x^2+1, -\frac{1}{2}\le x\le 4$.

Or

(b) (i) Let $f(x)=\begin{cases}x^2+1, \text{ if } x<1\\ (x-2)^2, \text{ if } x\ge 1\end{cases}$

(1) Find $\lim_{x\to 1^-} f(x)$ and $\lim_{x\to 1^+} f(x)$

(2) Does $\lim_{x\to 1} f(x)$ exist?

(3) Sketch the graph of f.

(ii) Find the local maximum and minimum values of the function $f(x)=x+2\sin x, 0\le x\le 2\pi$.

12. (a) (i) If $z=f(x,y)$, $x=e^u+e^{-v}$ and $y=e^{-u}-e^v$ prove that

$$\frac{\partial z}{\partial u}-\frac{\partial z}{\partial v}=x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}$$

(ii) Expand $e^x\log(1+y)$ in powers of x and y up to terms of third degree.

Or

(b) (i) Discuss the maxima and minima of $f(x, y) = x^3 y^2 (1 - x - y)$

(ii) The temperature T at any point (x, y, z) in space is $T = 400xyz^2$. Find the highest temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$.

13. (a) (i) Evaluate $\int_0^1 \ln x \, dx$.

(ii) Find $\int \frac{1}{x^2\sqrt{x^2+4}} dx$ using trigonometric substitutions.

Or

(b) (i) Evaluate $\int x^5 \sqrt{1+x^2} \, dx$ using substitution rules.

(ii) Find $\int \frac{2x^2 - x + 4}{x^3 + 4x} dx$ by using partial fraction.

14. (a) (i) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) \, dx \, dy$.

(ii) Calculate $\iint r^3 \, dr \, d\theta$ over the area included between the circles $r = 2\sin\theta$ and $r = 4\sin\theta$.

Or

(b) (i) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

(ii) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^1 \frac{dz \, dy \, dx}{\sqrt{x^2+y^2+z^2}}$.

15. (a) (i) Solve by the method of variation of parameters $y'' - 2y' + y = e^x \log x$.

(ii) Solve : $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin[\log(1+x)]$.

Or

(b) (i) Solve by method of undetermined coefficients $\frac{d^2y}{dx^2} + y = 2\cos x$.

(ii) Solve : $\frac{dx}{dt} = 5x + y, \frac{dy}{dt} = y - 4x$.