Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 40792

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Fourth/Fifth/Sixth Semester

Civil Engineering

MA 8491 – NUMERICAL METHODS

(Common to Aeronautical Engineering/Aerospace Engineering/ Agriculture Engineering/Electrical and Electronics Engineering/ Electronics and Instrumentation Engineering/Instrumentation and Control Engineering/Manufacturing Engineering/Mechanical Engineering (Sandwich)/ Mechanical and Automation Engineering/Chemical Engineering/Chemical and Electrochemical Engineering/Plastic Technology/Polymer Technology/ Textile Technology)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State the condition for convergence of Newton-Raphson method and the order of convergence.

2. Solve $x - 3y = -1;\ 3x + y = 2$ by Gauss-Jordan method.

3. Prove that $\mu + \frac{1}{2}\delta = E^{1/2}$.

4. If $f(x) = \frac{1}{x}$, $x = 1, 3, 4$ then find the second divided difference.

5. State the Newton's forward difference formulae for the first and second order derivatives at the value $x = x_0$ upto the fourth order difference term.

6. Evaluate $\int_1^2 \frac{1}{1 + x^3}\,dx$ using two point Gaussian quadrature formula.

7. By Taylor's series method find $y(1.1)$ given $\frac{dy}{dx} = x + y,\ y(1) = 0$.

8. What is the condition to apply Adams-Bashforth predictor corrector method?

9. Write down the Crank-Nicolson formula to solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ take $h = 1/4$ and $k = ah^2$.

10. Write down the Liebmann's iteration formula to solve the Laplace equation $u_{xx} + u_{yy} = 0$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find a real root of the equation $x^3 + x^2 = 100$ correct to 4 decimal places using fixed point iteration method. (6)

(ii) Use Jacobi method to find the eigen values and the corresponding eigen vectors of the matrix $\begin{pmatrix} 6 & \sqrt{3} \\ \sqrt{3} & 4 \end{pmatrix}$. (10)

Or

(b) (i) Solve the system of equations by Gauss-Seidel method $x - y + 4z = 12$, $x + 5y + 3z = 12$ and $3x - y - z = -2$. (8)

(ii) Using power method find the largest eigen value and the corresponding eigen vector of the matrix $\begin{pmatrix} 5 & 0 & 1 \\ 0 & -2 & 0 \\ 1 & 0 & 5 \end{pmatrix}$. (8)

12. (a) (i) Using Newton's divided difference formula, find the polynomial $f(x)$ and hence find $f(6)$ from the following data. (8)

x :	–2	–1	1	5
$f(x)$:	–84	–42	–6	42

(ii) Using Newton's backward interpolation formula, find the polynomial $f(x)$ from the following data and hence find $f(10)$. (8)

x :	–3	0	3	6	9
$f(x)$:	–1	–19	71	1403	7055

Or

(b) (i) The following values of x and y are given (10)

x :	1	2	3	4
y :	1	2	5	11

Find the cubic splines.

(ii) Using Newton's forward interpolation formula, find $y(-1)$ given (6)

x :	–2	0	2	4
$y(x)$:	–221	–59	7	25

13. (a) (i) The following data gives the velocity of a particle for 20 seconds at an interval of 5 seconds. Find the final acceleration using the entire data. (6)

t :	0	5	10	15	20
v :	0	3	14	69	228

(ii) Evaluate $\int_1^2 \frac{3x}{1+2x^2}\,dx$ using Romberg's method. (10)

Or

(b) (i) A jet fighter's position of an aircraft carrier's runway was timed during landing.

t	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y	7.989	8.403	8.781	9.129	9.451	9.750	10.031

where y is the displacement from the end of the carrier. Estimate the velocity and acceleration at $t = 1$. (8)

(ii) Evaluate $\int_2^3 \int_1^2 \frac{dx\,dy}{x^2 + y^2}$ using Simpson's rule with four sub-intervals in both directions. (8)

14. (a) (i) Apply Taylor's series method and find $y(0.1)$ and $y(0.2)$ correct to three decimal places if $\frac{dy}{dx} = y^2 + x$ and $y(0) = 1$. (8)

(ii) Apply Runge-Kutta method of order 4 to find an approximate value of y for $x = 0.2$ and $x = 0.4$, taking $h = 0.2$, if $\frac{dy}{dx} = x^2 + y^2$ given that $y = 1$ when $x = 0$. (8)

Or

(b) (i) Using modified Euler method, find $y(1.2)$ and $y(1.4)$ given $\frac{dy}{dx} = \log(x + y);\ y(1) = 2$. (8)

(ii) Solve $\frac{dy}{dx} = y - x^2$ at $x = 0.8$ by Milne's predictor and corrector method, given $y(0) = 1$, $y(0.2) = 1.12186$, $y(0.4) = 1.46820$ and $y(0.6) = 1.7379$. (8)

15. (a) Solve the equation $u_{xx} + u_{yy} = 0$ over a square region of side 4. Boundary conditions are $u(0, y) = 0$, $u(4, y) = 12 + y$, $u(x, 0) = 3x$, $u(x,4) = x^2$, $0 \le x \le 4$ and $0 \le y \le 4$. (16)

Or

(b) (i) Solve $16u_{xx} = u_{tt}, 0 < x < 5$, $t > 0$ given $u(0, t) = 0$, $u(5, t) = 0$ $u(x, 0) = x^2(5 - x)$ and $u_t(x, 0) = 0$. Compute u for 5 time steps with $h = 1$ and $k = 1/4$. (8)

(ii) Solve $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ subject to the conditions $u(0, t) = 0$, $u(8, t) = 0$ and $u(x, 0) = \frac{x}{2}(8 - x)$ using Bender-Schmidt formula. (8)