Catalog

UNIT - II

SECOND LAW AND AVAILABILITY ANALYSIS

First law of Thermodynamics:

In a cyclic process net heat transfer is equal to the net work transfer

$$\boxed{\oint Q = \oint W}$$

Second law of Thermodynamics:

(i) Clausius statement:

Heat can flow from hot body to cold body without any external aid, but heat cannot flow from cold body to hot body without any external aid.

(ii) Kelvin-Plank statement:

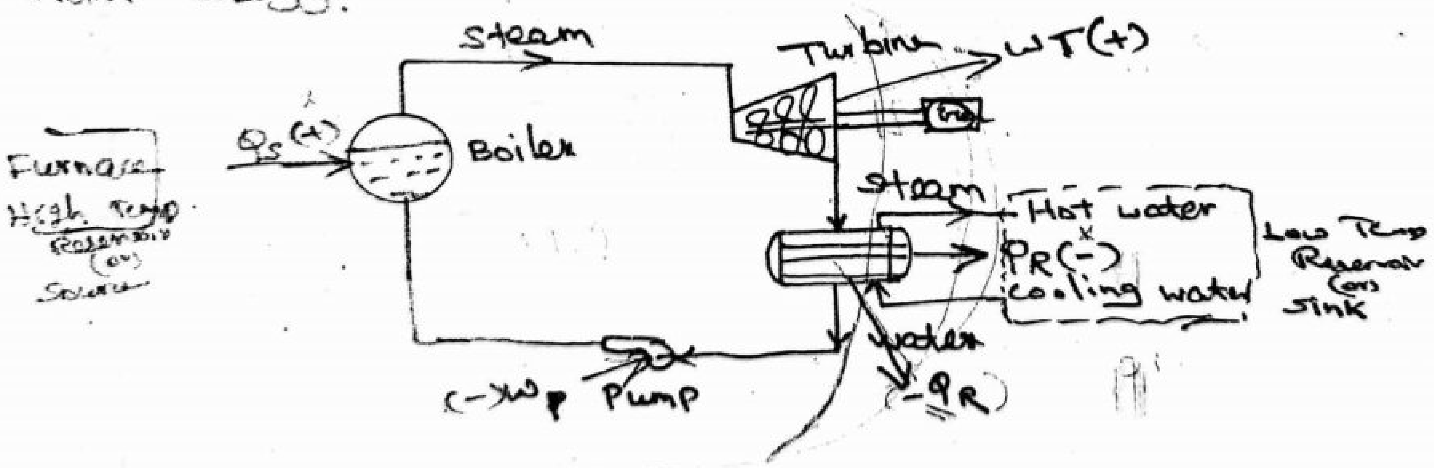
It is impossible to construct an engine working on a cyclic process, which converts the entire heat energy supplied into equivalent amount of work.

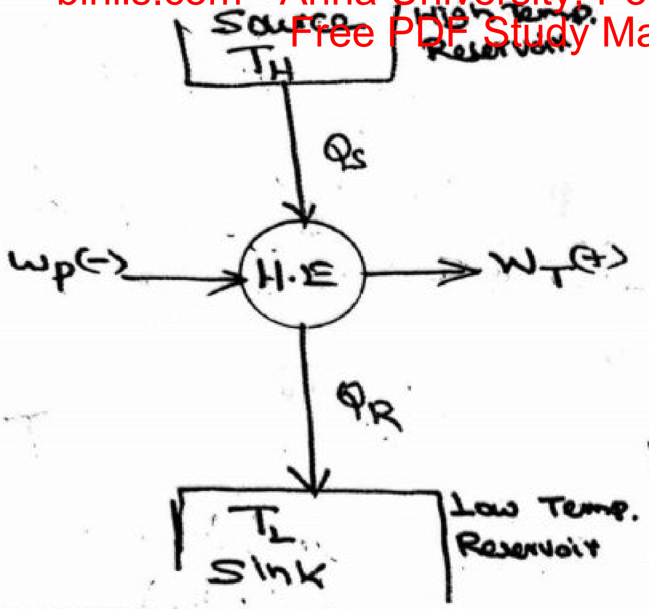
Application of second law of Thermodynamics:

(i) Heat engine
(ii) Refrigerator
(iii) Heat Pump.

Heat engine: (ex: I.C. Engine, Boiler)

is a device which converts the H. energy into work energy.





Work done = Heat sup − Heat Rej.

$W = Q_S - Q_R$

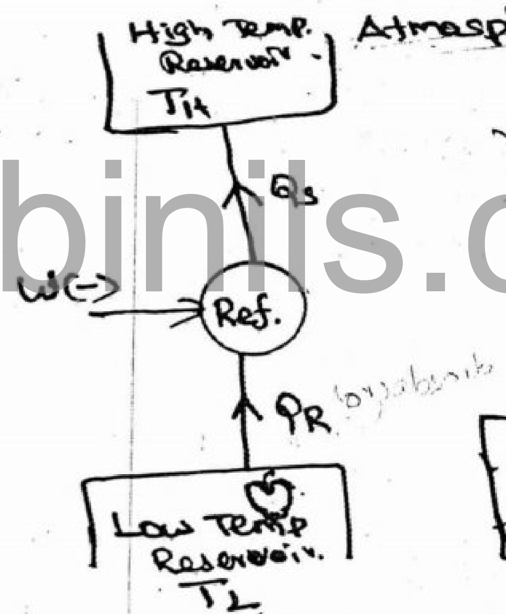
Efficiency = $\frac{\text{Work done}}{\text{Heat supplied}}$

$$\boxed{\eta_{H.E} = \frac{Q_S - Q_R}{Q_S}}$$

Heat (Q) is proportional to Temp

$$\therefore \boxed{\eta_{H.E} = \frac{T_H - T_L}{T_H}}$$

2. Refrigerator: is a device which is used to remove heat from a Cold system. In other words it is used to maintain the temp of the body lower than that of ~~body~~ surroundings. Ex. Air conditioner, Freezers.



η is expressed in terms of CoP.

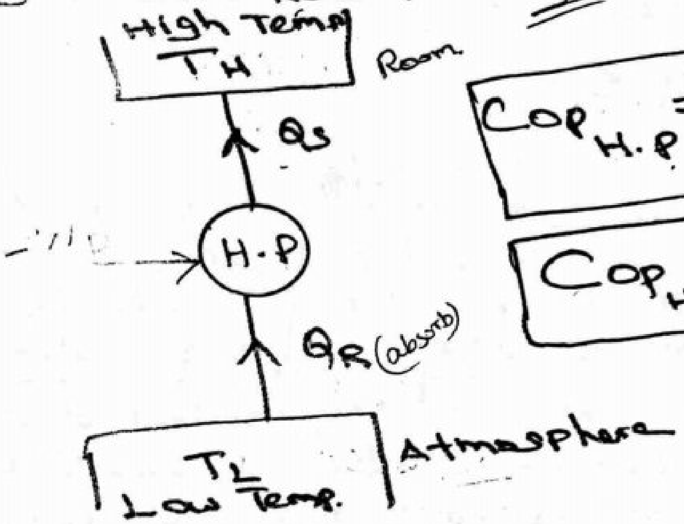
$$COP_R = \frac{\text{Refrigerating effect}}{\text{work done}} = \frac{R.E}{W}$$

$$= \frac{\text{Heat extracted (or) heat rejected}}{\text{Workdone}}$$

$$\boxed{COP_R = \frac{Q_R}{W} = \frac{Q_R}{Q_S - Q_R} = \frac{T_L}{T_H - T_L}}$$

CoP values always greater than 1

③ **Heat Pump:** is a device which is used to supply the heat to a hotter system. In other words it is used to maintain the body Temp higher than the surroundings. Ex: Room heater during winter season.



$$\boxed{COP_{H.P} = \frac{Q_s}{W} = \frac{Q_s}{Q_s - Q_R} = \frac{T_H}{T_H - T_L}}$$

$$\boxed{COP_{H.P} = COP_{Ref} + 1}$$

1. An irreversible heat engine with 66% efficiency of the max. possible is operating between 1000K and 300K. If it delivers 3KW of work, determine the heat extracted from the high Temp. reservoir and heat rejected to low temp. reservoir. (Ap. 13).

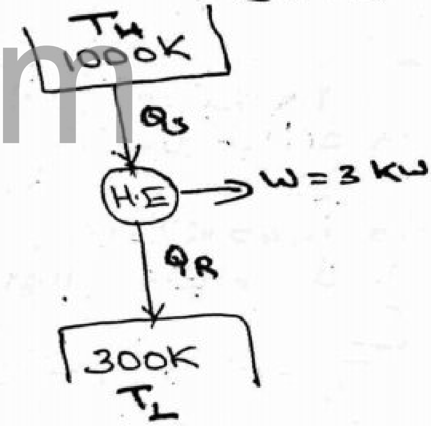
G.D:

$\eta_{HE} = 66\%$

$T_H = 1000K$

$T_L = 300K$

$W = 3KW$



To find: Q_s, Q_R

Solution:

$Q_s = ?$ $\quad \eta_{H.E} = \frac{W}{Q_s}$

$$Q_s = \frac{W}{\eta} = \frac{3}{0.66} = 4.54 kW$$

$$\boxed{Q_s = 4.54 KW}$$

Heat Rejected $Q_R = ?$

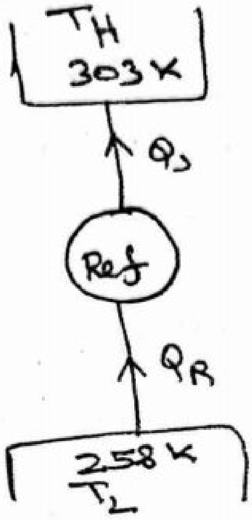
$W = Q_s - Q_R$

$3 = 4.54 - Q_R$

$$\boxed{Q_R = 1.54 KW}$$

AU.

2. A domestic food freezer maintains temperature of $-15°C$. The ambiant air is at $30°C$. If heat leaks into the freezer at a rate of $1.75\,KJ/s$. What is the least Power, necessary to pump.
Apr - 2003.



G.D:

$T_L = -15°C + 273 = 258K$

$T_H = 30°C + 273 = 303K$

$Q_R = 1.75\ KJ/s$

To find: $W = ?$

Solution:

$COP_{Ref} = \frac{Q_R}{W}$

$W = \frac{Q_R}{COP}$

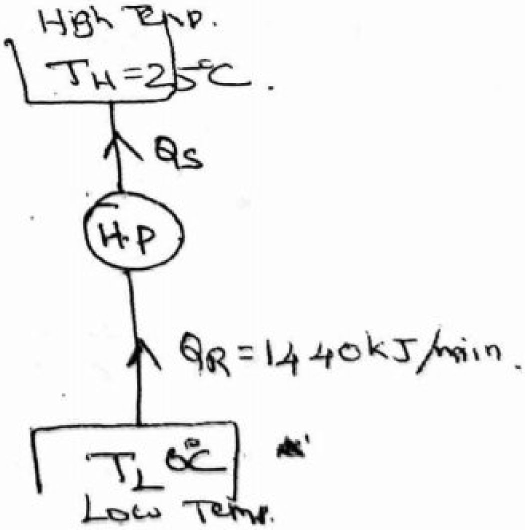
$Cop = \frac{T_L}{T_H - T_L} = \frac{258}{303 - 258} = 5.733$

$W = \frac{1.75}{5.733}$

$\boxed{W = 0.305\,KW}$

AU.

3. A reversible heat Pump is used to maintain a temp of $0°C$ in a refrigerator when it rejects the heat to the surroundings at $25°C$. If the heat removal rate from the refrigerator is $1440\,KJ/min$, Determine the Cop of the Refrigerator and Work input required.
June - 2014.



G.D:

$T_L = 0°C + 273 = 273K$

$T_H = 25°C + 273 = 298K$

$Q_R = 1440\,KJ/min \quad \frac{1440}{60} = 24\,KJ/s$

To find: $COP_{Ref} = ?$

$W = ?$

$(Cop)_{H.P} = (Cop)_{Ref} + 1$

$$\boxed{(Cop)_{Ref} = (Cop)_{H.P} - 1}$$

$(Cop)_{H.P} = \frac{T_H}{T_H - T_L} = \frac{298}{298 - 273} = 11.92$

$(Cop)_{Ref} = 11.92 - 1$

$\boxed{(Cop)_{Ref} = 10.92}$

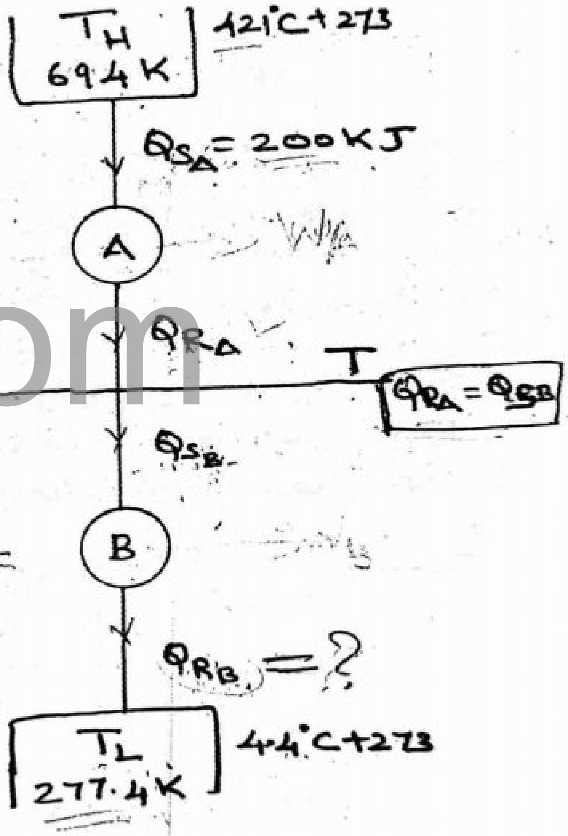
$W = ?$

$$(COP)_{Ref} = \frac{Q_R}{W} = \frac{T_L}{Q_S - Q_R} \qquad (Cop)_{HP} = \frac{Q_S}{W}$$

$$W = \frac{Q_R}{(COP)_{Ref}} = \frac{24}{10.92} = \frac{kJ}{s}$$

$$\boxed{W = 2.198\ KJ/s \ (or)\ KW}$$

An. 4. Two reversible heat engines A and B are arranged in series. A rejecting heat directly to B. Engine A receives 200kJ at a temp of 421°C from a hot source, while engine B is in communication with a cold sink at a temp of 4.4°C. If the work out put of A is twice that of B, find
(i) The intermediate temp between Engine A & B.
(ii) The efficiency of each engine.
(iii) The heat rejected to the cold sink.



G.D:

$T_H = 694K$

$T_L = 277.4K$

$Q_{SA} = 200KJ$

$W_A = 2 W_B$

$$\eta_A = \frac{T_H - T_B}{T_H}$$

To find:

$T, \eta_A, \eta_B, Q_{RB} = ?$

$$\eta_B = \frac{T - T_L}{T}$$

Solution:

$$\boxed{W_A = 2 W_B}$$

$$(Q_{SA} - Q_{RA}) = 2(Q_{SB} - Q_{RB})$$

$$Q_{SA} - Q_{RA} = 2(Q_{RA} - Q_{RB})$$

$$Q_{SA} - Q_{RA} = 2Q_{RA} - 2Q_{RB}$$

$$3Q_{RA} = 2Q_{RB} + Q_{SA} \quad (\because Q \text{ is proportional to } T)$$

$Q \propto T$

$$3T = 2T_L + T_H$$

$$T = \frac{2(277.4) + 694}{3}$$

$$\boxed{T = 416.26K}$$

$$\left[\because \eta_{H.E} = \frac{T_H - T_L}{T_H}\right]$$

$$\eta_A = \frac{T_H - T}{T_H} = \frac{694 - 416.26}{694} = 40\%$$

$$\eta_B = \frac{T - T_L}{T} = \frac{416.26 - 277.4}{416.26} = 33.39\%$$

$Q_{RB} = ?$

Consider engine "A"

$$\frac{T_H}{T} = \frac{Q_{SA}}{Q_{RA}} \quad [\because Q \propto T]$$

$$Q_{RA} = Q_{SA} \times \frac{T}{T_H} = 200 \times \frac{416.26}{694}$$

$$\boxed{Q_{RA} = 119.95\ KJ}$$

$$Q_{RA} = Q_{SB} = 119.95\ KJ$$

Consider engine "B"

$T \propto Q$

$$\frac{T}{T_L} = \frac{Q_{SB}}{Q_{RB}}$$

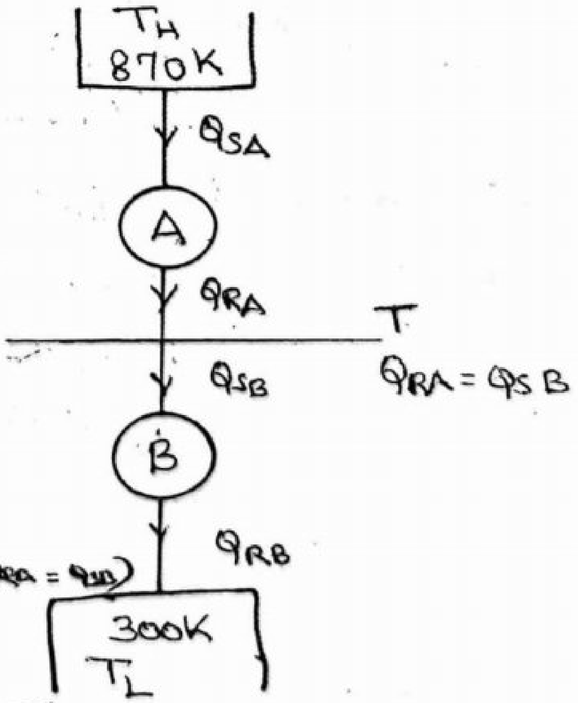
$$Q_{RB} = Q_{SB} \times \frac{T_L}{T} = 119.95 \times \frac{277.4}{416.26}$$

$$\boxed{Q_{RB} = 79.93\ KJ}$$

AU 5. Two – Carnot engines A and B are operated in series. The first one "A" receives heat at 870K and rejects to a reservoir at temperature T. The second engine "B" receives the heat rejected by the first engine and in turn rejects to a heat reservoir at 300K. Calculate the intermediate temp T in °C between two heat engines for the foll. cases.
(a) The work output of the two engines are equal
(b) The efficiencies of the two engines are equal.

G.D: $T_H = 870K$
$T_L = 300K$

To find:
$T \Rightarrow W_A = W_B$
$T \Rightarrow \eta_A = \eta_B$



Solution:

(i) $W_A = W_B$

$$Q_{SA} - Q_{RA} = Q_{SB} - Q_{RB}$$

$$Q_{SA} - Q_{RA} = Q_{RA} - Q_{RB} \quad (\because Q_{RA} = Q_{SB})$$

$$2Q_{RA} = Q_{RB} + Q_{SA}$$

$$2T = T_L + T_H \quad [Q \propto T]$$

(ii) $\eta_A = \eta_B$

$$\frac{Q_{S_A} - Q_{R_A}}{Q_{S_A}} = \frac{Q_{S_B} - Q_{R_B}}{Q_{S_B}}$$

$$\frac{T_H - T}{T_H} = \frac{T - T_L}{T} \quad (\because Q \propto T)$$

$$\frac{870 - T}{870} = \frac{T - 300}{T}$$

$$T(870 - T) = 870(T - 300)$$

$$\cancel{870T} - T^2 = \cancel{870T} - 261000$$

$$T^2 = 261000$$

$$T = 510.88\,K$$

$$\boxed{T = 237.88^\circ C}$$

6. A reversible heat engine operating between reservoirs at 900K and 300K drives a reversible refrigerator operating between reservoirs at 300K and 250K. The heat engine receives 1800KJ heat from 900K reservoir. The net output from the combined engine and refrigerator is 360KJ. Find the heat transferred to the refrigerator and the ~~heat~~ net heat rejected to the reservoir at 300K.

G.D:

$T_1 = 900K$

$T_2 = 300K$

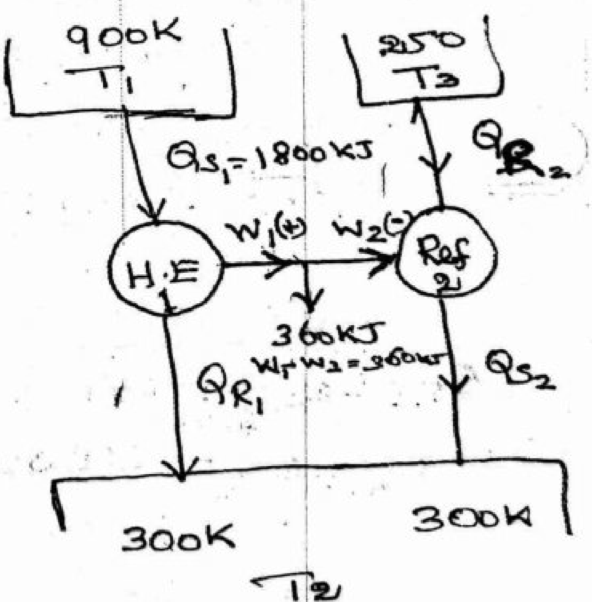
$T_3 = 250K$

$Q_{S_1} = 1800KJ$

$W_1 - W_2 = 360KJ$

To find: (1) Q_{R_2}, (2) $Q_{R_1} + Q_{S_2} = ?$

$Q_{R_1}, Q_{R_2}, Q_{S_2} = ?$



Solution:

$Q_{R_1} = ?$ Consider H.E:

$$\eta_{H.E} = \frac{W}{Q_S} = \frac{Q_{S_1} - Q_{R_1}}{Q_{S_1}}$$

$$\eta_{H.E} = \frac{Q_{S_1} - Q_{R_1}}{Q_{S_1}}$$

$$\eta_{H.E} = \frac{T_{H_1} - T_2}{T_1} = \frac{900 - 300}{900} = 0.66 = 66\%$$

$$0.66 = \frac{1800 - Q_{R1}}{1800}$$

$$1800 - Q_{R1} = 0.66\,(1800)$$

$$Q_{R1} = 1800 - 1200$$

$$\boxed{Q_{R1} = 600\,KJ}$$

$$\boxed{W_1 - W_2 = 360\,KJ}$$

$$W_1 = Q_{S1} - Q_{R1} = 1800 - 600$$

$$\boxed{W_1 = 1200\,KJ}$$

$$W_1 - W_2 = 360\,KJ$$

$$1200 - W_2 = 360\,KJ$$

$$W_2 = 1200 - 360$$

$$\boxed{W_2 = 840\,KJ}$$

$Q_{R_2} = ?$

Consider Refrigerator :

$$(COP)_{Ref} = \frac{Q_{R2}}{W_2}$$

$$Q_{R2} = (COP)_{Ref} \times W_2$$

$$Q_{R_2} = 5 \times 840$$

$$\boxed{Q_{R_2} = 4200\,KJ}$$

$$(COP)_{Ref} = \frac{T_1}{T_2 - T_1} = \frac{250}{300 - 250} = 5$$

250, 300

$Q_{S2} = ?$

$$(COP)_{Ref} = \frac{Q_{R2}}{Q_{S2} - Q_{R2}}$$

$$5 = \frac{4200}{Q_{S2} - 4200}$$

$$5(Q_{S2} - 4200) = 4200$$

$$5\,Q_{S_2} - 5 \times 4200 = 4200$$

$$\boxed{Q_{S2} = 5040\,KJ}$$

$$W_2 = Q_{S_2} - Q_{R_2}$$

$$Q_{S2} = W_2 + Q_{R_2} = 840 + 4200 = 4968\,KJ$$

$$Q_{R1} + Q_{S_2} = 648 + 4968$$

$$\boxed{Q_{R_1} + Q_{S_2} = 5580\,KJ}$$

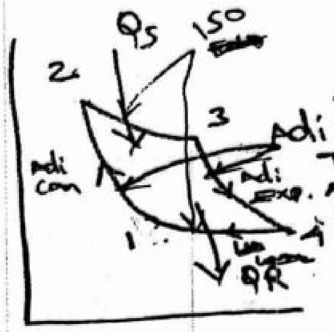
CARNOT CYCLE

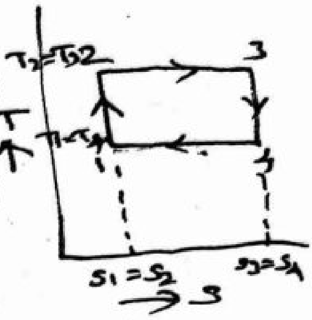
is also called as Constant Temperature Cycle.

It consists 4 processes.

Two Reversible Adiabatic process (or) Two Isentropic process

Two Isothermal (or) Two Const. Temp process





Process 1-2: Rev. Adi (or) Isentropic compression process:

During this process Air is compressed isentropically from state 1 → to state 2. In this process both pressure and Temp increases from P_1 to P_2 and T_1 to T_2 respectively. But the volume decreases from V_1 to V_2. In this process entropy remains constant. $(S_1 = S_2)$.

$$\text{compression Ratio} = \frac{V_1}{V_2}$$

Process 2-3: Isothermal heat Supplied process: (or) Iso. Exp.

During this process heat is supplied to the air at constant temperature. So $T_2 = T_3$

$$\text{Heat supplied} \quad \boxed{Q_{S_{2-3}} = T_2 ds = T_3 ds}$$

$\therefore ds = \frac{dQ}{T}$

$\left[\because ds = \frac{Q}{T}\right]$ for the process

$\left[\because Q = Tds\right]$

Process 3-4: Isentropic Expansion process

In this process pressure and Temp decreases. but vol will increases. and entropy remains constant. $\boxed{S_3 = S_4}$

$$\text{Expansion Ratio} = \frac{V_4}{V_3} = \text{Comp. Ratio} = \frac{V_1}{V_2}$$

Process 4-1: Isothermal heat Rejection process:

During this process heat is Rejected from the air at constant temp. So $T_4 = T_1$

$$\boxed{Q_{R_{4-1}} = T_1 ds = T_4 ds}$$

work done = Heat supplied − Heat Rejected.

$$= T_2 ds - T_1 ds$$

$$W = (T_2 - T_1) ds$$

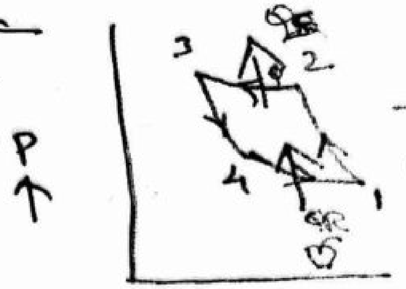
$$\text{Efficiency} = \frac{W}{Q_S} = \frac{(T_2-T_1)\,ds}{T_2\,ds} = \frac{T_2-T_1}{T_2} = \frac{T_H-T_L}{T_H}$$

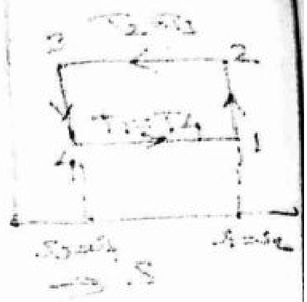
$$\boxed{\eta_{carnot} = \frac{T_2-T_1}{T_2} = \frac{T_H-T_L}{T_H}}$$

Reversed Carnot Cycle

It consists 4 Processes.

* Two Reversible Adiabatic (or) Isentropic processes.
* Two Isothermal Processes



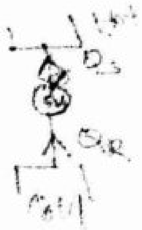


Process 1-2: Isentropic compression process:

(same as Carnot cycle)

Process 2-3: Isothermal heat Rejection (or) heat supplied to a hot body process:

$$Q_{R_{2-3}} = T_2 ds = T_3 ds$$



Process 3-4: Isentropic expansion process:

(same as Carnot cycle)

Process 4-1: Isothermal heat extraction process

$$Q_{R_{4-1}} = T_1 ds = T_4 ds$$

$$W = Q_S - Q_R = T_2 ds - T_1 ds$$

$$\boxed{W = (T_2 - T_1) ds}$$

$$Cop = \frac{\text{Heat extraction}}{\text{Work done}} = \frac{Q_R}{W} = \frac{Q_R}{Q_S - Q_R}$$

$$\boxed{Cop = \frac{T_L}{T_H - T_L}}$$

In a Carnot cycle the max pressure and Temp. are limited to 18 bar and 410°C. The volume ratio of isentropic compression is 6 and isothermal expansion is 1.5. Assume the volume of the air at the beginning of isothermal expansion as 0.18 m³. Show the cycle on P-V and T-s diagrams. and determine
(1) The Pr. and Temp. at main points.
(2) Find heat supplied and heat Rejected.
(3) Thermal efficiency of the cycle.

G..D

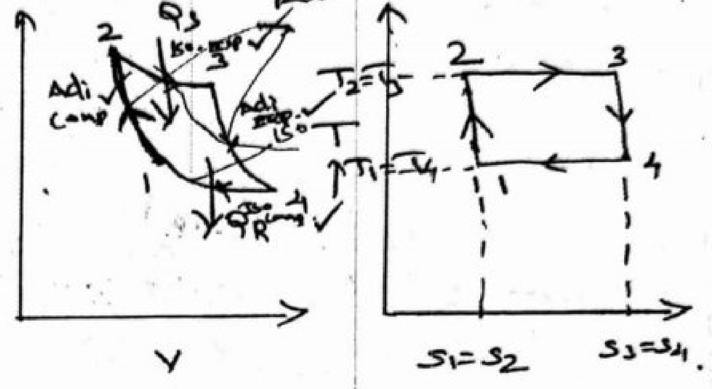
$P_2 = 18\ bar = 1800\ kN/m^2$

$T_2 = T_3 = 410°C + 273 = 683K$

$\frac{V_1}{V_2} = 6 = \frac{V_4}{V_3}$

$\frac{V_3}{V_2} = 1.5$

$V_2 = 0.18\ m^3$



indicate 4 process separate.

To find: (i) T_1, T_4, P_1, P_3, P_4
(ii) Q_S, Q_R
(iii) $\eta_{carnot} = \frac{Q_S - Q_R}{Q_S}$

Solution:
Consider process 1-2: (Adiabatic process) $PV^\gamma = C$

$PV \quad \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma$

$TV \quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

$TP \quad \frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^\gamma = (6)^{1.4}$

$P_1 = \frac{P_2}{(6)^{1.4}} = \frac{1800}{(6)^{1.4}}$

$\boxed{P_1 = 146\ kN/m^2}$

$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = (6)^{1.4-1}$

$T_1 = \frac{T_2}{(6)^{0.4}} = \frac{683}{(6)^{0.4}}$

$\boxed{T_1 = T_4 = 333.54K}$

Consider process 2-3: (Iso-thermal process) $T = C$
$(T_2 = T_3)$ $(T_2 = T_3)$

$\frac{P_2V_2}{\cancel{T_2}} = \frac{P_3V_3}{\cancel{T_3}}$

$P_2 . V_2 = P_3V_3$

$P_3 = P_2 \times \left(\frac{V_2}{V_3}\right) = 1800 \times \frac{1}{1.5} =$

$P_3 = 1200\ kN/m^2$

Consider 3-4: (Adiabatic process)

$$\frac{P_3}{P_4} = \left(\frac{V_4}{V_3}\right)^{\gamma} = (6)^{1.4}$$

$$P_4 = \frac{P_3}{(6)^{1.4}} = \frac{1200}{(6)^{1.4}}$$

$$\boxed{P_4 = 97.67\ kN/m^2}$$

PV $\frac{P_3}{P_4} = \left(\frac{V_4}{V_3}\right)^{\gamma}$

TV $\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{\gamma-1}$

TP $\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}$

(ii) Heat supplied (Q_s) =

Consider process 2-3: (Isothermal process)

$T = C$, $PV = C$

$$Q_s = P_2 V_2 \ln\left(\frac{V_3}{V_2}\right) = 1800 \times 0.18 \ln(1.5)$$

$$Q_s = 131.37\ KJ$$

Heat Rejected Q_R

Consider process 4-1 (Isothermal process)

$T_4 = T_1$

$$Q_R = P_1 V_1 \ln\left(\frac{V_4}{V_1}\right)$$

$$= 146 \times 1.08 \ln\left(\frac{P_1}{P_4}\right)$$

$$= 146 \times 1.08 \ln\left(\frac{146}{97.67}\right)$$

$$\boxed{Q_R = 63.38\ KJ}$$

$\frac{V_1}{V_2} = 6$

$V_1 = 6 \times 0.18 = 1.08\ m^3$

$T_4 = T_1$

$\frac{P_4 V_4}{T_4} = \frac{P_1 V_1}{T_1}$

$P_4 V_4 = P_1 V_1$

$\frac{V_4}{V_1} = \frac{P_1}{P_4}$

(iii) $\eta_{carnot} = \frac{W}{Q_s} = \frac{Q_s - Q_R}{Q_s}$

$$= \frac{131.37 - 63.38}{131.37}$$

$$\boxed{\eta = 51.78\%}$$

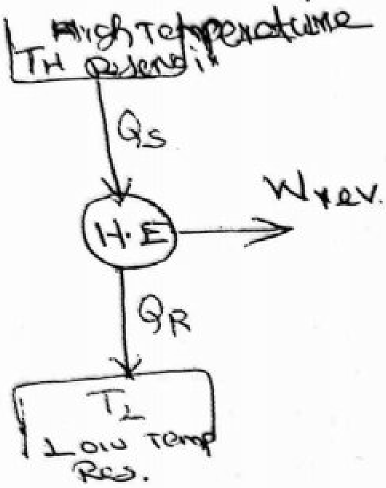
(or)

$$\eta = \frac{T_H - T_L}{T_H} = \frac{683 - 333}{683}$$

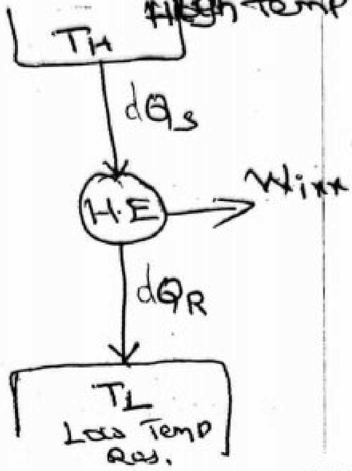
$$\boxed{\eta = 51.24\%}$$

Derivation of Clausius Inequality:

Clausius Inequality states that "When a system undergoes a cyclic process the summation of $\frac{dQ}{T}$ is less than or equal to zero."



Reversible H.E



Irreversible H.E

$W_{irr} < W_{rev}$.

Consider an engine operating between two fixed temp's ~~reservoirs~~ T_H and T_L. Let dQ_s units of heat be supplied at temp T_H and dQ_R units of heat rejected at temperature T_L in a cycle.

Thermal efficiency, $\eta_{irr} = \frac{dQ_s - dQ_R}{dQ_s}$

Thermal efficiency of any Reversible engine is

$$\eta_{rev} = \frac{T_H - T_L}{T_H}$$

The efficiency of actual/irreversible engine cycle must be less than the reversible engine.

$$\frac{dQ_s - dQ_R}{dQ_s} \leq \frac{T_H - T_L}{T_H}$$

$$\frac{dQ_s}{dQ_s} - \frac{dQ_R}{dQ_s} \leq \frac{T_H}{T_H} - \frac{T_L}{T_H}$$

$$1 - \frac{dQ_R}{dQ_s} \leq 1 - \frac{T_L}{T_H}$$

$$\frac{dQ_R}{dQ_s} \leq \frac{T_L}{T_H}$$

$$\frac{dQ_R}{T_L} \leq \frac{dQ_s}{T_H}$$

$$\frac{dQ_R}{T_L} - \frac{dQ_s}{T_H} \leq 0$$

If $\oint \frac{dQ}{T} = 0$; The cycle is Rev.
If $\oint \frac{dQ}{T} < 0$ The cycle is Irr.
If $\oint \frac{dQ}{T} > 0$ The cycle is impossible

For the cyclic process $\boxed{\oint \frac{dQ}{T} \leq 0}$

Clausius inequality

Entropy (S) kJ/kg K.

The loss of available energy (or) un availability (or) degradation of energy is called entropy.

The quantative measure of the amount of heat is not available to do work.

Change in entropy, ds;

$$ds = \frac{\text{Change of heat transfer}}{\text{Absolute temperature}}$$

$$= \frac{dQ}{T}$$

$$\Delta S = S_1 - S_2 = \int_1^2 \frac{dQ}{T}$$

⑤ Polytropic Process:

$$ds = \frac{\gamma - n}{\gamma - 1} \times mR\ln\left(\frac{V_2}{V_1}\right)$$

entropy Interms of T & V: $ds = mR\ln\left(\frac{V_2}{V_1}\right) + mC_v\ln\left(\frac{T_2}{T_1}\right)$

P & V: $ds = mC_p\ln\left(\frac{V_2}{V_1}\right) + mC_v\ln\left(\frac{P_2}{P_1}\right)$

P&T ⇒ $ds = mR\ln\left(\frac{P_1}{P_2}\right) + mC_p\ln\left(\frac{T_2}{T_1}\right)$.

1. Air in a closed vessel of fixed volume 0.15 m³ exerts pressure of 12 bar at 250°C. If the vessel is cooled so that the pressure falls to 3.5 bar, determine the final temperature, heat transfer and change in entropy.

G.D const. vol. process

$V = 0.15\,m^3$

$P_1 = 12\,bar = 1200\,kN/m^2$

$T_1 = 250°C + 273 = 523\,K$

$P_2 = 3.5\,bar = 350\,kN/m^2$

To find: T_2, Q, ds

Solution:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \qquad V_1 = V_2$$

$$T_2 = \frac{P_2}{P_1} \times T_1 = \frac{350}{1200} \times 523 = 152.5\,K$$

$Q = W + \Delta U$

$Q = \Delta U = m C_v (T_2 - T_1)$

Heat Transfer Q:

$Q = m C_v (T_2 - T_1)$

$Q = 1.2 \times 0.718 (152.54 - 523)$

$\boxed{Q = -319.18 \, kJ}$

$P_1 V_1 = m R T_1$

$m = \frac{P_1 V_1}{R T_1} = \frac{1200 \times 0.15}{0.287 \times 523} = 1.2 \, kg$

$\boxed{m = 1.2 \, kg}$

Change in entropy (ds):

$ds = m C_v \ln\left(\frac{T_2}{T_1}\right)$

$= 1.2 \times 0.718 \ln\left(\frac{152.54}{523}\right)$

$\boxed{ds = -1.06 \, kJ/K}$

1.6 kg of air compressed according to the law $PV^{1.3} = C$ from a pressure of 1.2 bar and temperature of 20°C to a pressure of 17.5 bar. Calculate (i) work done (ii) Heat transferred (iii) change in entropy.

(Polytropic process)

G.D

$m = 1.6 \, kg$

$PV^{1.3} = C$

$n = 1.3$

$P_1 = 1.2 \, bar = 120 \, kN/m^2$

$T_1 = 20°C + 273 = 293 \, K$

$P_2 = 17.5 \, bar = 1750 \, kN/m^2$

To find: (i) W (ii) Q (iii) ds

Work done (W) $= \frac{P_1 V_1 - P_2 V_2}{n-1}$

$V_1, V_2 = ?$

$P_1 V_1 = m R T_1$

$V_1 = \frac{m R T_1}{P_1} = \frac{1.6 \times 0.287 \times 293}{120} = 1.121 \, m^3$

$\boxed{V_1 = 1.121 \, m^3}$

$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^n$

$\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)^{1/n}$

$V_2 = \frac{V_1}{\left(\frac{P_2}{P_1}\right)^{1/n}} = \frac{1.121}{\left(\frac{1750}{120}\right)^{1/1.3}} = 0.142 \, m^3$

$\boxed{V_2 = 0.1427 \, m^3}$

$W = \frac{120 \times 1.121 - 1750 (0.1427)}{1.3 - 1}$

$\boxed{W = -384.02 \, kJ}$

(ii) Heat transfer (Q)

$$Q = \frac{\gamma - n}{\gamma - 1} \times W = \frac{1.4-1.3}{1.4-1} \times -384.02$$

$$\boxed{Q = -96\,kJ}$$

(iii) Change in entropy (ds)

$$ds = \frac{\gamma - n}{\gamma - 1} \times mR \ln\left(\frac{V_2}{V_1}\right)$$

$$= \frac{1.4-1.3}{1.4-1} \times 1.6 \times 0.287 \times \ln\left(\frac{0.1427}{1.021}\right)$$

$$\boxed{ds = -0.2366\,kJ/K}$$

(or)

$$ds = mC_v \ln\left(\frac{P_2}{P_1}\right) = mC_p \ln\left(\frac{V_1}{V_2}\right)$$

Tds equation.

First Tds equation
second Tds "
3rd Tds " Refer 4th unit.

Available energy:
The portion of the energy supplied as heat which can be converted into useful work by a reversible engine.

Un available energy:
The portion of the energy supplied as heat which cannot be converted into work due to friction is called as unavailable energy.

Availability: Max useful work obtained from the system is called availability.

$$1^{st} \text{ law efficiency} = \frac{\text{Net work output}}{\text{Heat supplied}}$$

$$\eta_{II} = \frac{\text{Change in the available energy of the system}}{\text{" " " " of the source.}}$$

High grade energy: The energy which can completely be converted into useful form of energy is called high grade energy.

Ex: i) Mechanical work
ii) Electrical energy
iii) water power
iv). wind power.

Low grade energy: The energy, which cannot completely be converted into useful form of energy is called Low grade energy.

Ex: Heat energy.

Available Energy: (or) Availability: Maximum useful work obtained from the system is called as Availability.

$$A.E = Q - T_0 \Delta S$$

Q = Heat transfer
$\Delta S = S_2 - S_1$
T_0 - atm Temp

Availability simbols ϕ (or) ψ

$\phi = Q - T_0 \Delta S \Rightarrow$ for closed system.

$\psi = (h_1 - h_2) - T_0 \Delta S \Rightarrow$ for open system.

$\psi = \psi_1 - \psi_2 = h_1 - T_0 s_1 - (h_2 - T_0 s_2)$
$\psi_1 = h_1 - T_0 s_1$
$\psi_2 = h_2 - T_0 s_2$

$$\boxed{A.E = W_{max\,(or)\,rev} = \phi \text{ (or) } \psi}$$

Un Available Energy (or) Irreversibility:

It is defined as the difference between Maximum work to the actual work

$$I = W_{max} - W_{act} = T_0 \Delta S$$

T_0 - Atm. Temp.

⊗ Energy loss is called un available energy.

Available Energy $= Q - T_0 \Delta S$

$$\boxed{AE = Q - UAE}$$

$$\boxed{UAE = Q - AE}$$

Availability formula for closed system $= Q - T_0 \Delta S$
PROCESS

Process	Q	ΔS
Constant Volume	$mC_v(T_2 - T_1)$	$mC_v \ln\left(\frac{P_2}{P_1}\right)$ (or) $mC_v \ln\left(\frac{T_2}{T_1}\right)$
Constant Pressure	$mC_p(T_2 - T_1)$	$mC_p \ln\left(\frac{V_2}{V_1}\right)$ (or) $mC_p \ln\left(\frac{T_2}{T_1}\right)$
Const. Temp. (or) Isothermal	$PV \ln\left(\frac{V_2}{V_1}\right)$	$mR \ln\left(\frac{V_2}{V_1}\right)$ (or) $mR \ln\left(\frac{P_1}{P_2}\right)$ (or) $\frac{Q}{T_1}$ (or) $\frac{Q}{T_2}$
Adiabatic (or) Isentropic	0	0
Polytropic	$W \times \frac{\gamma - n}{\gamma - 1}$	$mC_p \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{P_2}{P_1}\right)$

Availability formula for open system:

1) Turbine: $(+W)$

$$\psi = W_{max\,(or)\,rev} = (h_1 - h_2) - T_0(S_1 - S_2)$$

Irreversibility $I = T_0 \Delta S$ $\quad \therefore \Delta S = S_1 - S_2$

Second Law Efficiency $\eta_{II} = \frac{W_{max}}{W_{act}}$ (or) $\frac{W_{rev}}{W_{act}}$

2) $$\boxed{W_{act} = m(h_1 - h_2) + Q}$$

1) SFEE $m\left[h_1 + \frac{C_1^2}{2} + Z_1 g\right] + Q = m\left[h_2 + \frac{C_2^2}{2} + Z_2 g\right] + W$

4) Availability at Inlet $\psi_1 = (h_1 - T_0 S_1)$
outlet $\psi_2 = h_2 - T_0 S_2$

3) W_{max} (or) W_{rev} $= \psi_1 - \psi_2 = (h_1 - h_2) - T_0(S_1 - S_2)$

2) Compressor / Pump (−W)

$$\eta_{II} = \frac{W_{max}}{W_{act}}$$

SFEE

$$m\left[h_1 + \cancel{\frac{c_1^2}{2}} + \cancel{z_1 g}\right] + Q = m\left[h_2 + \cancel{\frac{c_2^2}{2}} + \cancel{z_2 g}\right] + W$$

$$\boxed{W_{act} = m(h_2 - h_1) - Q}$$

Compressor and Pump are work absorbing devices.

$$W_{max} = (h_2 - h_1) - T_0 \Delta S \quad [\because \Delta S = S_2 - S_1]$$

$$\text{Irreversibility } I = T_0 \Delta S \quad [\because \Delta S = S_2 - S_1]$$

AU 3.6 / pg 2.117

1. One kg of air is contained in a piston cylinder assembly at 10 bar pressure and 500K temperature. The piston moves outwards and the air expands to 2 bar pressure and 350 K temperature. Determine the maximum work obtainable. Assume the environmental conditions to be 1 bar and 290K. Also make calculations for the availability in the Initial and final states.

G.D

$m = 1\ kg$

$P_1 = 10\ bar$

$T_1 = 500\ K$

$P_2 = 2\ bar$

$T_2 = 350\ K$

$P_0 = 1\ bar$

$T_0 = 290\ K.$

To find:

$$W_{max} = \psi = \psi_1 - \psi_2 = ?$$

$$\psi_1,\ \psi_2 = ?$$

(4)

Solution:

Availability in the Initial state

$\psi_1 = h_1 - T_0 S_0$

$(\because \psi = \psi_1 - \psi_2 = (h_1 - h_2) - T_0 \Delta s)$

$= (h_1 - T_0 S_1) - (h_2 - T_0 S_2)$

$\psi_1 = m[h_1 - T_0 (S_1 - S_0)]$

$= m\left[C_p T_1 - T_0\left[C_p \ln\left(\frac{T_1}{T_0}\right) - R \ln\left(\frac{P_1}{P_0}\right)\right]\right]$

(T, P)

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$

$\left[\because \Delta s = mC_p \ln \frac{T_2}{T_1} - mR \ln\left(\frac{P_2}{P_1}\right)\right]$

$\psi_1 = 1\left[1.005 \times 500 - 290\left[1.005 \ln\left(\frac{500}{290}\right) - 0.287 \ln\left(\frac{1000}{100}\right)\right]\right]$

$\boxed{\psi_1 = 535.38 \text{ kJ}}$

Availability in the Final state:

$\psi_2 = h_2 - T_0 S_2$

$\psi_2 = m[h_2 - T_0 (S_2 - S_0)]$

$= m\left\{C_p T_2 - T_0\left[C_p \ln\left(\frac{T_2}{T_0}\right) - R \ln\left(\frac{P_2}{P_0}\right)\right]\right\}$

$= 1\left\{1.005 \times 350 - 290\left[1.005 \ln\left(\frac{350}{290}\right) - 0.287 \ln\left(\frac{200}{100}\right)\right]\right\}$

$\boxed{\psi_2 = 354.63 \text{ kJ}}$

Maximum work: W_{max}

$W_{max} = \psi_1 - \psi_2$

$= 535.38 - 354.63$

$\boxed{W_{max} = 180.75 \text{ kJ}}$

to atmosphere at 101.325 KPa, 25°C. Heat transfer amounts to 7 KJ/kg. Calculate the entering steam availability, leaving steam availability and the maximum work. For helium, $C_p = 5.2$ KJ/kgK and molecular weight $= 4.003$ kg/kg-mol.

Given data:

$P_1 = 300\,KPa = 300\,KN/m^2$

$T_1 = 300°C + 273 = 573\,K$

$P_2 = 100\,KPa = 100\,KN/m^2$

$T_2 = 150°C + 273 = 423\,K$

$P_0 = 101.325\,KN/m^2$

$T_0 = 25°C + 273 = 298\,K$

$Q = 7\,KJ/kg$

$m = 1\,kg$

$C_p = 5.2\,kJ/kgK$

$M = 4.003\,kg/kg\text{-}mol$

To find:

$\psi_1, \psi_2 = ?$

$W_{max} = ?$

Sol:

Gas constant $R = \dfrac{\bar{R}}{M_w} = \dfrac{8.314}{4.003} = 2.077\,KJ/kgK$

Entering steam Availability:

$\psi_1 = h_1 - T_0\, s_1$

$= m[h_1 - T_0(s_1 - s_0)]$

$= m\left[C_p T_1 - T_0\left[C_p \ln\left(\frac{T_1}{T_0}\right) - R\ln\left(\frac{P_1}{P_0}\right)\right]\right]$

(T, P)

$\Delta s = mC_p \ln\left(\frac{T_2}{T_1}\right) - mR\ln\left(\frac{P_2}{P_1}\right)$

$\psi_1 = 1\left[5.2 \times 573 - 298\left[5.2\ln\left(\frac{573}{298}\right) - 2.077\ln\left(\frac{300}{101.325}\right)\right]\right]$

$\psi_1 = 2979.6 - 298\,[3.39 - 2.25]$

$\boxed{\psi_1 = 2641.21\,kJ/kg.}$

Leaving steam availability:

$$\psi_2 = h_2 - T_0 \Delta s_2$$

$$\psi_2 = \cancel{m}\left[h_2 - T_0(s_2 - s_0)\right]$$

$$= \cancel{m}\left[C_p T_2 - T_0\left[C_p \ln\left(\frac{T_2}{T_0}\right) - R\ln\left(\frac{P_2}{P_0}\right)\right]\right]$$

$$= 1\left[5.2\times 423 - 298\left[5.2\ln\left(\frac{423}{298}\right) - 2.077\ln\left(\frac{100}{101.325}\right)\right]\right]$$

$$\psi_2 = 2199.6 - 298\left[1.8214 - (-0.02773)\right]$$

$$= 2199.6 - 550.62$$

$$\boxed{\psi_2 = 1648.97\ KJ/Kg}$$

Maximum Work: W_{max}

$$W_{max} = \psi_1 - \psi_2$$

$$= 2641.2 - 1648.97$$

$$\boxed{W_{max} = 992.24\ KJ/Kg}$$

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1. A single stage air turbine is to operate with air inlet pressure and Temperature of 1bar and 600K. During the Expansion, the turbine losses are 20KJ/Kg to the surroundings which is at 1bar and 300K. For 1Kg of mass flow rate determine (i) decrease is availability (ii) Maximum work (iii) the irreversibility.

$P_1 = 1 bar$

$T_1 = 600K$

$Q = -20 KJ/kg$

$P_2 = 1 bar = P_0 = 100 kN/m^2$

$T_2 = 300K = T_0$

$m = 1kg$

To find:

$\psi_1 - \psi_2$, W_{max}, I

Decrease in Availability:

$$\psi_1 - \psi_2 = \cancel{m}\left[(h_1 - h_2) - T_0(s_1 - s_2)\right]$$

$$= \cancel{m}\left[C_p(T_1 - T_2) - T_0(s_1 - s_2)\right]$$

Δs:

$$\Delta s = s_1 - s_2 = m\left[C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)\right]$$

$$= 1\left[1.005 \ln\left(\frac{300}{600}\right) - 0.287 \ln\left(\frac{1}{1}\right)\right]$$

$$s_1 - s_2 = \boxed{\Delta s = -0.697 KJ/kgK}$$

$$\psi_1 - \psi_2 = 1\left[1 \times 1.005(600 - 300) - 300(-0.697)\right]$$

$$\boxed{\psi_1 - \psi_2 = 510.6 KJ/kg}$$

Maximum Work:

W_{max} = maximum work = decrease in Availability $= \psi$

$\psi = \psi_1 - \psi_2$

$$\boxed{W_{max} = 510.6 KJ/kg}$$

Irreversibility:

$$I = W_{max} - W_{act}.$$

Q7

$W_{act} = ?$

From SFEE

$$m\left(g\cancel{z_1} + \frac{\cancel{c_1^2}}{2} + h_1 + Q\right) = m\left(g\cancel{z_2} + \frac{\cancel{c_2^2}}{2} + h_2 + W_{act}\right)$$

$z_1 = z_2$
$c_1 = c_2$

$$m(h_1 + Q) = m(h_2 + W_{act})$$

$$W_{act} = m[(h_1 - h_2) + Q]$$

$$W_{act} = m\left[C_p(T_1 - T_2) + Q\right]$$

$$= 1\left[1.005(600-300) - 20\right].$$

$$\boxed{W_{act} = 281.5\ KJ/Kg}$$

Irreversibility $I = W_{max} - W_{act}$

$$I = 510.6 - 281.5$$

$$\boxed{I = 229.1\ KJ/Kg}$$