Reg. No. :

Question Paper Code : 40781

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Second Semester

Civil Engineering

MA 8251 – ENGINEERING MATHEMATICS – II

(Common to : Aeronautical Engineering/Aerospace Engineering/ Agriculture Engineering/Automobile Engineering/Biomedical Engineering/ Computer Science and Engineering/Computer and Communication Engineering/ Electrical and Electronics Engineering/Electronics and Communication Engineering/Electronics and Instrumentation Engineering/Electronics and Telecommunication Engineering/Environmental Engineering/Geoinformatics Engineering/Industrial Engineering/Industrial Engineering and Management/ Instrumentation and Control Engineering/Manufacturing Engineering/Material Science and Engineering/Mechanical Engineering/Mechanical Engineering (Sandwich)/Mechanical and Automation Engineering/Mechatronics Engineering/Medical Electronics/Petrochemical Engineering/Production Engineering/Robotics and Automation/Safety and Fire Engineering/ Bio Technology/Biotechnology and Biochemical Engineering/ Chemical Engineering/Chemical and Electrochemical Engineering/Fashion Technology/Food Technology/Handloom and Textile Technology/ Information Technology/Petrochemical Technology/Petroleum Engineering/ Pharmaceutical Technology/Plastic Technology/Polymer Technology/ Textile Chemistry/Textile Technology)

(Regulations 2017)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If 2 and 3 are two eigen values of $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$, find the eigen values of A^{-1}.

2. If 1, 1 and –2 are eigen values of the matrix A, then state its signature and index.

3. Find the unit normal vector to the surface $xyz = 2$ at $(2, 1, 1)$.

4. State Stokes theorem.

5. Is $f(z) = z^3$ analytic? Justify your answer.

6. Find the fixed points of the transformation $w = \dfrac{2z+6}{z+7}$.

7. Find the nature of the singularity of $f(z) = \sin\left(\dfrac{1}{z+1}\right)$.

8. Find the residue of $\cot z$ at the pole $z = 0$.

9. Find $L\left[\dfrac{1}{\sqrt{t}}\right]$.

10. If $L[f(t)] = \dfrac{s}{(s+2)^3}$, using initial value theorem find $\lim_{t\to 0}[f(t)]$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 4 & -20 & -10 \\ -2 & 10 & 4 \\ 6 & -30 & -13 \end{bmatrix}$. (8)

(ii) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ and hence find its inverse. (8)

Or

(b) Reduce the quadratic form $3x^2 + 3y^2 + 3z^2 + 2xy + 2xz - 2yz$ to canonical form. Discuss its nature. (16)

12. (a) (i) Find the values of a and b so that the surfaces $ax^3 - by^2z = (a+3)x^2$ and $4x^2y - z^3 = 11$ may cut orthogonally at $(2, -1, -3)$. (6)

(ii) Verify Green's theorem in the plane for $\int_C \left(xy + y^2\right)dx + x^2\,dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. (10)

Or

(b) (i) Prove that $\vec{F} = (y+z)\vec{i} + (x+z)\vec{j} + (x+y)\vec{k}$ is irrotational. Find its scalar potential. (6)

(ii) Verify Gauss divergence theorem for $\vec{F} = x^3\vec{i} + y^3\vec{j} + z^3\vec{k}$ taken over the cube bounded by $x = 0$, $x = a$, $y = 0$, $y = a$, $z = 0$, $z = a$. (10)

13. (a) (i) Prove that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic. Find the conjugate harmonic function and the corresponding analytic function $f(z)$. (8)

(ii) Find the bilinear transformation which maps the points $z = 0, 1, \infty$ onto the points $w = -5, -1, 3$. (8)

Or

(b) (i) If $f(z) = u + iv$ is analytic function, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log|f(z)| = 0$. (8)

(ii) Find the image of $1 < x < 2$ under the transformation $w = \frac{1}{z}$. (8)

14. (a) (i) Evaluate $\int_C \frac{z+1}{z^2 + 2z + 4} dz$, where C is the circle $|z + 1 + i| = 2$ using Cauchy's integral formula. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{1}{2 - \cos\theta} d\theta$ using contour integration. (8)

Or

(b) (i) Find Laurent's series expansion of $\frac{6z+5}{z(z+1)(z-2)}$ in $1 < |z+1| < 3$. (8)

(ii) Using Cauchy's residue theorem evaluate $\int_C \frac{12z - 7}{(z-1)^2 (2z+3)} dz$ where C is $|z| = 2$. (8)

15. (a) (i) Find the Laplace transform of the periodic function $f(t)=\begin{cases}1, & 0\le t\le a\\ -1, & a\le t\le 2a\end{cases}$ and $f(t+2a)=f(t)$. (8)

(ii) Find $L^{-1}\left[\frac{1}{(s^2+4)^2}\right]$ by using Convolution theorem. (8)

Or

(b) (i) Find $L\left[\frac{e^{at}-\cos 6t}{t}\right]$. (8)

(ii) Solve $\frac{d^2y}{dt^2}+y=\sin 2t$, $y(0)=0$, $y'(0)=0$ using Laplae transform. (8)

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