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Question Paper Code : 40789

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Fourth Semester

Computer Science and Engineering

MA 8402 — PROBABILITY AND QUEUEING THEORY

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Test whether the function $f(x)=\begin{cases}|x|, & -1 \le x \le 1\\ 0, & otherwise\end{cases}$ can be the probability density function of a continuous random variable.

2. A fair coin was tossed two times. Given that the first toss resulted in heads, what is the probability that both tosses resulted in heads?

3. The joint pdf of a two-dimensional random variable (*X*, *Y*) is given $f(x,y)=\begin{cases}kxe^{-y}; & 0 \le x \le 2, y > 0\\ 0; & otherwise\end{cases}$. Find the value of '*k*'.

4. In a partially destroyed laboratory. a record of an analysis of correlated data, the following results only are legible: Variance of $X = 9$; Regression equations are $8X - 10Y + 66 = 0$ and $40X - 18Y = 214$. What are the mean values of *X* and *Y*?

5. Is Poisson process stationary? Justify.

6. A random process is defined by $X(t) = K \cos \omega t$; $t \ge 0$, where 'ω' is a constant and *K* is uniformly distributed between 0 and 2. Determine $E\{X(t)\}$.

7. What do the letters in the symbolic representation (*a*/*b*/*c*) : (*d*/*e*) of a queueing model represent?

8. Write down the steady state condition for $M/M/3/\infty$ queueing model.

9. State Jackson's theorem for an open network.

10. What do you mean by E_k in the $M/E_k/1$ queueing model?

PART B — (5 × 16 = 80 marks)

11. (a) (i) The CDF of the random variable Y is given by $F_Y(y) = \begin{cases} 0; & y < 0 \\ K(1 - e^{-2y}); & y \geq 0 \end{cases}$. For what value of K is the function a valid CDF. With the above value of K, what is $P(1 \leq Y \leq 5)$ and what is $P(Y > 2)$. (8)

(ii) The quarterback of a certain football team has a good game with probability 0.6 and a bad game with probability 0.4. When he has a good game, he throws at least one interception with a probability of 0.2; and when he has a bad game, he throws at least one interception with a probability of 0.5. Given that he threw at least one interception in particular game, what is the probability that he had a good game? (8)

Or

(b) (i) The weights in pounds of parcels arriving at a package delivery company's warehouse can be modelled by an N(5,16) normal random variable X. (1) What is the probability that a randomly selected parcel weighs between 1 and 10 pounds? (2) What is he probability that a randomly selected parcel weighs more than 12 pounds? (8)

(ii) State and Prove "Forgetfulness property" of the exponential distribution. (8)

12. (a) (i) Find the bivariate probability distribution of (X,Y) given below, find $P(X \leq 1)$, $P(Y \leq 3)$, $P(X \leq 1, Y \leq 3)$, $P(X \leq 1/Y \leq 3)$, $P(Y \leq 3/ X \leq 1)$ and $P(X+Y \leq 4)$. (10)

X \ Y	1	2	3	4	5	6
0	0	0	1/32	2/32	2/32	3/32
1	1/16	1/16	1/8	1/8	1/8	1/8
2	1/32	1/32	1/64	1/64	0	2/64

(ii) If $X_1, X_2, X_3, \ldots X_n$ are Poisson variates with parameter $\lambda = 2$, use the central limit theorem to estimate $P(120 \leq S_n \leq 160)$, where $S_n = X_1 + X_2 + X_3 + \ldots X_n$ and $n = 75$. (6)

Or

(b) Find the regression equation of Y on X from the below data. (16)

X:	3	5	6	8	9	11
Y:	2	3	4	6	5	8

13. (a) (i) A random process $X(t)$ is defined by $X(t) = A\cos t + B\sin t$, $-\infty < t < \infty$, where A and B are independent random variables, each of which has a value –2 with probability 1/3 and a value 1 with probability 1/3. Show that $X(t)$ is a wide-sense stationary process. (8)

(ii) Consider a Markov chain on (0, 1, 2) having transition matrix given by $P = \begin{pmatrix} 0 & 1 & 0 \\ 1/6 & 1/2 & 1/3 \\ 0 & 2/3 & 1/3 \end{pmatrix}$. Find the steady state probability distribution. (8)

Or

(b) (i) A random process $X(t)$ is given by $X(t) = A\cos t + (B+1)\sin t$, $-\infty < t < \infty$, where A and B are independent random variables with $E(A) = E(B) = 0$ and $E(A^2) = E(B^2) = 1$. Is $X(t)$ wide-sense stationary process? Justify. (8)

(ii) Studies indicate that the probability that three cars will arrive at a parking lot in a 5 minute interval is 0.14. If cars arrive according to a Poisson process, determine the following: (1) The average arrival rate of cars (2) The probability that no more than 2 cars arrive in a 10 minute interval. (8)

14. (a) (i) An airport has a single runway. Airplanes have been found to arrive at the rate of 15 per hour. It is estimated that each landing takes 3 minutes. Assuming a Poisson process for arrivals and an exponential distribution for landing times. Find the expected number of airplanes waiting to land, expected waiting time. What is the probability that the waiting will be more than 5 minutes? (8)

(ii) Obtain the expressions for the steady state probabilities of a single server queuing model with finite waiting room. (8)

Or

(b) (i) Arrivals at a telephone booth are considered to be Poisson with an average time of 12 mins between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 mins.

(1) Find the average number of persons waiting in the system

(2) What is the probability that a person arriving at the booth will have to wait in the queue? (8)

(ii) Obtain the expressions for the steady state probabilities of a multi server queuing model with infinite capacity. (8)

15. (a) Derive Pollaczek — Khintchine formula. (16)

Or

(b) Write a short note about (i) series queues (ii) Open queueing network. (16)