UNIT- II
INTERPOLATION AND APPROXIMATION

PROBLEMS BASED ON NEWTON FORWARD AND BACKWARD DIFFERENCE FORMULA

Newton's Forward Formula

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \cdots \ldots \ldots \ldots \ldots ..$$

$u = \frac{x-x_0}{h}$ *and* $h = x_1 - x_0$

Newton's Backward Formula

$$y(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots \ldots \ldots \ldots \ldots ..$$

$h = x_1 - x_0$ $v = \frac{x-x_n}{h}$

1. ***Find the value of*** y ***when*** $x = 5$ ***for the following data.***

x	4	6	8	10
y	1	3	8	10

Given :

$x_0 = 4 \qquad y_0 = 1$
$x_1 = 6 \qquad y_1 = 3$
$x_2 = 8 \qquad y_2 = 8$
$x_3 = 10 \qquad y_3 = 10$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
4	1			
		$3 - 1 = 2$ (Δy_0)		
6	3		$5 - 2 = 3(\Delta^2 y_0)$	
		$8 - 3 = 5$		$-3 - 3 = -6(\Delta^3 y_0)$
8	8		$2 - 5 = -3$	
		$10 - 8 = 2$		
10	10			

$h = x_1 - x_0 = 6 - 4 = 2$

$$u = \frac{x - x_0}{h} = \frac{x - 4}{2} = \frac{5 - 4}{2} = \frac{1}{2} = 0.5$$

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$y(5) = 1 + \frac{0.5}{1!}(2) + \frac{0.5(0.5-1)}{2!}(3) + \frac{.5(.5-1)(.5-2)}{3!}(-6)$$

$$y(5) = 1 + 1 + \frac{0.5(-0.5)}{2}(3) + \frac{.5(-0.5)(-1.5)}{6}(-6)$$

$$y(5) = 1 + 1 + \frac{-0.25}{2}(3) - (-0.25)(-1.5)$$

$$y(5) = 1 + 1 - 0.375 - 0.375 = 1.25$$

2. ***Find the value of y when x = 2 for the following data.***

x	0	5	10	15
y	14	379	1444	3584

Given :

$x_0 = 0$ $\quad y_0 = 14$

$x_1 = 5$ $\quad y_1 = 379$

$x_2 = 10$ $\quad y_2 = 1444$

$x_3 = 15$ $\quad y_3 = 3584$

$$h = x_1 - x_0 = 5 - 0 = 5$$

$$u = \frac{x - x_0}{h} = \frac{x - 0}{5} = \frac{2 - 0}{5} = \frac{2}{5} = 0.4$$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	14			
		$379-14=365$ (Δy_0)		
5	379		$1065-365=700$ $(\Delta^2 y_0)$	
		$1444-379=1065$		$1075-700=375$ $(\Delta^3 y_0)$
10	1444		$2140-1065=1075$	
		$3584-1444=2140$		
15	3584			

$$y(x)=y_0+\frac{u}{1!}\Delta y_0+\frac{u(u-1)}{2!}\Delta^2 y_0+\frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

$$y(2)=14+\frac{0.4}{1!}(365)+\frac{0.4(0.4-1)}{2!}(700)+\frac{0.4(0.4-1)(0.4-2)}{3!}(375)$$

$$y(2)=14+146+\frac{(-0.24)}{2}(700)+\frac{(-0.384)}{6}(375)$$

$$y(2)=14+146-84+24=100$$

Newton's Backward Formula

$$y(x)=y_n+\frac{v}{1!}\nabla y_n+\frac{v(v+1)}{2!}\nabla^2 y_n+\frac{v(v+1)(v+2)}{3!}\nabla^3 y_n+\cdots\ldots\ldots\ldots\ldots$$

$h=x_1-x_0$ $\quad v=\frac{x-x_n}{h}$

3. *Find the value of* y *when and* $x=43$ & $x=84$ *for the*

following data.

***Solution*:**

$x_0 = 40 \qquad y_0 = 184$

$x_1 = 50 \qquad y_1 = 204$

$x_2 = 60 \qquad y_2 = 226$

$x_3 = 70 \qquad y_3 = 250$

$x_4 = 80 \qquad y_4 = 276$

$x_5 = 90 \qquad y_5 = 304$

x	40	50	60	70	80	90
y	184	204	226	250	276	304

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
40	184					
		$204 - 184 = 20$				
50	204		2			
		$226 - 204 = 22$				
60				0		
	226		2			
70					0	
		$250 - 226 = 24$		0		0
	250				0	
80			2			
	276			0		
		$276 - 250 = 26$				
90			2			
	304	$304 - 276 = 28$				

(i) when $x = 43$

$h = x_1 - x_0 = 5 - 40 = 10$

$$u = \frac{x - x_0}{h} = \frac{x - 40}{10} = \frac{43 - 40}{10} = \frac{3}{10} = 0.3$$

$$y(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0$$

43=184+0.31!(20)+0.30.3−12!(2)+0.30.3−10.3−23!(0)

$$y(43) = 184 + 6 + \frac{0.3(-0.7)}{2}(2) + 0$$

$$y(43) = 184 + 6 - 0.21$$

$$y(43) = 189.79$$

(ii) when $x = 84$

$h = x_1 - x_0 = 50 - 40 = 10$

$$v = \frac{x - x_n}{h} = \frac{x - 90}{10}\quad v = \frac{84 - 90}{10} = \frac{-6}{10} = -0.6$$

$$y(x) = y_n + \frac{v}{1!}\nabla y_n + \frac{v(v+1)}{2!}\nabla^2 y_n + \frac{v(v+1)(v+2)}{3!}\nabla^3 y_n + \cdots \ldots\ldots\ldots\ldots$$

$$y(84) = 304 + \frac{(-0.6)}{1!}(28) + \frac{(-0.6)(-0.6+1)}{2!}(2) + \frac{(-0.6)(-0.6+1)(-0.6+2)}{3!}(0) + \cdots$$

$$y(84) = 304 - 16.8 + \frac{(-0.6)(0.4)}{2}(2) + 0$$

$$y(84) = 304 - 16.8 - 0.24$$

$$y(84) = 286.96$$

UNIT- II
INTERPOLATION AND APPROXIMATION

PROBLEMS BASED ON CUBIC SPLINES FORMULA

4.4 Cubic Splines

Interpolating with a cubic spline

The cubic spline interpolation formula is

$$S(x) = y(x) = y = \frac{1}{6h}[(x_i - x)^3 M_{i-1} + (x - x_{i-1})^3 M_i]$$
$$+ \frac{1}{h}(x_i - x)[y_{i-1} - \frac{h^2}{6}M_{i-1}] + \frac{1}{h}(x - x_{i-1})[y_i - \frac{h^2}{6}M_i]$$

where

$$M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}[y_{i-1} - 2y_i + y_{i+1}]$$

n = number of data

i = number of intervals [i.e., $i = 1, 2, 3, , (n-1)$]

h = length of interval = interval length.

Note : If M_i and y_i'' values are not given, then assume $M_0 = M_n = 0$ $\left[\text{or } y_0'' = y_n'' = 0\right]$, and find $M_1, M_2, \cdots, M_{n-1}$ in 1st interval, 2nd interval, $\cdots$, $(n-1)^{th}$ interval value.

Note : Order of convergence of the cubic spline is 4.

Find the cubic spline approximation for the function f(x) given by the data:

x	0	1	2	3
$y=f(x)$	1	2	33	244

with $M_0 = 0 = M_3$. Hence estimate the value $f(0.5), f(1.5), f(2.5)$. {AU2010}

Solution: We know that cubic spline interpolation formula for $x_{i-1} \le x < x_i, i = 1,2,3$ is

$$S_i(x) = y(x) = y = \frac{1}{6h}\left[(x_i - x)^3 M_{i-1} + (x - x_{i-1})^3 M_i\right]$$

$$+\frac{1}{h}(x_i - x)\left[y_{i-1} - \frac{h^2}{6}M_{i-1}\right]$$

$$+\frac{1}{h}(x - x_{i-1})\left[y_i - \frac{h^2}{6}M_i\right] \qquad (1)$$

$$\text{where } M_{i-1} + 4M_i + M_{i+1} = \frac{6}{h^2}\left[y_{i-1} - 2y_i + y_{i+1}\right] \qquad (2)$$

n = number of data = 4

i = number of intervals = 3 i.e., $i = 1,2,3$.

h = length of inteval = 1

Solving (3)&(4), $(3) \Rightarrow 4M_1 + M_2 = 180$

$$4 \times (4) \Rightarrow 4M_1 + 16M_2 = 4320$$

$$\text{i.e., } (3) + 4 \times (4) \Rightarrow -15M_2 = 4140$$

$$\Rightarrow M_2 = 276$$

$$(3) \Rightarrow 4M_1 = 180 - 276$$

$$\Rightarrow M_1 = -24$$

To find Cubic spline

When $i = 1$,

Cubic spline in $x_{i-1} \le x \le x_i$

$$\text{i.e., } x_0 \le x \le x_1$$

$$\text{i.e., } 0 \le x \le 1$$

i.e., Cubic spline in $0 \le x \le 1$ is

$$y_1(x) = S_1(x) = \frac{1}{6(1)}\left[(x_1 - x)^3 M_0 + (x - x_0)^3 M_1\right]$$

$$+ \frac{1}{1}(x_1 - x)\left[y_0 - \frac{1}{6}M_0\right] + \frac{1}{1}(x - x_0)\left[y_1 - \frac{1}{6}M_1\right]$$

$$= \frac{1}{6}\left[(1 - x)^3 (0) + (x - 0)^3 (-24)\right]$$

$$+ (1 - x)[1 - 0] + (x - 0)[2 - (-24)]$$

$$= -4x^3 + (1 - x) + 6x$$

$$= -4x^3 + 5x + 1$$

When $i = 2$,

Cubic spline in $x_{i-1} \le x \le x_i$

i.e., $x_1 \le x \le x_2$

i.e., $1 \le x \le 2$

i.e., Cubic spline in $1 \le x \le 2$ is

$$y_2(x) = S_2(x) = \frac{1}{6(1)}\left[(x_2 - x)^3 M_1 + (x - x_1)^3 M_2\right]$$

$$+ \frac{1}{1}(x_2 - x)\left[y_1 - \frac{1}{6}M_1\right] + \frac{1}{1}(x - x_1)\left[y_2 - \frac{1}{6}M_2\right]$$

$$= \frac{1}{6}\left[(2 - x)^3(-24) + (x - 1)^3(276)\right]$$

$$+ (2 - x)\left[2 - \frac{1}{6}(-24)\right] + (x - 1)\left[33 - \frac{1}{6}(276)\right]$$

$$= -4(2 - x)^3 + 46(x - 1)^3 + 6(2 - x) - 13(x - 1)$$

$$= 50x^3 - 162x^2 + 162x - 53$$

When $i = 3$,

Cubic spline in $x_{i-1} \le x \le x_i$

i.e., $x_2 \le x \le x_3$

i.e., $2 \le x \le 3$

i.e., Cubic spline in $2 \le x \le 3$ is

$$y_3(x) = S_3(x) = \frac{1}{6(1)}\left[(x_3 - x)^3 M_2 + (x - x_2)^3 M_3\right]$$

$$+ \frac{1}{1}(x_3 - x)\left[y_2 - \frac{1}{6}M_2\right] + \frac{1}{1}(x - x_2)\left[y_3 - \frac{1}{6}M_3\right]$$

$$= \frac{1}{6}\left[(3 - x)^3 (276) + 0\right]$$

$$+ (3 - x)\left[33 - \frac{1}{6}(276)\right] + (x - 2)[244 - 0]$$

$$= 46\left(27 - x^3 + 9x^2 - 27x\right) - 13(3 - x) + 244x - 488$$

$$= -46x^3 + 414x^2 - 985x + 715$$

$\therefore$ Cubic spline is

$$S(x) = \begin{cases} S_1(x) = y_1(x) = -4x^3 + 5x + 1, & 0 \le x \le 1 \\ S_2(x) = y_2(x) = 50x^3 - 162x^2 + 167x - 53, & 1 \le x \le 2 \\ S_3(x) = y_3(x) = -46x^3 + 414x^2 - 985x + 715, & 2 \le x \le 3 \end{cases}$$

When $x = 0.5, y_1(x = 0.5) = S_1(x = 0.5) = -4(0.5)^3 + 5(0.5)^2 + 1 = 3$

When $x = 1.5, y_2(x = 1.5) = S_2(x = 1.5) = 50(1.5)^3 - 162(1.5)^2 + 167(1.5) - 53 = 1.75$

When $x = 2.5, y_3(x = 2.5) = S_3(x = 2.5) = -46(2.5)^3 + 414(2.5)^2 - 985(2.5) + 715 = 121.25$

From the following table

x	1	2	3
$y = f(x)$	−8	−1	18

. Find cubic spline and compute $y(1.5), y'(1), y(2.5)$ and $y'(3)$.

Solution:

$$S(x) = \begin{cases} S_1(x) = y_1(x) = 3(x-1)^3 + 4x - 12, & 1 \le x \le 2 \\ S_2(x) = y_2(x) = 3(3-x)^3 + 22x - 48, & 2 \le x \le 3 \end{cases}$$

&

$$y(x = 1.5) = S_1(x = 1.5) = -\frac{45}{8}, y'(x = 1) = S_1'(x = 1) = 4$$

$$y(x = 2.5) = S_2(x = 2.5) = 7.375, y'(x = 3) = S_2'(x = 3) = 22.$$

Solution: Hint :

$$S(x) = y(x) = \begin{cases} S_1(x) = y_1(x) = -4x^3 + 5x + 1, & x \in [0,1] \\ S_2(x) = y_2(x) = 50x^3 - 162x^2 + 1670x - 53, & x \in [1,2] \\ S_3(x) = y_3(x) = -46x^3 + 414x^2 - 985x + 715, & x \in [2,3] \end{cases}$$

$$f(x) = -46x^3 + 414x^2 - 985x + 715 \quad x \in [2,3]$$

$$f'(x) = -138x^2 + 828x - 985$$

$$f'(x = 2.5) = -138(2.5)^2 + 828(2.5) - 985$$

$$= 222.5$$

Fit a natural cubic spline for the following data:

x	0	1	2	3
$y = f(x)$	1	4	0	–2

{AU 2008}

Solution: Assume $M_0 = 0 = M_3$.

$$S(x) = \begin{cases} S_1(x) = y_1(x) = -2x^3 + 5x + 1, & [0,1] \\ S_2(x) = y_2(x) = 3x^3 - 15x^2 + 20x - 4, & [1,2] \\ S_3(x) = y_3(x) = -x^3 + 9x^2 - 28x + 28, & [2,3] \end{cases}$$

4.4.3 Anna University Questions

1. If $f(0) = 1, f(1) = 2, f(2) = 33$ and $f(3) = 244$, find a cubic spline approximation, assuming $M(0) = M(3) = 0$. Also, find $f(2.5)$. (AM10)

4. The following values of x and y are given:

x:	1	2	3	4
y:	1	2	5	11

Find the cubic splines and evaluate $y(1.5)$ and $y'(3)$ (MJ12)

Solution: Hint :

$$S(x) = y(x) = \begin{cases} S_1(x) = y_1(x) = \frac{1}{3}\left(x^3 - 3x^2 + 5x\right), & x \in [1,2] \\ S_2(x) = y_2(x) = \frac{1}{3}\left(x^3 - 3x^2 + 5x\right), & x \in [2,3] \\ S_3(x) = y_3(x) = \frac{1}{3}\left(-2x^3 + 24x^2 - 76x + 81\right), & x \in [3,4] \end{cases}$$

$$y(x) = \frac{1}{3}\left(x^3 - 3x^2 + 5x\right) \Rightarrow y(1.5) = 1.375, \quad x \in [1,2]$$

$$y'(x) = \frac{1}{3}\left(3x^2 - 6x + 5\right) \Rightarrow y'(3) = 4.666666667, \quad x \in [2,3]$$

(or)

$$y'(x) = \frac{1}{3}\left(-6x^2 + 48x - 76\right) \Rightarrow y'(3) = 4.666666667, \quad x \in [3,4]$$

5. Obtain the cubic spline for the following data to find $y(0.5)$. (ND12)

x:	−1	0	1	2
y:	−1	1	3	35

Solution: Hint :

$$S(x) = y(x) = \begin{cases} S_1(x) = y_1(x) = -2x^3 - 6x^2 - 2x + 1, & x \in [-1,0] \\ S_2(x) = y_2(x) = 10x^3 - 6x^2 - 2x + 1, & x \in [0,1] \\ S_3(x) = y_3(x) = -8x^3 + 48x^2 - 56x + 19, & x \in [1,2] \end{cases}$$

6. Using cubic spline, compute $y(1.5)$ from the given data. (MJ13)

x :	1	2	3
y :	−8	−1	18

Solution: Hint :

$$S(x) = y(x) = 3x^3 - 9x^2 + 13x - 15$$

$$y(1.5) = 3(1.5)^3 - 9(1.5)^2 + 13(1.5) - 15 = -\frac{45}{8} = -5.625, \quad x \in [1,2]$$

9. Obtain the cubic spline approximation for the function $y = f(x)$ from the following data, given that $y_0'' = y_3'' = 0$. (ND14)

x	−1	0	1	2
y	−1	1	3	35

10. Find the cubic Spline interpolation. (AU N/D, 2007,AM2014)

x	1	2	3	4	5
$f(x)$	1	0	1	0	1

11. Given the following table, find $f(2.5)$ using cubic spline functions : (AU May/June 2007)

x	1	2	3	4
$f(x)$	0.5	0.3333	0.25	0.2

Solution : [Ans: $S_2(2.5) = 0.2829$]

UNIT- II

INTERPOLATION AND APPROXIMATION

4.1 Introduction

The process of finding the value of a function inside the given range of discrete points are called interpolation. We have

1. Interpolation with unequal intervals
2. Interpolation with equal intervals

Methods of Equal or Unequal intervals	Methods of only Equal intervals
Lagrange's interpolation (Lagrange & Inverse Lagrange)	Newton's forward difference method
Newton's divided difference interpolation	Newton's backward difference method
Cubic Splines	

4.2 Lagrange's interpolation

Lagrangian Polynomials(Equal and unequal intervals):

Let $y = f(x)$ be a function which takes the values $y = y_0, y_1, \cdots, y_n$ corresponding to $x = x_0, x_1, \cdots, x_n$.

Lagrange's interpolation formula(x given, finding y in terms of x)

$$y = y(x) = f(x) = \frac{(x - x_1)(x - x_2)\ldots(x - x_n)}{(x_0 - x_1)(x_0 - x_2)\ldots(x_0 - x_n)} y_0 + \frac{(x - x_0)(x - x_2)\ldots(x - x_n)}{(x_1 - x_0)(x_1 - x_2)\ldots(x_1 - x_n)} y_1 + \ldots + \frac{(x - x_0)(x - x_1)\ldots(x - x_{n-1})}{(x_2 - x_0)(x_2 - x_1)\ldots(x - x_{n-1})} y_n$$

Inverse Lagrange's interpolation formula (y given, finding x in terms of y)

$$x = x(y) = f(y) = \frac{(y-y_1)(y-y_2)\ldots(y-y_n)}{(y_0-y_1)(y_0-y_2)\ldots(y_0-y_n)}x_0 + \frac{(y-y_0)(y-y_2)\ldots(y-y_n)}{(y_1-y_0)(y_1-y_2)\ldots(y_1-y_n)}x_1 + \ldots + \frac{(y-y_0)(y-y_1)\ldots(y-y_{n-1})}{(y_2-y_0)(y_2-y_1)\ldots(y-y_{n-1})}x_n$$

Note: Lagrange's interpolation formula can be used for equal and unequal intervals.

PROBLEMS BASED ON LAGRANGIAN POLYNOMIALS

(1) Using Lagrange's interpolation, calculate the profit in the year 2000 from the following data

Year :	1997	1999	2001	2002
Profit in lakhs of Rs	43	65	159	248

Solution : $x_0 = 1997$ $x_1 = 1999$ $x_2 = 2001$ $x_3 = 2002$

$y_0 = 1997$ $y_1 = 1999$ $y_2 = 2001$ $y_3 = 2002$

By Lagrange's interpolation formula we have

$$Y=f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

$$Y=f(x) = \frac{(x-1999)(x-2001)(x-2002)}{(1997-1999)(1997-2001)(1997-2002)}43 + \frac{(x-1997)(x-2001)(x-2002)}{(1999-1997)(1999-2001)(1999-2002)}65 + \frac{(x-1997)(x-1999)(x-2002)}{(2001-1997)(2001-1999)(2001-2002)}159 + \frac{(x-1997)(x-1999)(x-2001)}{(2002-1997)(2002-1999)(2002-2001)}248$$

Y=f(x) =$\frac{(2000-1999)(2000-2001)(2000-2002)}{(1997-1999)(1997-2001)(1997-2002)}43+\frac{(2000-1997)(2000-2001)(2000-2002)}{(1999-1997)(1999-2001)(1999-2002)}65+$

2000−19972000−19992000−20022001−19972001−19992001−2002159+2000−19972000−19992000−20012002−19972002−19992002−2001248

Y=f(x) =$\frac{(1)(-1)(-2)}{(-2)(-4)(-5)}43+\frac{(3)(-1)(-2)}{2(-2)(-3)}65+\frac{(3)(1)(-2)}{4(2)(-1)}159+\frac{(3)(1)(-1)}{5(3)(1)}248$

Y=f(x) =$\frac{-43}{20}+\frac{65}{2}+\frac{477}{4}-\frac{248}{5}$

=100

2. For the given values evaluate f(9) using Lagrange's interpolation

Solution :

x	5	7	11	13	17
f(x)	150	392	1452	2366	5202

Solution :

x_0 =5 x_1 =7 x_2 =11 x_3 =13 x_4 =17

y_0 =150 y_1 =392 y_2 =1452 y_3 =2366 y_4 =5202

By Lagrange's interpolation formula we have

Y=f(x) = $\frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)}y_0+\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)}y_1+$

$\frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}y_2$

$+\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)}y_3+\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)}y_4$

$$Y=f(9)=\frac{(9-7)(9-11)(9-13)(9-17)}{(5-7)(5-11)(5-13)(5-17)}150+\frac{(9-5)(9-11)(9-13)(9-17)}{(7-5)(7-11)(7-13)(7-17)}392+$$

$$\frac{(9-5)(9-7)(9-13)(9-17)}{(11-5)(11-7)(11-13)(11-17)}1452$$

$$+\frac{(9-5)(9-7)(9-11)(9-17)}{(13-5)(13-7)(13-11)(13-17)}2366+\frac{(9-5)(9-7)(9-11)(9-13)}{(17-5)(17-7)(17-11)(17-13)}5202$$

$$=\frac{(2)(-2)(-4)(-8)}{(-2)(-6)(-8)(-12)}$$

$$=-\frac{50}{3}+\frac{3136}{15}+\frac{3872}{3}$$

=810

4. Using Lagrange interpolation formula, find $f(4)$ given that $f(0) = 2, f(1) = 3, f(2) = 12, f(15) = 3587$.

Solution: Given

x	$x_0 = 0$	$x_1 = 1$	$x_2 = 2$	$x_3 = 15$
$y = f(x)$	$y_0 = 2$	$y_1 = 3$	$y_2 = 12$	$y_3 = 3587$

Lagrange interpolation formula is

$$y = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} y_0 + \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} y_1 + \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} y_2 + \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} y_3$$

$$f(4) = \frac{(4-1)(4-2)(4-15)}{(0-1)(0-2)(0-15)}(2) + \frac{(4-0)(4-2)(4-15)}{(1-0)(1-2)(1-15)}(3) + \frac{(4-0)(4-1)(4-15)}{(2-0)(2-1)(2-15)}(12) + \frac{(4-0)(4-1)(4-2)}{(15-0)(15-1)(15-2)}(3587)$$

$$= \frac{(3)(2)(-11)}{(-1)(-2)(-15)}(2) + \frac{(4)(2)(-11)}{(1)(-1)(-14)}(3) + \frac{(4)(3)(-11)}{(2)(1)(-13)}(12) + \frac{(4)(3)(2)}{(15)(14)(13)}(3587)$$

$$= 77.99 = 78$$

Using Lagrange's formula, prove that

$$y_1 = y_3 - 0.3(y_5 - y_{-3}) + 0.2(y_{-3} - y_{-5}).$$

Solution: From the equation, the values of x are

$x_0 = -5$	$x_1 = -3$	$x_2 = 3$	$x_3 = 5$
$y_0 = y_{-5}$	$y_1 = y_{-3}$	$y_2 = y_3$	$y_3 = y_5$

The x values are not equally space, so use Lagrange's formula to find $y = f(x)$.

Lagrange's formula for a set of 4 pair of values is

$$y = y_x = f(x) = \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} y_0 + \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} y_1$$

$$+ \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} y_2 + \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)} y_3$$

$$= \frac{(x + 3)(x - 3)(x - 5)}{(-5 + 3)(-5 - 3)(-5 - 5)} y_{-5} + \frac{(x + 5)(x - 3)(x - 5)}{(-3 + 5)(-3 - 3)(-3 - 5)} y_{-3}$$

$$+ \frac{(x + 5)(x + 3)(x - 5)}{(3 + 5)(3 + 3)(3 - 5)} y_3 + \frac{(x + 5)(x + 3)(x - 3)}{(5 + 5)(5 + 3)(5 - 3)} y_5$$

Put $x = 1$, we get

$$y_1 = \frac{(1 + 3)(1 - 3)(1 - 5)}{(-5 + 3)(-5 - 3)(-5 - 5)} y_{-5} + \frac{(1 + 5)(1 - 3)(1 - 5)}{(-3 + 5)(-3 - 3)(-3 - 5)} y_{-3}$$

$$+ \frac{(1 + 5)(1 + 3)(1 - 5)}{(3 + 5)(3 + 3)(3 - 5)} y_3 + \frac{(1 + 5)(1 + 3)(1 - 3)}{(5 + 5)(5 + 3)(5 - 3)} y_5$$

$$= -0.2y_{-5} + 0.5y_{-3} + y_3 - 0.3y_5$$

$$= -0.2y_{-5} + 0.2y_{-3} + 0.3y_{-3} + y_3 - 0.3y_5$$

$$y_1 = y_3 - 0.3(y_5 - y_{-3}) + 0.2(y_{-3} - y_{-5})$$

Anna University Questions

1. Use Lagrange's formula to find a polynomial which takes the values $f(0) = -12$, $f(1) = 0$, $f(3) = 6$ and $f(4) = 12$. Hence find $f(2)$. (AM10)

   Solution: $\left[f(x) = x^3 - 7x^2 + 18x - 12 \Rightarrow f(2) = 4\right]$

2. Using Lagrange's interpolation formula to fit a polynomial to the given data $f(-1) = -8$, $f(0) = 3$, $f(2) = 1$ and $f(3) = 12$. Hence find the valur of $f(1)$. (ND10)

   Solution: $\left[f(x) = 2x^3 - 6x^2 + 3x + 3 \Rightarrow f(1) = 2\right]$

3. Find the expression of $f(x)$ using Lagrange's formula for the followin data. (AM11)

x :	0	1	4	5
$f(x)$:	4	3	24	39

   Solution: $\left[f(x) = 2x^2 - 3x + 4\right]$