

IDEAL AND REAL GASES.....1

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## Ideal and Real gases; Thermodynamic Relations

Ideal gas: is a gas which obey the equation of state

$$PV = RT \quad (\text{or}) \quad PV = RT$$

only air consider as ideal gas.

Real gas: ① is a gas which doesn't obey the eqn. of state  
② equation of state of real gas

$$PV = ZRT$$

$$Z = \frac{PV}{RT}$$

② all the gases considered as real gas  
Z - compressibility factor.

### Difference between ideal and Real gases

#### Thermodynamic functions:

- ① Enthalpy  $h = U + PV$
- ② Helmot's function  $\phi = U - TS$
- ③ Gibbs function  $g = h - TS$

#### 1. Boyle's law (at const. Temp)

$$V \propto \frac{1}{P} \quad PV \propto 1 \quad PV = C \text{ at const. Temp.}$$

#### 2. Charles law ( $P \neq 0$ )

At const. P.

$$V \propto T$$

At const. Vol.

$$P \propto T$$

#### 3. Joule's law: internal energy of gas only depends on the Temp.

$$\Delta U = m C_v \Delta T \quad \therefore U = m C_v T$$

#### ④ Regnault's law: the two sp. heats $C_p$ and $C_v$ of a gas do not change with the change of Temp and P.

Thermodynamic Relations

Max well's Relations

From, first law of thermodynamics.

$$Q = W + \Delta U$$

Re arranging the parameters.  $ds = \frac{Q}{T}$

$$Q = \Delta U + W$$

$$Tds = du + pdv$$

$$\boxed{du = Tds - pdv} \quad \text{--- ①}$$

$$\left[ \because W = pdv \right]$$

$$ds = \frac{Q}{T}$$

$$\left[ \because Q = Tds \right]$$

w.k.t. Enthalpy

$$h = U + PV$$

differentiate both side

$$dh = du + d(pv)$$

$$dh = du + vdp + pdv$$

substitute eq. no. ①

$$dh = Tds - pdv + vdp + pdv$$

$$\boxed{dh = Tds + vdp} \quad \text{--- ②}$$

$$\left[ \because du = Tds - pdv \right]$$

w.k.t Helmot's function

$$a = U - TS$$

differentiate both side

$$da = du - d(Ts)$$

$$da = du - Tds - sdT$$

$$da = Tds - pdv - Tds - sdT \quad \left[ \because du = Tds - pdv \right]$$

$$\boxed{da = -pdv - sdT} \quad \text{--- ③}$$

We know that

Gibbs function

$$g = h - TS$$

differentiate both side

$$dg = dh - d(Ts)$$

$$dg = dh - Tds - sdT$$

$$dg = Tds + vdp - Tds - sdT \quad \left[ \because dh = Tds + vdp \right]$$

$$\boxed{dg = vdp - sdT} \quad \text{--- ④}$$

by Inverse exact differential

$$du = T ds - p dv \quad \text{--- (1)}$$

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v \quad \text{--- I}$$

$$dh = T ds + v dp \quad \text{--- (2)}$$

$$\left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p \quad \text{--- II}$$

$$da = -p dv - s dt \quad \text{--- (3)}$$

$$\left(\frac{\partial p}{\partial t}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T \quad \text{--- III}$$

$$dg = v dp - s dt \quad \text{--- (4)}$$

$$\left(\frac{\partial v}{\partial t}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T \quad \text{--- IV}$$

Cyclic Relation of PTV

P T V

$$\left(\frac{\partial p}{\partial t}\right)_v \left(\frac{\partial t}{\partial v}\right)_p \left(\frac{\partial v}{\partial p}\right)_T = -1$$

$$du = T ds - p dv \quad \text{--- (1)}$$

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v$$

$$dh = T ds + v dp$$

$$\left(\frac{\partial T}{\partial p}\right)_s = \left(\frac{\partial v}{\partial s}\right)_p$$

② First Tds equation (or) Entropy relation:

Entropy is a function of Temp & pressure.  
w.k.t  $S_2 - S_1 = m c_p \ln\left(\frac{T_2}{T_1}\right) + m R \ln\left(\frac{P_1}{P_2}\right)$

$S = f(T, P)$   
 $ds = \left(\frac{\partial s}{\partial T}\right)_P dT + \left(\frac{\partial s}{\partial P}\right)_T dP$  ————— ①

multiply T by both side

$T ds = T \left(\frac{\partial s}{\partial T}\right)_P dT + T \left(\frac{\partial s}{\partial P}\right)_T dP$  ————— ②

w.k.t

$\Delta H = m C_p \Delta T$

$h = m C_p T$

$h = f(T, P)$

$dh = \left(\frac{\partial h}{\partial T}\right)_P dT + \left(\frac{\partial h}{\partial P}\right)_T dP$  ————— ③

$\Delta H = m C_p \Delta T$

$\frac{\partial h}{\partial T} = C_p$   
 $C_p = \left(\frac{\partial h}{\partial T}\right)_P$

w.k.t

$dh = C_p dT + \left(\frac{\partial h}{\partial P}\right)_T dP$  ————— ③

$dh = T ds + v dp$  ————— ④

equate eqn. ③ and ④

$T ds + v dp = C_p dT + \left(\frac{\partial h}{\partial P}\right)_T dP$

$T ds = C_p dT + \left(\frac{\partial h}{\partial P}\right)_T dP - v dp$

$ds = \frac{C_p}{T} dT + \frac{1}{T} \left[ \left(\frac{\partial h}{\partial P}\right)_T - v \right] dP$  ————— ⑤

Compare eqn ① and ⑤

$\left(\frac{\partial s}{\partial T}\right)_P = \frac{C_p}{T}$  ————— ⑥

From Maxwell's relation - 4.

$\left(\frac{\partial s}{\partial P}\right)_T = - \left(\frac{\partial v}{\partial T}\right)_P$  ————— ⑦

subst. eqn ⑥ and ⑦ in eqn no. ②

$T ds = T \left(\frac{C_p}{T}\right) dT + T \left(-\frac{\partial v}{\partial T}\right)_P dP$

$T ds = C_p dT - T \left(\frac{\partial v}{\partial T}\right)_P dP$

Side = dT  
dP

Second Tds equation

w.k.T

$$s_2 - s_1 = m C_v \ln\left(\frac{T_2}{T_1}\right) + m R \ln\left(\frac{V_2}{V_1}\right)$$

$$s = f(T, v)$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv \quad \text{--- ①}$$

multiply T by both side by T

$$T ds = T \left(\frac{\partial s}{\partial T}\right)_v dT + T \left(\frac{\partial s}{\partial v}\right)_T dv \quad \text{--- ②}$$

w.k.T

$$\Delta U = m C_v \Delta T$$

$$U = m C_v T$$

$$U = f(T, v)$$

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv$$

$$du = c_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv \quad \text{--- ③}$$

$$\Delta U = m C_v \Delta T$$

$$\partial u = c_v dT$$

$$c_v = \left(\frac{\partial u}{\partial T}\right)_v$$

partial derivatives form

w.k.T

$$du = T ds - p dv \quad \text{--- ④}$$

equating eqs ③ and ④

$$T ds - p dv = c_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv$$

$$T ds = c_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv + p dv$$

$$ds = \frac{c_v}{T} dT + \left[\frac{\left(\frac{\partial u}{\partial v}\right)_T + p}{T}\right] dv \quad \text{--- ⑤}$$

Compare eqn ① and ⑤

$$\left(\frac{\partial s}{\partial T}\right)_v = \frac{c_v}{T} \quad \text{--- ⑥}$$

From Maxwell's relation III

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v \quad \text{--- ⑦}$$

sub eqn 6 and 7 in eqn no. ⑤

$$T ds = \cancel{T} \frac{c_v}{\cancel{T}} dT + T \left(\frac{\partial p}{\partial T}\right)_v dv$$

$$T ds = c_v dT + T \left(\frac{\partial p}{\partial T}\right)_v dv$$

Tds equation (or) entropy Relation

### Third Tds

Entropy is the function of pressure and volume.

$$s = s(p, v)$$

$$ds = \left(\frac{\partial s}{\partial p}\right)_v dp + \left(\frac{\partial s}{\partial v}\right)_p dv \quad \text{--- (1)}$$

Consider  $\left(\frac{\partial s}{\partial p}\right)_v$  <sup>First part</sup>  $\Rightarrow$  it is the function of  $(s, p, T)$

Chain Rule of  $(s, p, T)$

$$\left(\frac{\partial s}{\partial p}\right)_v \left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial T}{\partial s}\right)_v = 1$$

$$\left(\frac{\partial s}{\partial p}\right)_v = \frac{1}{\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial T}{\partial s}\right)_v} = \left(\frac{\partial T}{\partial p}\right)_v \left(\frac{\partial s}{\partial T}\right)_v$$

From 2<sup>nd</sup> Tds eqn. eqn no. 6

$$= \left(\frac{\partial T}{\partial p}\right)_v \frac{C_v}{T} \quad \text{--- (2)}$$

Consider 2<sup>nd</sup> part

$\left(\frac{\partial s}{\partial v}\right)_p \Rightarrow$  it is the function of  $s, v, T$

Chain Rule of  $(s, v, T)$

$$\left(\frac{\partial s}{\partial v}\right)_p \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial s}\right)_p = 1$$

$$\left(\frac{\partial s}{\partial v}\right)_p = \frac{1}{\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial s}\right)_p} = \left(\frac{\partial T}{\partial v}\right)_p \frac{C_p}{T} \quad \text{--- (3)}$$

substitute eqn. 2 & 3 in eqn. no. 1

$$ds = \frac{C_v}{T} \left(\frac{\partial T}{\partial p}\right)_v dp + \frac{C_p}{T} \left(\frac{\partial T}{\partial v}\right)_p dv$$

$$T ds = C_v \left(\frac{\partial T}{\partial p}\right)_v dp + C_p \left(\frac{\partial T}{\partial v}\right)_p dv$$

$$T ds = C_v dT + C_p dT$$

1<sup>st</sup> Tds

2<sup>nd</sup> Tds

3<sup>rd</sup> Tds



Differentiate between  $C_p$  and  $C_v$   
 Differentiate between heat Capacities:

 $C_p - C_v$ 

W.K.T

First Tds equation

$$T ds = C_p dT - T \left( \frac{\partial v}{\partial T} \right)_p dp$$

Second Tds equation

$$T ds = C_v dT + T \left( \frac{\partial p}{\partial T} \right)_v dv$$

equate First and second Tds equations.

$$C_p dT - T \left( \frac{\partial v}{\partial T} \right)_p dp = C_v dT + T \left( \frac{\partial p}{\partial T} \right)_v dv$$

$$(C_p - C_v) dT = T \left( \frac{\partial p}{\partial T} \right)_v dv + T \left( \frac{\partial v}{\partial T} \right)_p dp$$

$$dT = \frac{T \left( \frac{\partial p}{\partial T} \right)_v dv}{C_p - C_v} + \frac{T \left( \frac{\partial v}{\partial T} \right)_p dp}{C_p - C_v} \quad \text{--- ①}$$

We know that

$$p v = n R T$$

$$T = f(p, v)$$

$$dT = \left( \frac{\partial T}{\partial p} \right)_v dp + \left( \frac{\partial T}{\partial v} \right)_p dv \quad \text{--- ②}$$

Compare equation ① and ②

$$\left( \frac{\partial T}{\partial p} \right)_v = \frac{T \left( \frac{\partial v}{\partial T} \right)_p}{C_p - C_v} \quad \text{--- ③} \quad \left( \frac{\partial T}{\partial v} \right)_p = \frac{T \left( \frac{\partial p}{\partial T} \right)_v}{C_p - C_v} \quad \text{--- ④}$$

Consider eqn ③

$$C_p - C_v = \frac{T \left( \frac{\partial v}{\partial T} \right)_p}{\left( \frac{\partial T}{\partial p} \right)_v}$$

$$C_p - C_v = T \left( \frac{\partial v}{\partial T} \right)_p \left( \frac{\partial p}{\partial T} \right)_v \quad \text{--- ⑤}$$

W.K.T.

Cyclic Relation of p, T, v

$$\left( \frac{\partial p}{\partial T} \right)_v \left( \frac{\partial T}{\partial v} \right)_p \left( \frac{\partial v}{\partial p} \right)_T = -1$$

$$\left( \frac{\partial p}{\partial T} \right)_v = \frac{-1}{\left( \frac{\partial T}{\partial v} \right)_p \left( \frac{\partial v}{\partial p} \right)_T} = \left( \frac{\partial v}{\partial T} \right)_p \left( \frac{\partial p}{\partial v} \right)_T$$

$$\left( \frac{\partial p}{\partial T} \right)_v = - \left[ \left( \frac{\partial v}{\partial T} \right)_p \left( \frac{\partial p}{\partial v} \right)_T \right] \quad \text{--- ⑥}$$

Substitute eqn ⑥ in eqn. no. ⑤

$$C_p - C_v = T \left( \frac{\partial v}{\partial T} \right)_p \left( - \left( \frac{\partial v}{\partial T} \right)_p \left( \frac{\partial p}{\partial v} \right)_T \right)$$

$$C_p - C_v = - T \left( \frac{\partial v}{\partial T} \right)_p^2 \left( \frac{\partial p}{\partial v} \right)_T \quad \text{--- ⑦}$$

Derive the expression for ratio of specific heats:

(11) Prove that  $\frac{C_p}{C_v} = \left(\frac{\partial P}{\partial V}\right)_S \left(\frac{\partial V}{\partial P}\right)_T$

From 1<sup>st</sup> Tds equation, eqn no: 6.

$$C_p = T \left(\frac{\partial S}{\partial T}\right)_P$$

From second Tds equation, eqn no: 6

$$C_v = T \left(\frac{\partial S}{\partial T}\right)_V$$

$$\gamma = \frac{C_p}{C_v} = \frac{T \left(\frac{\partial S}{\partial T}\right)_P}{T \left(\frac{\partial S}{\partial T}\right)_V} \quad \text{--- (1)}$$

Consider the Term

$\left(\frac{\partial S}{\partial T}\right)_P$ ; Variables is the function of entropy, Temp and pressure.

Let  $f(S, T, P)$

Cyclic Relation.

$$\left(\frac{\partial S}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_S \left(\frac{\partial P}{\partial S}\right)_T = -1$$

$$\left(\frac{\partial S}{\partial T}\right)_P = \frac{-1}{\left(\frac{\partial T}{\partial P}\right)_S \left(\frac{\partial P}{\partial S}\right)_T}$$

$$\left(\frac{\partial S}{\partial T}\right)_P = - \left(\frac{\partial P}{\partial T}\right)_S \left(\frac{\partial S}{\partial P}\right)_T \quad \text{--- (2)}$$

Consider the Term.

$\left(\frac{\partial S}{\partial T}\right)_V$ ; Variables is the function of entropy, Temp and volume.

Let  $f(S, T, V)$

Cyclic Relation

$$\left(\frac{\partial S}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_S \left(\frac{\partial V}{\partial S}\right)_T = -1$$

$$\left(\frac{\partial S}{\partial T}\right)_V = \frac{-1}{\left(\frac{\partial T}{\partial V}\right)_S \left(\frac{\partial V}{\partial S}\right)_T}$$

$$\left(\frac{\partial S}{\partial T}\right)_V = - \left[\left(\frac{\partial V}{\partial T}\right)_S \left(\frac{\partial S}{\partial V}\right)_T\right] \quad \text{--- (3)}$$

Substitute eqn. 2 and 3 in equation no. 1.

$$\begin{aligned} \gamma = \frac{C_p}{C_v} &= \frac{T \left(\frac{\partial P}{\partial T}\right)_S \left(\frac{\partial S}{\partial P}\right)_T}{T \left(\frac{\partial V}{\partial T}\right)_S \left(\frac{\partial S}{\partial V}\right)_T} = \left(\frac{\partial P}{\partial T}\right)_S \left(\frac{\partial S}{\partial P}\right)_T \frac{\left(\frac{\partial T}{\partial V}\right)_S \left(\frac{\partial V}{\partial P}\right)_T}{\left(\frac{\partial V}{\partial T}\right)_S \left(\frac{\partial S}{\partial V}\right)_T} \\ &= \frac{\left(\frac{\partial P}{\partial T}\right)_S \left(\frac{\partial S}{\partial P}\right)_T \left(\frac{\partial T}{\partial V}\right)_S \left(\frac{\partial V}{\partial P}\right)_T}{\left(\frac{\partial V}{\partial T}\right)_S \left(\frac{\partial S}{\partial V}\right)_T} \end{aligned}$$

$$\boxed{\frac{C_p}{C_v} = \left(\frac{\partial P}{\partial V}\right)_S \left(\frac{\partial V}{\partial P}\right)_T} \quad \checkmark$$

Clausius Clapeyron equation

Clapeyron equation which involves the relationship between the saturation pressure, saturation temperature, the enthalpy of evaporation and the specific volume of the two phases involved.

Let, the entropy is the function of Temp & volume.

$$s = f(T, v)$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv \quad \text{--- (1)}$$

When the phase is changing from saturated liquid to saturated vapour, temperature remains constant ( $T = C$ )

So ds equation reduces to

$$ds = \left(\frac{\partial s}{\partial v}\right)_T dv \quad \text{--- (2) } [\because dT = 0]$$

From Maxwell's relation we know that

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$$

substituting in eqn. no. (2)

$$ds = \left(\frac{\partial p}{\partial T}\right)_v dv \quad \text{--- (3)}$$

$\left(\frac{\partial p}{\partial T}\right)_v$  is the slope of the saturation curve.

Integrating the above equation between sat. liquid and sat. vapour. (g)

$$\int_{s_f}^{s_g} ds = \left(\frac{dp}{dT}\right)_v \int_{v_f}^{v_g} dv$$

$$[s]_{s_f}^{s_g} = \left(\frac{dp}{dT}\right)_v [v]_{v_f}^{v_g}$$

$$s_g - s_f = \left(\frac{dp}{dT}\right)_v [v_g - v_f]$$

$$\frac{dp}{dT} = \frac{s_g - s_f}{v_g - v_f} = \frac{s_{fg}}{v_{fg}} \quad \text{--- (4)}$$

From 2<sup>nd</sup> Law of thermodynamics

$$ds = \frac{dq}{T}$$

For constant pressure process

$$dq = dh$$

$$m C_p (T_2 - T) = m C_p (T_2 - T)$$

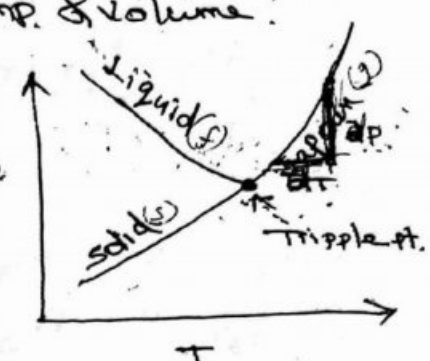
$$ds = \frac{dh}{T}$$

$$s_g - s_f = \frac{h_g - h_f}{T}$$

$$s_{fg} = \frac{h_{fg}}{T}$$

Substitute  $s_{fg}$  values in eqn. no. (4)

$$\frac{dp}{dT} = \frac{s_g - s_f}{v_g - v_f} = \frac{s_{fg}}{v_{fg}} = \frac{h_{fg}}{T v_{fg}}$$

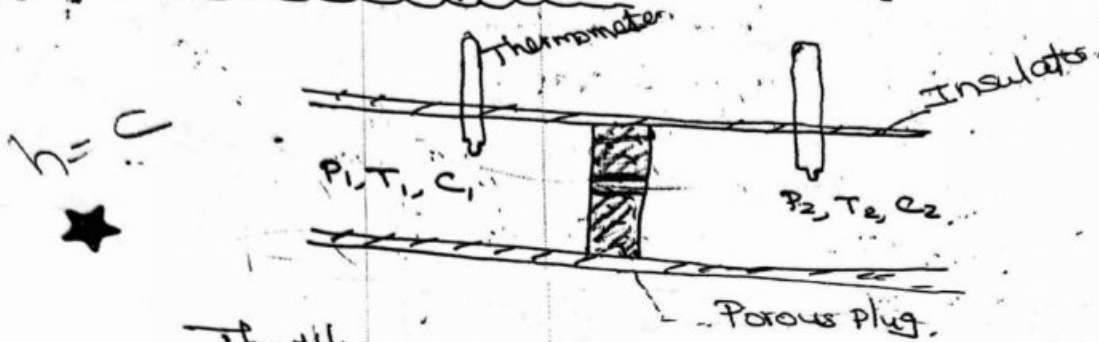


## Joule-Thomson Co-efficient

Joule Thomson Coefficient is defined as the change in Temperature with change in pressure keeping the enthalpy remains constant. It is denoted by  $(\mu)$ .

$$\mu = \left[ \frac{\partial T}{\partial P} \right]_h \quad \text{--- (1)}$$

## Joule Thomson Experiment:



Throttling process takes place in Joule Thomson experiment. Throttling process is defined as the fluid expansion through a minute orifice, its pressure and temperature are reduced. But there is no heat transfer and no work done by the system. In this process Enthalpy remains constant.

$P_1$  - Inlet gas pressure  
 $T_1$  - Inlet gas temperature  
 $C_1$  - Inlet gas velocity.

and  $P_2, T_2, \text{ or } C_2$  are the respective outlet values of gas.

The whole apparatus is completely insulated.

$$\therefore Q = 0 \text{ (No heat transfer)}$$

$$W = 0 \text{ (No shaft work)}$$

We know that

steady flow energy equation

$$gz_1 + \frac{C_1^2}{2} + h_1 + Q = gz_2 + \frac{C_2^2}{2} + h_2 + W \quad \text{--- (2)}$$

since there is no considerable change in velocity.

$$C_1 = C_2 \checkmark$$

$$Z_1 = Z_2 \checkmark$$

$$W = 0$$

$$Q = 0$$

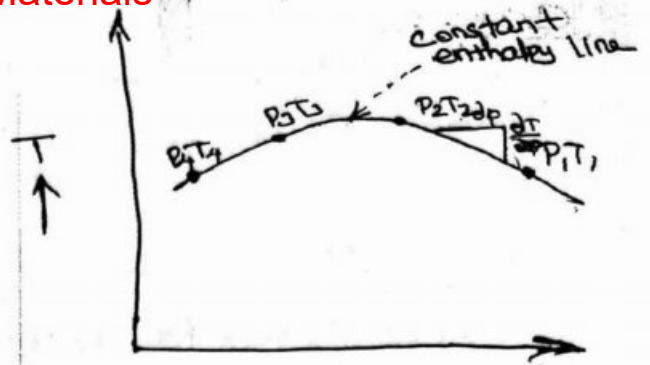
FREE  
Equation becomes

$$h_1 = h_2$$

Enthalpy at Inlet = Enthalpy at outlet.

It indicates that the enthalpy is constant for throttling process.

This Temp. and Pr. diagram shows the constant enthalpy line. The slope of the constant enthalpy line is known as Joule Thomson Coefficient ( $\mu$ ).



$$\mu = \left[ \frac{\partial T}{\partial P} \right]_h$$

For Ideal gas,  $\mu$  may be either ~~positive~~,  $\rightarrow$  P depending upon ~~it~~

The generalized equation of the Joule Thomson Coefficient can be derived by using change in enthalpy equation.

We know that

$$dh = Tds + vdp \quad \text{--- (1)}$$

W.k.T First Tds eqn

$$Tds = C_p dT - T \left( \frac{\partial v}{\partial T} \right)_p dp \quad \text{--- (2)}$$

substitute eqn (2) in eqn (1)

$$dh = C_p dT - T \left( \frac{\partial v}{\partial T} \right)_p dp + vdp$$

$$dh - vdp = C_p dT - T \left( \frac{\partial v}{\partial T} \right)_p dp$$

Divide  $C_p$  by both side

$$\frac{dh}{C_p} - \frac{vdp}{C_p} = dT - \frac{T}{C_p} \left( \frac{\partial v}{\partial T} \right)_p dp$$

$$dT = \frac{dh}{C_p} + \frac{T}{C_p} \left( \frac{\partial v}{\partial T} \right)_p dp - \frac{vdp}{C_p}$$

$$dT = \frac{dh}{C_p} + \frac{1}{C_p} \left[ T \left( \frac{\partial v}{\partial T} \right)_p - v \right] dp \quad \text{--- (3)}$$

differentiating this eqn. with respect to pressure at constant enthalpy.

$$\mu = \left[ \frac{\partial T}{\partial P} \right]_h = \frac{1}{C_p} \left[ T \left( \frac{\partial v}{\partial T} \right)_p - v \right] \quad \text{--- (4)}$$

From this equation we can determine the Joule Thomson Coefficient.

For Ideal gas,  $\mu$  - may be either positive or negative depending upon the thermodynamic state of the gas.

Case 1: if  $\mu$  is (+ve)

throttling process produces cooling effect.

Case 2: if  $\mu$  is (-ve)

throttling process produces heating effect.

Case 3:  $\mu = 0$

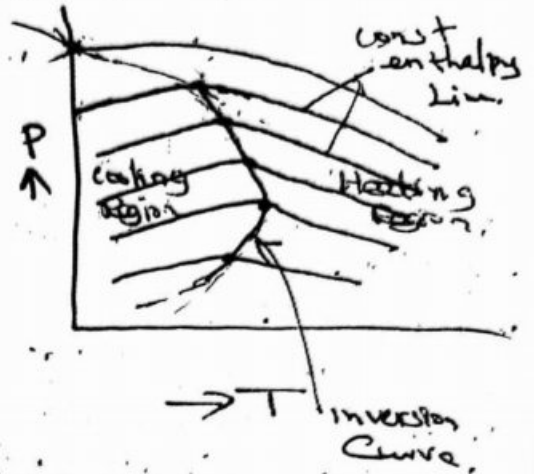
Case 3: if  $\mu = 0$

In throttling process Temp remains constant. This temperature is called inversion Temperature.

Inversion Curve:

The Maximum point on each curve is called inversion point. And the locus of the inversion point is called inversion curve.

max. inversion Temp



show that Joule Thomson Coefficient is zero for an Ideal gas:

Joule Thomson Coefficient

$$\mu = \left[ \frac{\partial T}{\partial P} \right]_h = \frac{1}{C_p} \left[ T \left( \frac{\partial v}{\partial T} \right)_P - v \right]$$

W.K.T

The equation of state

Ideal gas equation

$$PV = RT$$

$$v = \frac{R}{P} T$$

Differentiate the above equation w.r. to "T" by keeping pressure P constant:

$$\left( \frac{\partial v}{\partial T} \right)_P = \frac{R}{P} = \frac{v}{T}$$

$$Pv = RT$$

$$\frac{R}{P} = \frac{v}{T}$$

$$\mu = \left[ \frac{\partial T}{\partial P} \right]_h = \frac{1}{C_p} \left[ T \left( \frac{v}{T} \right) - v \right]$$

$$\mu = \left( \frac{\partial T}{\partial P} \right)_h = 0 \text{ for Ideal gas.}$$

What is coefficient of volume expansion? "β"

$$\beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_P$$

Coefficient of volume expansion is defined as the change in volume with change in temperature per unit volume by pressure remains constant.

2. Isothermal Compressibility: "K"

$$K = -\frac{1}{V} \left( \frac{\partial V}{\partial P} \right)_T$$

Isothermal compressibility is defined as the change in volume with change in pressure per unit volume by temperature remains constant.

show (or) prove  $C_p - C_v = R$

First law of Thermodynamics

$$Q = W + \Delta U$$

$$m C_p (T_2 - T_1) = m R (T_2 - T_1) + m C_v (T_2 - T_1)$$

$$W = P(V_2 - V_1) = P_2 V_2 - P_1 V_1$$

$$[ \because W = m R (T_2 - T_1) ]$$

$$\boxed{C_p - C_v = R}$$

1. Using Clausius - Clapeyron's equation, estimate the enthalpy of evaporation at 200°C,  $v_g = 0.127 \text{ m}^3/\text{kg}$ ,  $v_f = 0.00157 \text{ m}^3/\text{kg}$ ,  $\frac{dP}{dT} = 32 \text{ kPa/K}$ .

G.D:

$$T = 200^\circ\text{C} + 273 = 473 \text{ K}$$

$$v_g = 0.127 \text{ m}^3/\text{kg}$$

$$v_f = 0.00157 \text{ m}^3/\text{kg}$$

$$\frac{dP}{dT} = 32 \text{ kPa/K}$$

Clausius - Clapeyron's equation.

$$\frac{dP}{dT} = \frac{h_{fg}}{T v_{fg}}$$

$$\frac{dP}{dT} = \frac{h_{fg}}{T (v_g - v_f)}$$

$$32 = \frac{h_{fg}}{473 \times (0.127 - 0.00157)}$$

$$\boxed{h_{fg} = 1910.814 \text{ kJ/kg}}$$

2. One kg of ideal gas heated from  $50^{\circ}\text{C}$  to  $150^{\circ}\text{C}$ .  
If  $R = 280 \text{ J/kgK}$  and  $\gamma = 1.32$  for the gas  
determine.

- 1)  $C_p$  and  $C_v$
- 2)  $\Delta U$
- 3)  $\Delta H$ .

G. D.:

$$T_1 = 50 + 273 = 323 \text{ K}$$

$$T_2 = 150 + 273 = 423 \text{ K}$$

$$R = 280 \text{ J/kgK} = 0.28 \text{ kJ/kgK}$$

$$\gamma = 1.32$$

Sol:

$$C_p - C_v = R = 0.28$$

$$\gamma = \frac{C_p}{C_v} = 1.32$$

$$C_p = 1.32 C_v$$

$$1.32 C_v - C_v = 0.28$$

$$C_v [1.32 - 1] = 0.28$$

$$C_v = \frac{0.28}{0.32} = 0.875 \text{ kJ/kgK}$$

$$C_p = 1.155 \text{ kJ/kgK}$$

$$2) \Delta U = m C_v (T_2 - T_1) = 1 \times 0.875 (423 - 323)$$

$$\Delta U = 87.5 \text{ kJ}$$

$$3) \Delta H = m C_p (T_2 - T_1) = 1 \times 1.155 (423 - 323)$$

$$\Delta H = 115 \text{ kJ}$$