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Question Paper Code : 12198

M.E./M.Tech. DEGREE EXAMINATIONS, JANUARY 2022.

First Semester

Computer Integrated Manufacturing

MA 4155 — APPLIED PROBABILITY AND STATISTICS FOR MANUFACTURING ENGINEERING

(Common to M.E. Manufacturing Engineering)

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. A box contains 4 bad and 6 good tubes. Two are drawn out from the box at a time. One of them is tested and found to be good. What is the probability that the other one also good?

2. If a Poisson variate X is such that $P(X=1)=2P(X=2)$. Find $P(X=0)$.

3. Let X and Y have the joint probability mass function.

Y \ X	0	1	2
0	0.1	0.4	0.1
1	0.2	0.2	0

Find $P(X+Y>1)$.

4. Let X and Y are bivariate distribution with correlation coefficient $\rho_{xy}=1/2, \sigma_x=2, \sigma_y=3$. Find Var $(2X-4Y+3)$.

5. Define Standard error in the sampling distribution.

6. A sample of size 13 gave an estimated population variance of 3.0 ; while another sample of size 15 gave an estimate of 2.5. Could both samples be from populations with the same variance?

7. An observed random sample of size 9 from a $N(\mu, \sigma = 2)$ has a mean 50. Obtain 95% confidence interval for μ.

8. If x is a value assumed by a random variable having the probability density function $f(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}}, x > 0, \theta > 0 \\ 0, \quad otherwise \end{cases}$, find k so that the interval from 0 to kx is $(1-\alpha)$ confidence interval for the parameter θ.

9. Write down the ANOVA table for one criteria classification.

10. What is a Latin square design? Show that a 2 × 2 Latin square is not possible.

PART B — (5 × 13 = 65 marks)

11. (a) (i) A and B alternately throw a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, show that his chance of winning is 30/61. (6)

(ii) A random variable X has the following probability distribution

X :	−2	−1	0	1	2	3
P (x) :	0.1	k	0.2	$2k$	0.3	$3k$

Find k, $P(X < 2)$ and $E(X)$. (7)

Or

(b) (i) A bag contains 5 balls and it is not known how many of them are white. Two balls are drawn at random from the bag and they are noted to be white. What is the chance that all the balls in the bag are white? (6)

(ii) The mean yield for one acre plots is 662 kgs with standard deviation 32. Assuming normal distribution, how many one acre plots in a batch of 10000 plots would you expect to yield,

(1) over 700 kgs

(2) below 650 kgs. (7)

12. (a) The joint probability density function of the two dimensional random variable (X, Y) is given by $f(x, y) = \begin{cases} \frac{8}{9} xy, & 0 \le x \le y \le 2 \\ 0, & otherwise \end{cases}$. Find the marginal densities of X and Y. Also find the conditional density functions $f(x/y)$ and $f(y/x)$. (13)

Or

(b) Calculate the correlation coefficient for the following heights in inches of fathers (x) and their sons (y). (13)

x	65	66	67	67	68	69	70	72
y	67	68	65	68	72	72	69	71

13. (a) Two independent samples of sizes 8 and 7 contained the following values :

Sample 1	19	17	15	21	16	18	16	14
Sample 2	15	14	15	19	15	18	16	–

Is the difference between the sample means significant? (13)

Or

(b) (i) Theory predicts that the proportion of beans in 4 groups A, B, C, D should be 9:3:3:1. In an experiment among 1600 beans, the numbers in the 4 groups were 882, 313, 287 and 118. Does the experiment support the theory? (6)

(ii) A cubical die is thrown 9000 times and a throw of 3 or 4 is observed 3240 times. Show that the die cannot be regarded as an unbiased one, and find the extreme limits between which the probability of a throw of 3 or 4 lies. (7)

14. (a) Given one observation from a population with probability density function $f(x, \theta) = \frac{2}{\theta^2}(\theta - x),\ 0 \le x \le \theta$. Obtain $100(1-\alpha)\%$ confidence interval for θ. (13)

Or

(b) If p is the observed proportion of success in n independent Bernoullian trials, prove that 95% fiducial limits for the population proportion P are, for the large samples, given by $p \pm 1.96\sqrt{\frac{p-p^2}{n}}$. (13)

15. (a) On a feeding experiment, a farmer has four types of hogs denoted by I, II, III and IV. These types are each divided into three groups which are fed varietal A, B and C. The following results are obtained, the numbers in the table being the gains n weight in pounds in the various groups,

	I	II	III	IV
A	7.0	16.0	10.5	13.5
B	14.5	15.5	15.0	21.0
C	8.5	16.5	9.5	13.5

Perform an analysis of variance on these data and test the significance of variation between rations and between types. (13)

Or

(b) A varietal trial was conducted on wheat with 4 varieties in a Latin square design. The plan of the experiment and the per plot yield are given below : (13)

C 25	B 23	A 20	D 20
A 19	D 19	C 21	B 18
B 19	A 14	D 17	C 20
D 17	C 20	B 21	A 15

Perform a three way analysis of variance.

PART C — (1 × 15 = 15 marks)

16. (a) If the joint probability density function of a two dimensional random variable (X, Y) is given by $f(x, y) = \begin{cases} x^2 + \frac{xy}{3}, & 0 < x < 1;\ 0 < y < 2 \\ 0, & otherwise \end{cases}$

Find

(i) $P\left(X > \frac{1}{2}\right)$

(ii) $P(Y > 1)$

(iii) $P(Y < X)$

(iv) $P\left(Y < \frac{1}{2} \middle| X < \frac{1}{2}\right)$

(v) $P(X + Y > 1)$

(vi) Find the conditional density functions. (15)

Or

(b) Below is given the plan and yields of 2^2-factorial experiment involving 2 factors N and S each at two levels 0 and 1. Analysis the design. (15)

Blocks	I	(1) 117	s 106	ns 125	n 124
	II	ns 124	(1) 120	s 117	n 124
	III	(1) 111	n 127	s 114	ns 126
	IV	ns 125	n 131	s 112	(1) 108
	V	ns 95	s 97	(1) 73	n 138
	VI	n 158	(1) 81	ns 125	s 117