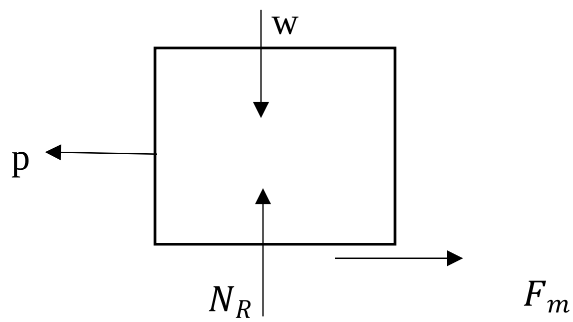
Unit –V

FRICTION

Friction:

When the two surface contact with each other and one surface tense to move with respect to another surface.

The tangent force developed in the contact6 surface and in the opposite reaction.



Types of Friction:

- Dry Friction or coulomb friction
- Fluid friction

Dry friction:

It is referred to the friction which is developed b/w two dry surface

Dry friction types

1. Static friction
2. Dynamic friction

Static friction:

It is the friction which experience by object when it is rest.

Dynamic friction:

It is the friction which experience by object in moving condition.

Dynamic friction further classified by two types

1. Sliding friction
2. Rolling friction

Coefficient of Friction:

It is the ratio b/w limiting friction and the normal reaction [normal friction] is called coefficient of friction.it is denoted by letter μ

$$\mu = \frac{F_N}{R}$$

Coulomb Law of Friction [Dry Law]

1. Law of static friction
2. Law of dynamic friction

Law of static friction:

* The frictional force is always act opposite direction to be movement of the object or body
* The frictional force does not depends upon the size and shape of the object
* The frictional force depends the degree of roughness of the contact surface b/w the two object
* The frictional force is equal to force applied to the body or object
* Limiting friction is directly propositional to the normal friction

$$F_N \alpha N_R$$

Coulomb Law of Dynamic Friction:

* The frictional force is always opposite to the movement of the object.
* The magnitude of the dynamic friction the contact ratio to normal reaction b/w two system
* The coefficient of kinetic friction less then coefficient of static friction

Angle of friction:

Angle of friction is Angle made by the resultant with the normal reaction and frictional force.

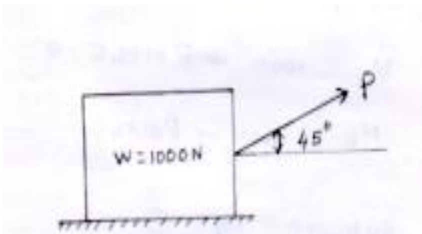
$$tan\emptyset = \frac{F_N}{N_R}$$

$$\emptyset = tan^{-1}\left[\frac{F_N}{N_R}\right]$$

Problem based on Friction:

1. A body weighting 1000N is lying on a horizontal plane. Determine the necessary force to move the body along the plane if the force is applied at angle of 45° to the horizontal with the coefficient of friction 0.24

Given:

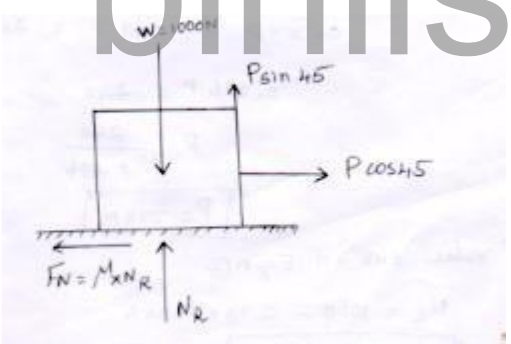


Coefficient of friction $\mu = 0.24$

Weight of body $w = 1000\ N$

To find

Soln



Sum of X Direction Force + ⟶ - ⟵

$\sum F_x = 0$

$Pcos45 - F_N = 0$

$Pcos45 - \mu \times N_R = 0$

$Pcos45 - 0.24 \times N_R = 0$ ---------------- (1)

Sum of Y direction force ↑ + ↓ -

$\sum F_y = 0$

$N_R - 1000 + Psin45 = 0$

$N_R = 1000 + psin45 = 0$

N_R value sub in Eqn(1)

$Pcos45 - 0.24 \times [1000 - Psin45] = 0$

$Pcos45 - 240 + [Psin45 \times 0.24] = 0$

$0.707P + 0.169\, P = 240$

$0.876P = 240$

$$P = \frac{240}{0.876}$$

$P = 273N$

P value sub in eqn (2)

$N_R = 1000 - 273 \times sin45$

$N_R = 806.95\, N$

Problem: 2

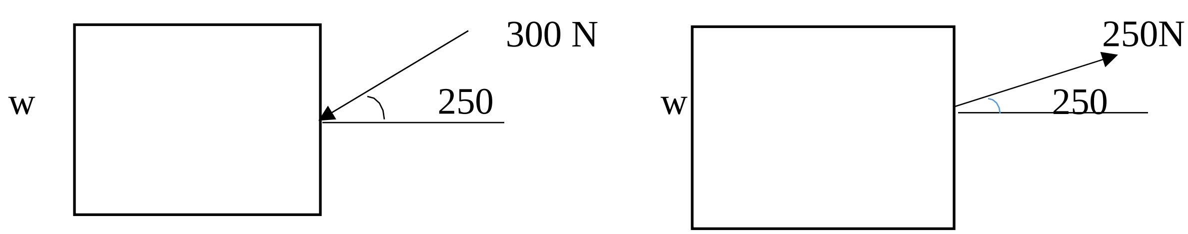
A pull of 250N inclined at 25° to the horizontal plane is required just to move a body kept on a rough horizontal plane. But the push required just to move the body is 300N. If the push is inclined at 25° to the horizontal. Find the weight of the body and the coefficient of friction b/w the body and the plane.

Given:

$P_1 = pull\ load = 250\ N\ at\ 25°$

$P_2 = push\ load = 300\ N\ at\ 25°$

To find

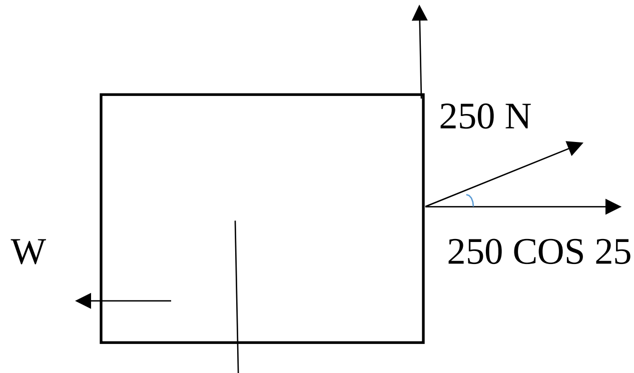


Weight and coefficient of friction

Soln

Case (i)

Free body diagram



Sum of x directional force + → ← -

$\sum F_x = 0$

$250 \cos 25° - F_N = 0$

$250 \cos 25° - \mu \times N_{R1} = 0$

$226.57 - \ \mu \times N_{R1} = 0$

$-\mu \times N_{R1} \ = -226.57$

$\mu \times N_{R1} \ = 226.57$------------------- (1)

Sum of Y directional force ↓ - ↑ +

$\sum F_y = 0$

$N_{R1} - W + 250 \ sin25 = 0$

$N_{R1} \ = W - 250 sin25$

$N_{R1} \ = W - 105.65$----------------- (2)

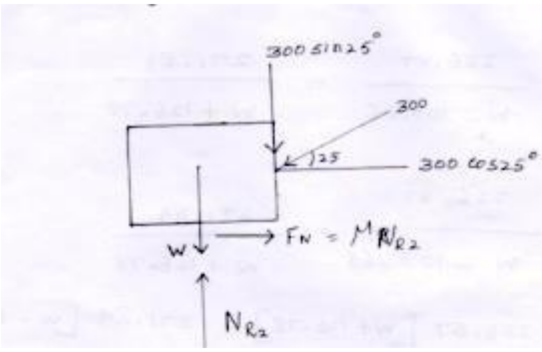
Sub eqn (1), from eqn (1)

$\mu N_{R1} \ = 226.57$

$$\mu = \frac{226.57}{N_{R1}}$$

$\mu = \frac{226.57}{W - 105.65}$ -------------- (3)

Case (2) Free body diagram



Sum of X Directional force $\sum F_x = 0$

$F_N - 300cos25° = 0$

$\mu N_{R2} = 300cos25°°$

$\mu N_{R2} = 271.89N$------------ (4)

Sum of vertical force[Y direction] $\sum F_y = 0$

$N_{R2} - W - 300sin25° = 0$

$N_{R2} = W + 300sin25°$

$N_{R2} = W + 126.78$

N_{R2} value sub in Eqn (4)

$\mu N_{R2} = 271.89$

$$\mu = \frac{271.89}{N_{R2}}$$

$\mu = \frac{271.89}{W+126.78}$ ---------- (5)

Eqn (3) =Eqn (5)

$$\frac{226.57}{W - 105.65} = \frac{271.89}{W + 126.78}$$

$226.57[W + 126.78] = 271.89[W - 105.65]$

$226.57W + 28.72 \times 10^3 = 271.89W - 28.72 \times 10^3$

$226.57W - 271.89W = -28.72 \times 10^3 - 28.72 \times 10^3$

$-45.32W = -57.44 \times 10^3$

$$W = \frac{57.44 \times 10^3}{45.32}$$

$W = 1267.43N$

W value sub in eqn (3)

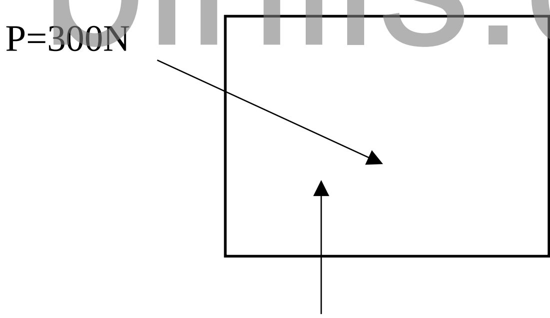
$$\mu = \frac{226.57}{W - 105.65}$$

$$\mu = \frac{226.57}{1267.43 - 105.65}$$

$\mu = 0.195$

Problem 3

Calculate the static coefficient of friction μ_sb/w the block shown in fig having a mass of 75kg and the surface. Also find the magnitude and direction of the friction force if the force P applied is inched at 45° to the horizontal and $\mu_s = 0.30$



Given:

Weight= $75\ kg = 75 \times 9.81 = 735.75N$

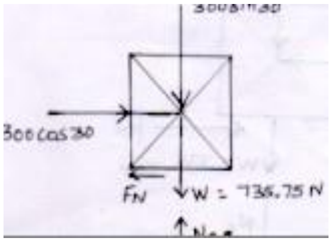
To find

Case (i) Coefficient of friction μ_s

Case (ii) Frictional force 'F_N', Direction Ø

Soln

Case (i) free body diagram



Sum of all the X direction force $\sum F_x = 0$

$300\cos 30 - F_N = 0$

$300\cos 30 - \mu N_R = 0$

$\mu N_R = -300 cos30$

$\mu N_R = 259.8$---------------- (1)

Sum of all the Y direction force $\sum F_y = 0$

$-300\sin 30 - 735.75 + N_R = 0$

$N_R = 300 sin30 + 735.75$

$N_R = 885.75\ N$

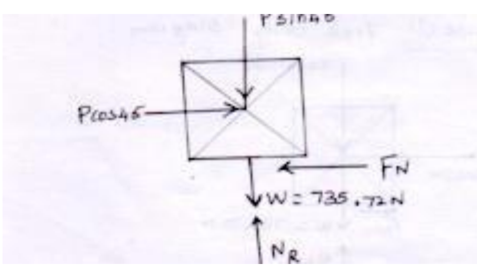
N_R value sub in eqn(i)

$\mu N_R = 259.8$

$$\mu = \frac{259.8}{N_R} = \frac{259.8}{885.75}$$

$\mu = 0.29$

Case (ii) free body diagram



Sum of all the horizontal force [X direction] $\sum F_x = 0$

$P\cos 45 - F_N = 0$

$-F_N = -P \cos 45$

$F_N = P \cos 45$

$\mu N_R = p\ cos45$

$N_R = \frac{pcos45}{\mu} = \frac{pcos45}{0.3}$

$N_R = 2.35p$

Sum of all the Direction force $\sum F_y = 0$

$-p\ sin45 - 732.72 + N_R = 0$

$-p \times 0.7 - 735.72 + 2.35p = 0$

$-0.7p + 2.35p = 735.72$

$1.64p = 735.72$

$p = \frac{735.72}{1.64}$

$p = 447.81N$

$N_R = 2.35p$

$N_R = 2.35 \times 447.81$

$N_R = 1052.37N$

$F_N = \mu \times R = 0.3 \times 1052.37$

F_N=315.71N

Direction Ø

$\emptyset = tan^{-1} [\frac{F_N}{N_R}]$

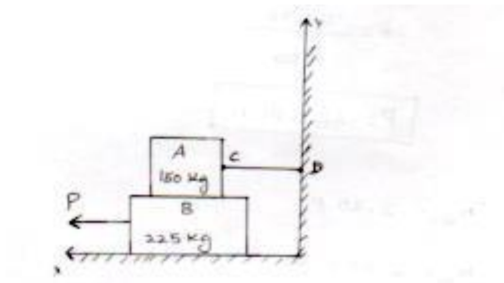
$\emptyset = tan^{-1} [\frac{315.71}{1052.37}]$

$\emptyset = 16°41'$

Problem 4:

Determine the smallest force P required to move the block B shown n fig below (i) block A is restrained by cable CD as shown in fig. (ii) Cable CD is removed. Take

$\mu_s = 0.30 \; and \;\; \mu_k = 0.25$



Given:

$W_A = 150kg = 150 \times 9.81 = 1471.5N$

$W_B = 225kg = 225 \times 9.81 = 2207.25N$

$\mu_s = 0.3$

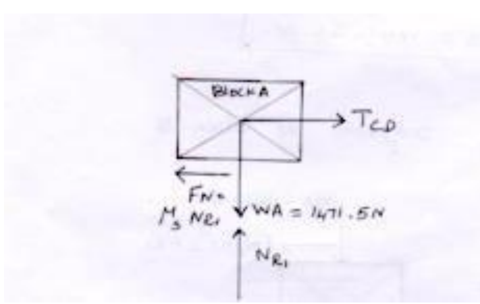
$\mu_k = 0.25$

To find

Force P

Soln

Block A is restrained by cable CD



Sum of X direction force $\sum F_x = 0$

$T_{CD} - F_N = 0$

$T_{CD} - \mu_s N_{R1} = 0$

$T_{CD} = \mu_s N_{R1}$

$T_{CD} = 0.3 N_{R1}$------------- (1)

Sum of vertical [Y direction force] $\sum F_y = 0$

$N_{R1} - W_A = 0$

$N_{R1} = W_A$

$N_{R1} = 1471.5N$

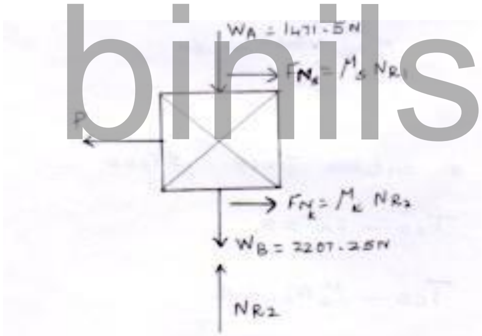
N_{R1} value sub in eqn(i)

$T_{CD} = 0.3\ R_1$

$T_{CD} = 0.3 \times 1471.3$

$T_{CD} = 441.45\ N$

Free body diagram of block B



Sum of X direction force

$F_{N_s} + F_{N_k} - p = 0$

$\mu_s N_{R1} + \mu_k N_{R2} - p = 0$

$p = \mu_s N_{R1} + \mu_k N_{R2}$-------- (1)

Sum of Y direction force

$-1471.5 - 2207.25 + N_{R2} = 0$

$N_{R2} = 1471.5 + 2207.25$

$N_{R2} = 3678.75N$

N_{R2} value sub in Eqn (1)

$p = \mu_s N_{R1} + \mu_k N_{R2}$

$p = 0.3 \times 1471.5 + 0.25 \times 3678.75$

$p = 1361.14\ N$

(i) Cable CD is removed

Both block is removed

Both the block will consider as a single body

$p = F_N$

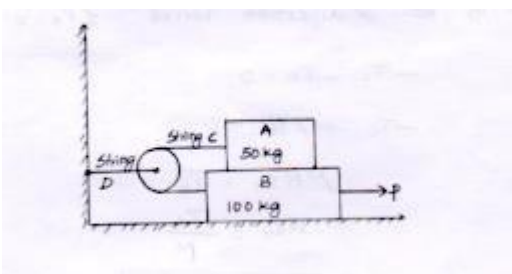
$p = \mu_k N_{R2} = 0.25 \times 3678.75$

$p = 919.68N$

Problem-6

Two blocks A and B of mass 5o kg and 10 kg respectively are connected by a string c which passes through a frictionless pulley connected with the fixed wall by another string D as shown in fig. Find the force P required to pull the block B. also find the tension in the string D.

Take coefficient of friction at all contact surface as 0.3°



Given:

Weight of block A $W_A = 50kg = 50 \times 9.81 = 490.5N$

Weight of block B $W_B = 100kg = 100 \times 9.81 = 981N$

Coefficient of friction$\mu = 0.3$

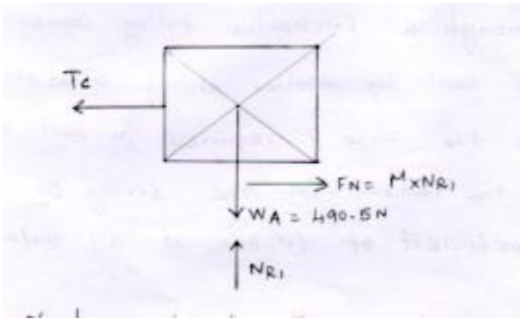
To find

(i) Force P

(ii) Tension in string T_D

Soln

Free body diagram of block A



Sum of x direction forces $\sum F_x = 0$

$-T_c$-$F_N = 0$

$-T_c + \mu N_{R1} = 0$

$\mu N_{R1} = T_c$

$N_{R1} = \frac{T_c}{\mu}$

$N_{R1} = \frac{T_c}{0.3}$ ----------- (1)

Sum of Y direction force $\sum F_Y = 0$

$-W_A + N_{R1} = 0$

$N_{R1} = W_A$

$N_{R1} = 490.5\ N$

N_{R1} value sub in Eqn (1)

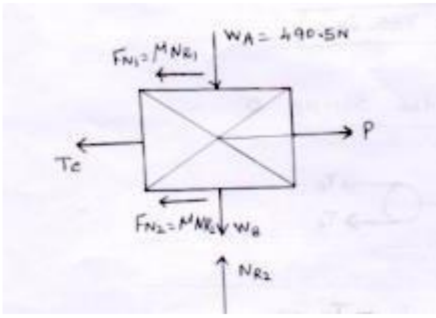
$N_{R1} = \frac{T_c}{0.3}$

$490.5 = \frac{T_c}{0.3}$

$T_c = 147.15\ N$

Consider block B

free body diagram



Sum of X direction force $\sum F_x = 0$

$p - T_c - F_{N1} - F_{N2} = 0$

$p - \mu N_{R1} + \mu N_{R2} = 0$

$p - 147.15 - 0.3 \times 490.15 - 0.3 \times N_{R2} = 0$

$p - 147.15 - 147.04 - 0.3 \times N_{R2} = 0$

$p - 294.19 - 0.3N_{R2} = 0$

$p = 294.19 - 0.3N_{R2} = 0$

$P = 294.19 + N_{R2}$------------- (2)

Sum of Y direction force $\sum F_Y = 0$

$N_{R2} - W_B - W_A = 0$

$N_{R2} - 981 - 490.5 = 0$

$N_{R2} = 1471.5N$

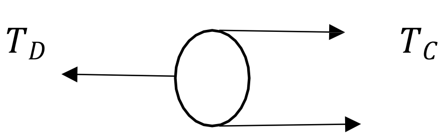
N_{R2} value sub in eqn(2)

$p = 294.19 + 0.3N_{R2}$

$p = 294.19 + 0.3 \times 1471.5$

$p = 735.64N$

Tension in the string D:



$\sum F_x = 0$

$T_c + T_c - T_D = 0$

$2T_c - T_D = 0$

$2 \times 147.15 - T_D = 0$

$294.3 - T_D = 0$

$-T_D = -294.3$

$T_D = 294.3N$

Problem -7

A force of 300 n is required just to move a block up a plane inclined at 20°.To the horizontal, the force being applied parallel to the plane shown in fig. if the inclination of the plane is increased to 25°, the force required just to move the block up is 340 N, [the force is acting parallel to the plane]. Determine the weight of the block and coefficient of friction.

Given:

Case (i)

Weight of body w=?

Force on body P= 300 N at 20°inclined on plane horizontal

Case (ii)

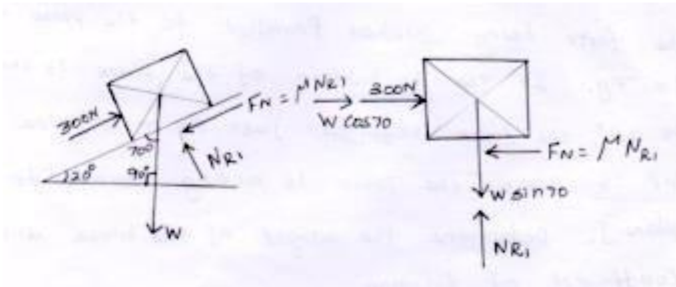
Force on body P= 340 N at 25°

To find:

Weight of body & coefficient of friction

Soln:

Case (i) free body diagram



Sum of X directional force $\sum F_X = 0$

$300 + wcos70 - F_N = 0$

$300 + wcos70 - \mu N_{R1} = 0$

$-N_{R1} = -[300 + wcos70]$

$\mu N_{R1} = 300 + w\cos 70$

$\mu = \frac{300+wcos70}{N_{R1}}$--------- (1)

Sum of all Y direction force $\sum F_y = 0$

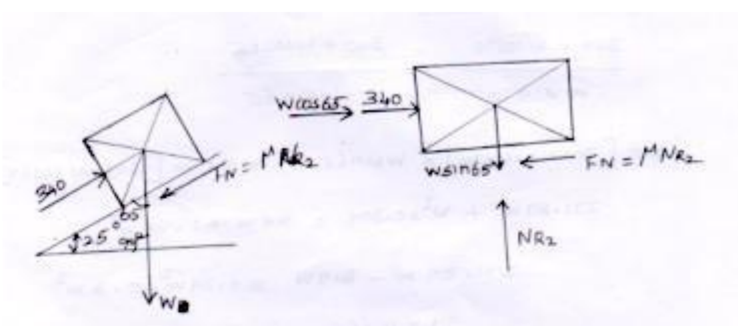
$N_{R1} - wsin70 = 0$

$N_{R1} = wsin70$------------ (2)

N_{R1} value sub in eqn (1)

$\mu = \frac{300+wcos70}{wsin70}$--------- (3)

Case (ii) consider block 2



Sum of all the X direction force $\sum F_X = 0$

$340 + w\cos 65 - F_N = 0$

$340 + wcos\ 65 - \mu N_{R2} = 0$

$-\mu N_{R2} = -[340 + wcos65]$

$\mu = \frac{340+wcos65}{N_{R2}}$--------- (4)

Sum of all the Y direction force $\sum F_Y = 0$

$N_{R2} - wsin65 = 0$

$N_{R2} = wsin65$-------------- (5)

Eqn (5) sub in eqn (4)

$\mu = \frac{340+w\cos 65}{w\sin 65}$------------- (6)

Eqn(3) =eqn (6)

$$\frac{300 + wcos70}{wsin70} = \frac{340 + w\cos 65}{w\sin 65}$$

$[300 + wcos70] \times wsin65 = wsin70[340 + w\ cos65]$

$271.89w + w^2 \times 0.309 = 319w + 0.39w^2$

$271.86w - 319w = 0.39w^2 - 0.3w^2$

$-47.11\ w = 0.09w^2$

$-47.11\ w = 0.09w$

$$w = \frac{-47.11}{0.09}$$

ans $w = -523.47\ N$

w value sub in eqn(3)

$$\mu = \frac{300 + wcos70}{wsin70}$$

$$\mu = \frac{300 + (-523.47)cos70}{(-523.47)sin70}$$

Ans $\mu = -0.24$

Problem 8

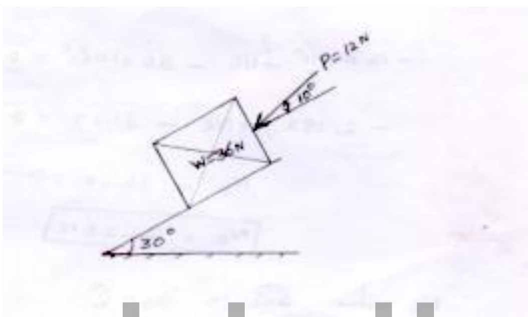
A block weighting 360 n is resting on a rough inclined plane having an inclination of 30°. A force of 12 N is applied at an angle of 10°up and the block is just on the point of moving down the plane. Determine the coefficient of friction

Given:

Block weight w=36 N

Inclination of the plane $\emptyset = 30°$

Force on block P=12 N at 10°

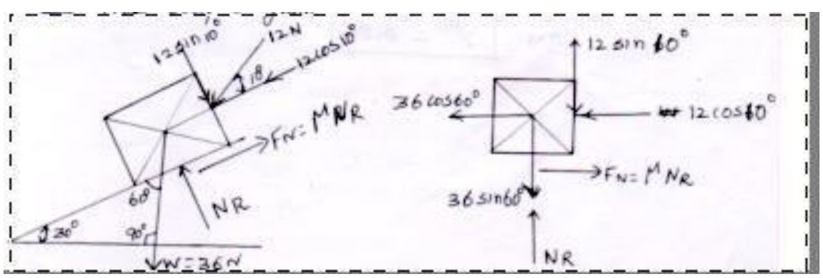


To find

Coefficient of frictionμ

Soln:

Free body diagram



Sum of X direction force $\sum F_X = 0$

$-12\ cos10 - 36cos60 + F_N$=0

$-11.87 - 18 + \mu N_R = 0$

$-29.87 + \mu N_R = 0$

$\mu N_R = 29.87$

$\mu = \frac{29.87}{N_R}$------ (1)

Sum of all the Y direction force $\sum F_y = 0$

$-12\ sin10° + N_R - 36\ sin60° = 0$

$-2.083 + N_R - 31.17 = 0$

$N_R - 33.25 = 0$

$N_R = 33.25\ N$

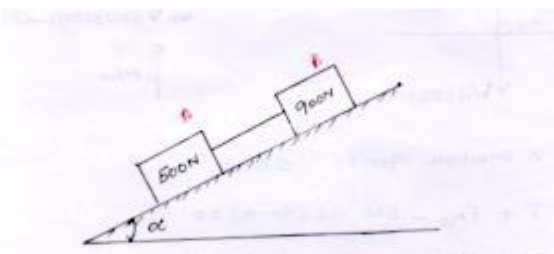
N_R value sub in Eqn (1)

$\mu = \frac{29.87}{N_R} = \frac{29.87}{33.25}$

$ans\ \mu = 0.89$

Problem 9

Two block of weight 500 N and 900 N connected by a rod are kept in an inclined plane is shown in fig. the rod is parallel to the plane. The coefficient of friction b/w 500 N block and the plane is 0.3 and that b/w 900 N block and the plane is 0.4 find the inclination of the plane with the horizontal and the rod when the motion down the plane is just about the start.



Given:

Weight of block A $W_A = 500\ N$

Weight of block B $W_B = 900\ N$

Coefficient of friction $\mu_A = 0.3$

Coefficient friction at block B $\mu_B = 0.4$

To find

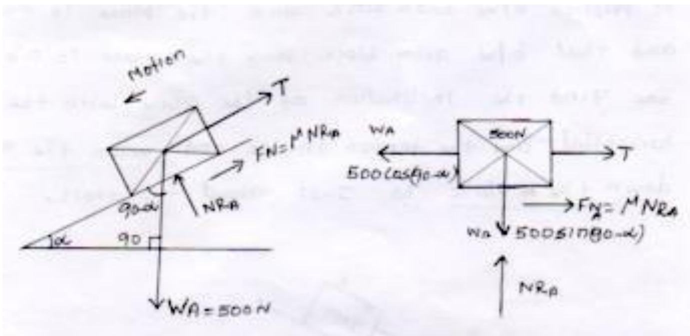
1. Inclination of the plane α

2. Tension

Soln:

Consider block 'A'

Free body diagram



Sum of X direction force $\Sigma F_X = 0$

$T + F_{N_A} - 500\cos(90-\alpha) = 0$

$T + \mu_A N_{RA} = 500\cos(90-\alpha)$

$T + \mu_A N_{RA} = 500 sin\alpha$----------- (1)

Sum of Y direction force $\Sigma F_Y = 0$

$N_{RA} - 500\sin(90-\alpha) = 0$

$N_{RA} = 500\sin(90-\alpha)$

$N_{RA} = 500\cos\alpha$ --------------- (2)

$T + 0.3\ N_{RA} = 500\ sin\alpha$

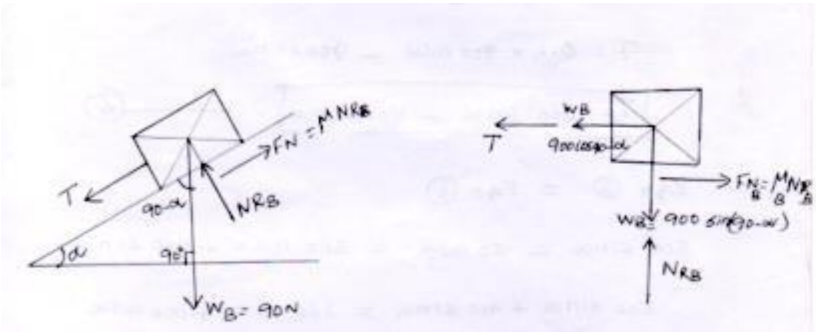
N_{RA} value sub in Eqn (1)

$T + 0.3 \times 500\ cos\alpha = 500\ sin\alpha$

$T + 150 cos\alpha = 500\sin\alpha$

$T = 500\sin\alpha - 150\ cos\alpha$ ------------ (3)

Consider block B



Sum of X direction force $\Sigma F_X = 0$

$F_{NB} - T - 900\cos(90-\alpha) = 0$

$\mu_B N_{RB} - T - 900 sin\alpha = 0$

$0.4N_{RB} - T - 900\sin\alpha = 0$

$-T = -0.4N_{RB} + 900\ sin\alpha$

$-T = -[0.4N_{RB} - 900 sin\alpha]$

$T = [0.4N_{RB} - 900 sin\alpha]$-------------- (4)

Sum of vertical [Y direction force] $\Sigma F_y = 0$

$N_{RB} - 900\sin(90-\alpha) = 0$

$N_{RB} = 900\sin(90-\alpha)$

$N_{RB} = 900\cos\alpha$ ---------(5)

N_{RB}value sub in Eqn(4)

$T = 0.4 \times N_{RB} - 900\ sin\alpha$

$T = 0.4 \times 900\ cos\alpha - 900 sin\alpha$

$T = 360\ cos\alpha - 900\ sin\alpha$--------------- (6)

Eqn (3) = eqn(6)

$500\sin\alpha - 150\cos\alpha = 360\ \ cos\alpha - 900 sin\alpha$

$500\sin\alpha + 900\ sin\alpha = 360 cos\alpha + 150 cos\alpha$

$1400 \sin \alpha = 510 \cos \alpha$

$\frac{sin\alpha}{cos\alpha} = \frac{510}{1400}$

$tan\alpha = \frac{510}{1400}$

$\alpha = tan^{-1}[\frac{510}{1400}]$

Ans $\alpha = 20°$

α value sub in Eqn (3)

$T = 500\ sin\alpha - 150\ cos\alpha$

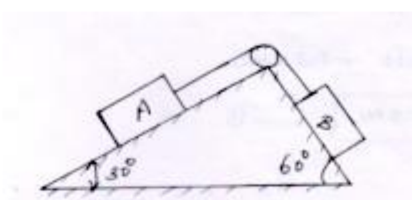
$T = 500 \sin 20 - 150\ cos20$

$T = 171 - 140$

$T = 31\ N$

Problem 10

Two block A and B are placed on inclined planes as shown in fig. the block a weight 1000N. Determine the minimum weight of the block b for maintaining the equilibrium of the system. Assume that the blocks are connected by an inextensible string passing over a frictionless pulley. Coefficient of friction μ_A b/w the block A and the plane is 0.25. Assume the same value for μ_B



Given:

Weight of block A $W_A = 1000N$

Coefficient of friction in block A & B $\mu_A = \mu_B = 0.25$

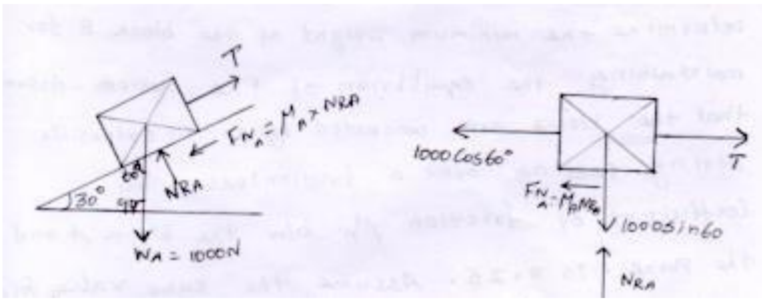
To find

Weight of block 'B' W_B

Solution:

Consider block 'A'

Free body diagram



Sum of X direction force Σ

$T - 1000\cos 60 - F_{NA} = 0$

$T - 1000\cos 60 - \mu_A N_{RA} = 0$

$T = 1000\ cos60 + 0.25\ N_{RA}$

$T = 500 + 0.25 N_{RA}$-------------- (1)

Sum of all Y direction forces

$N_{RA} - W_A = 0$

$N_{RA} - 100\ sin60 = 0$

$N_{RA} = 1000\ sin60$

$N_{RA} = 866\ N$

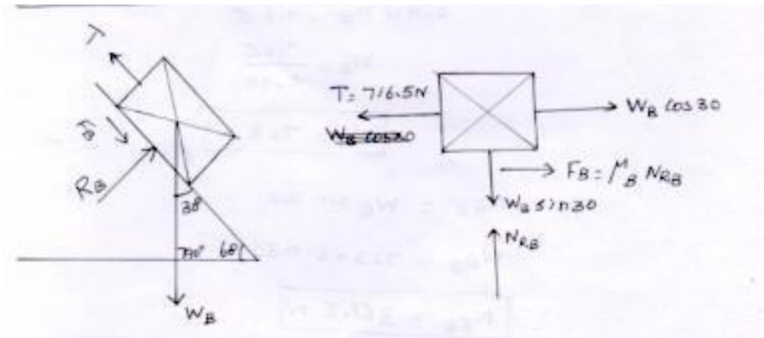
N_{RA} value sub in eqn (1)

$T = 500 + 0.25\ N_{RA}$

$T = 500 + 0.25\ \times 866$

$T = 716.5\ N$

Consider block B



Sum of X direction $\Sigma F_x = 0$

$-716.5 + W_B cos30 + \mu_B N_{RB} = 0$

$-716.5 + W_B cos30 + 0.25 \times N_{RB} = 0$

$W_B cos30 = 716.5 - 0.2 \times N_{RB}$---------- (2)

Sum of Y direction Force $\Sigma F_Y = 0$

$N_{RB} = W_B \sin 30 = 0$

$N_{RB} = W_B \sin 30$

N_{RB} value sub in Eqn (2)

$W_B cos30 = 716.5 - 0.2 \times N_{RB}$

$W_B cos30 = 716.5 - 0.2 \times [W_B \sin 30]$

$W_B cos30 = 716.5 - 0.125[W_B]$

$0.866W_B = 716.5 - 0.125W_B$

$0.866W_B + 0.125W_B = 716.5$

$0.991W_B = 716.5$

$$W_B = \frac{716.5}{0.991}$$

$$W_B = 723\,N$$

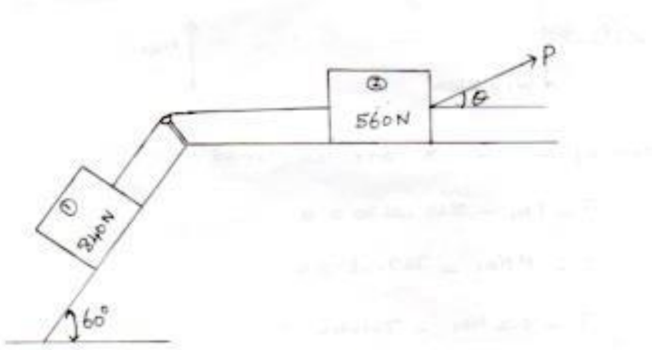
$N_{RB} = W_B \sin 30$

$N_{RB} = 723 \times \sin 30$

$N_{RB} = 361.5N$

Problem: 11

Determine the least value of P required to cause the motion impend the system shown in fig below. Assume coefficient of friction on all the contact surface as 0.2



Given:

Weight of block (1) W_1=840 N

Weight of block (2) $W_2 = 560\ N$

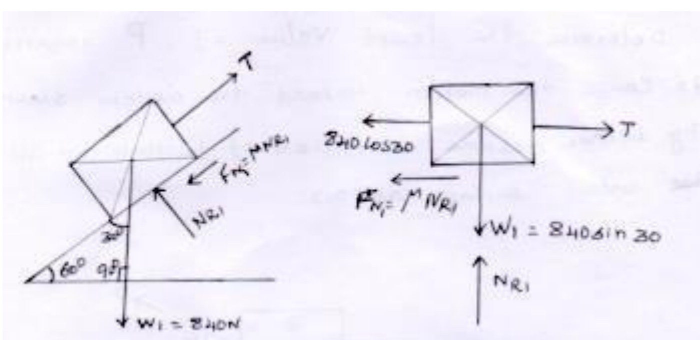
Coefficient of friction $\mu\ = 0.2$

To find

Load on block (2) P

Soln

Free body diagram of block



Sum of all the X direction forces

$T - F_{N1} - 840 \cos 30 = 0$

$T - \mu N_{R1} - 727.46 = 0$

$T - 0.2\ N_{R1} - 721.46 = 0$

$T = 0.2\ N_{R1} + 721.46$---------- (1)

Sum of All the Y direction force

$N_{R1} - w_1 \sin 30 = 0$

$N_{R1} - 840 \sin 30 = 0$

$N_{R1} - 420 = 0$

$N_{R1} = 420\ N$

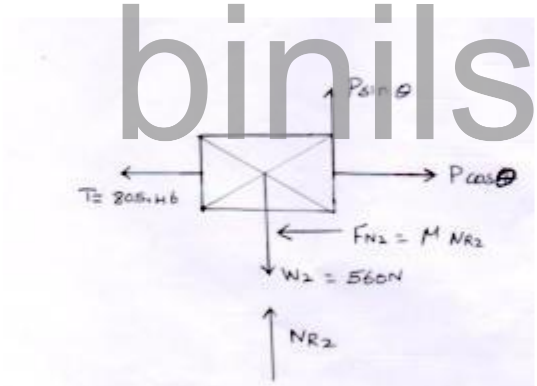
N_{R1} value sub in eqn (1)

$T = 0.2\ \times 420 + 721.46$

$T = 805.46\ N$

Consider block (2)

Free body diagram



Sum of X direction Force $\Sigma F_X = 0$

$-T - F_{N2} + p\ cos\theta = 0$

$-805.46 - \mu N_{R2} + Pcos\theta = 0$

Problem 12(H.W)

Determine whether the block shown in fig below having a mass of 40 kg is equilibrium and find the magnitude and direction of the friction force. Take $\mu_s =$ 0.40 and $\mu_k = 0.3$

Simple Contact Friction:

The friction force is the resisting force developed at the contact surface of two bodies due to their roughness and when the surface of one body moves over the surface of an another body.

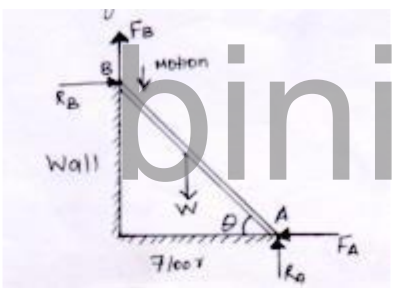
Some of important Engg application of simple contact friction are,

- Ladder friction
- Wedge friction
- Screw friction
- Belt friction

1. Ladder friction:

A ladder is a device used for climbing on roof or wall

Consider a ladder AB of length ‘l’ and weight ‘w’ resting on the ground at A and leaning against a rough wall at B as shown in fig.



Let the angle of ladder with horizontal be θ when the angle θ exceeds angle of friction (or) when the weight of the man on the ladder makes instability to the ladder, the ladder slips down.

During sliding the upper end of the ladder tends to slip downwards, hence the friction force at B, F_B will act upwards. If the coefficient of friction of wall surface B is μ_B

Then $\mu_B = \frac{F_B}{N_{RB}}$ (or) $F_B = \mu_B N_{RB}$

At the same time the lower end of the ladder A tends to slip away from the wall, hence the frictional force at A, F_A will act towards the wall if the coefficient of friction of floor surface A is μ_A

$\mu_A = \frac{F_A}{N_{RA}}$ (or) $F_A = \mu_A N_{RA}$

Condition:

$$\Sigma F_X = 0$$

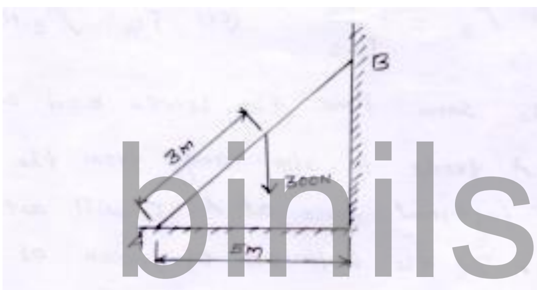
$$\Sigma F_Y = 0$$

$$\Sigma M = 0$$

$$F = \mu N_R$$

Problem 1:

A ladder is 8m long and weight 300N. The center of gravity of the ladder is 3m along the length of from the bottom end. The ladder rests against a vertical wall at B and on the horizontal floor at A as shown below. Determine the safe height upto which a man weighting 900N can climb without making the ladder slip. The coefficient of friction b/w ladder and floor is 0.4 and ladder top and wall is 0.3



Given:

Length of ladder l=8m

Weight of ladder w= 300N at 3m length from lower end

$\mu_A = 0.4$

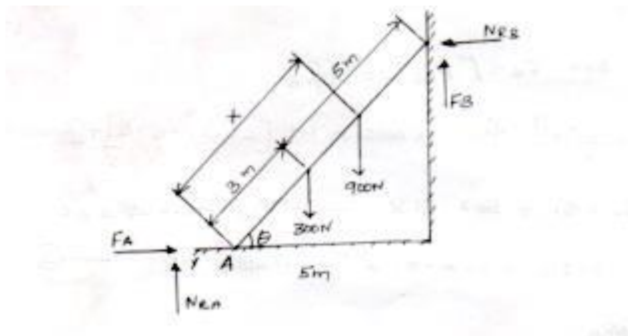
μ_B=0.3

To find

Safe height 'H'

Soln:

Free body diagram



Sum of X direction force $\Sigma F_X = 0$

$F_A - N_{RB} = 0$

$F_A - N_{RB}$-------------- (1)

Sum of Y direction force $\Sigma F_y = 0$

$N_{RA} + F_B - 300 - 900 = 0$

$N_{RA} + F_B = 300 + 900 = 0$

$N_{RA} + F_B = 1200$--------------- (2)

$\Sigma M_A = 0$ Moment about ‘A’

Now find out the θ

$\theta = cos^{-1}(\frac{5}{8})$

$\theta = 51°19'$

$\Sigma M_A = 0$ ↑ - ↓ +

$(300 \times 3cos\theta) + (900 \times cos\theta) + [F_B \times 5] + [-N_{RB} \times 8sin\theta] = 0$

$[300 \times 3cos51°19'] + [900 \times cos51°19'] + [-F_B \times 5] + [-N_{RB} \times sin51°19' = 0$

Sub $F_B = \mu N_{RB} = 0.3N_{RB}$

$[300 \times 3cos51°19'] + [900 \times cos51°19'] + [-0.3NR_B \times 5] + [-N_{RB} \times 8sin51°19' = 0$

$562.51 + 562.51 \times -1.5N_{RB} - 6.24N_{RB} = 0$

$562.51 + 562.21 \times -7.74 N_{RB} = 0$-------------- (3)

To find N_{RB}

From eqn (2)

$N_{RB} = F_A \quad F_A = \mu \times N_{RA}$

$F_A = 0.4 \times N_{RA}$

$N_{RA} = \frac{F_A}{0.4}$

$N_{RA} + F_B = 1200$--------------- (2)

$\frac{F_A}{0.4} + 0.3 \times N_{RB} = 1200$

$\frac{N_{RB}}{0.4} + 0.3 \times N_{RB} = 1200$

$2.5 N_{RB} + 0.3\ N_{RB} = 1200$

$2.8 N_{RB} = 1200$

$N_{RB} = \frac{1200}{2.8}$

$N_{RB} = 428.57 N$

N_{RB}value sub in Eqn(3)

$562.51 + 562.51 \times -7.74 N_{RB} = 0$

$562.51 + 562.51 \times -7.74 \times 428.57 = 0$

$562.51 \times = [7.74 \times 428.57] - [562.51]$

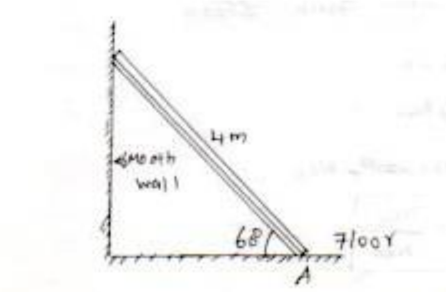
$562.51 \times = 2754.63$

$X = \frac{2754.63}{562.51}$

Ans $X\ =\ 4.89 m$from the floor of Ladder

Problem 2

A ladder of weight 1000 N and length 4m rest as shown in fig. if the 750 n weight is applied at distance of 3m from the top of ladder, it is at the point of sliding. Determine the coefficient of friction b/w ladder and the floor.



Given:

Length of Ladder l = 4m

Weight of ladder w = 1000N

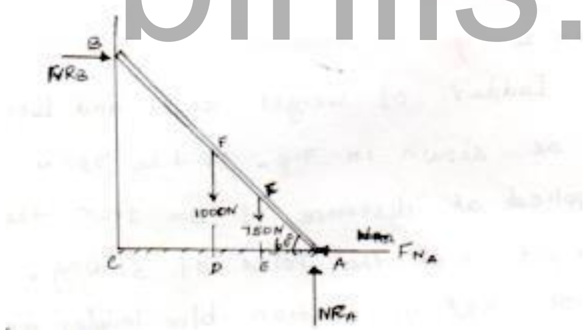
Weight of man = 750 N at 3m from toped

To find:

Coefficient of friction

Soln:

Free body diagram



Sum of X direction force $\Sigma F_X = 0$

$N_{RB} - F_{NA} = 0$

$N_{RB} - \mu_A N_{RA} = 0$

$-\mu_A N_{RA} = -N_{RB}$

$\mu_A = \frac{N_{RB}}{N_{RA}}$-------- (1)

Sum of all Y direction force

$N_{RA} - 1000 - 750 = 0$

$N_{RA} - 1750 = 0$

$N_{RA} = 1750N$

Take moment about 'A' ↓ + ↑ -

$\Sigma M_A = 0$

$\Sigma M_A = [N_{RB} \times BC] + [-1000 \times AD] + [-750 \times AE] = 0$

$BC = 4\ sin6$ $AD = 2\ cos60$ $AE = -1\cos 60$

$[N_{RB} \times 4\sin 60] + [-1000 \times 2\cos 60] + [-750 \times -1cos60] = 0$

$3.46N_{RB} - 1000 - 375 = 0$

$3.46N_{RB} - 1375 = 0$

$N_{RB} = \frac{1375}{3.46}$

$N_{RB} = 397.39\ N$

N_{RA} & N_{RB} value sub in Eqn (1)

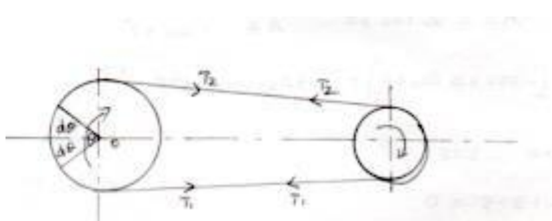
$\mu = \frac{N_{RB}}{N_{RA}} = \frac{397.39}{1750}$

$ans\ \mu = 0.22$

Belt friction:

Power is transmitted through a belt that is running round the two plugs.This is used in laths, diesel engine, and rise mills etc for power transmission.

The power is transmission is due to the friction existing b/w the belt and the pulley surface. The friction is called belt friction.



$\frac{T_2}{T_1} = e^{MQ}$ ⟶ Tension Ratio

T_2= tension in fight side

T_1 = tension in slack side

θ = angle of contact

μ = coefficient of friction

$T_2 > T_1$

θ value sub in radian

θ in radian $= {}^{\pi}/_{180} \times \theta\ value\ in\ degree$

Power $P = [T_2 - T_1] \times V$

$V = \frac{\pi dN}{60}\ m/s$

N= speed of drum

D = diameter of drum

V= belt speed (or) velocity of belt

Problem 1

A flat belt develops a tight side tension of 2000 N during power transmission the coefficient of friction b/w pulley and belt is 0.3, the angle of lap on smaller pulley is 165° and the belt speed is 18 m/s. determine the power that can be transmitted, if the belt is assumed to be perfectly elastic and without mass.

Given data:

Tension in tight side $T_1 = 2000N$

Coefficient of friction $\mu = 0.3$

Angle of contact $\theta = 165°$

Velocity of belt $V = 18\frac{m}{s}$

To find:

Power 'P'

Soln:

Power $P = [T_2 - T_1] \times V$

$T_2 = 2000, V = 18\, {}^{m}/_{s}\ given$

By Tension Ratio

$\frac{T_2}{T_1} = e^{MQ}, \mu = 0.3$

$\theta = 165°$

$\theta = 165° \times {}^{\pi}/_{180}$

$\theta = 2.87\ radian$

$\frac{2000}{T_1} = e^{0.3\times2.87}$

$T_1 = \frac{2000}{e^{0.3\times2.87}} = \frac{2000}{e^{0.861}} = \frac{2000}{2.36}$

$T_1 = 845.47\ N$

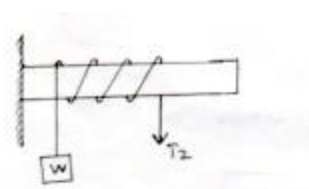
Power $P = [T_2 - T_1] \times V$

$P = [2000 - 845.47] \times 18$

$P = 20 \times 10^3 w$

Problem 2

A rope is wrapped three time around a rod shown in fig



Determine the force T required on the free end of the rope. To support a load of W= 20KN. Take μ *as* 0.3.

Given Data:

Weight w=20 KN $w = T_2$ $w = T_2$

$\mu = 0.3.$ $T_2 = T_1$ $T_1 = T$

To find:

Tension 'T'

Soln:

Tension Ratio

$$\frac{T_2}{T_1} = e^{MQ}$$

$$\theta = 3 \times {}^{\pi}/_{180} \times 360°$$

$$\theta = 18.84\, radian$$

$$\frac{T_2}{T_1} = e^{0.3 \times 18.84}$$

$$\frac{20}{\pi} = e^{0.3 \times 18.84}$$

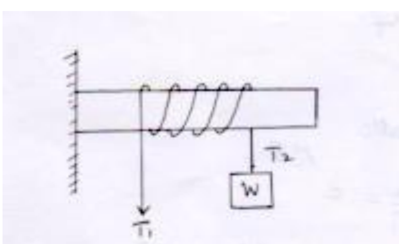
$$T_1 = \frac{20}{e^{0.3 \times 18.84}}$$

$$T_1 = 0.07\, KN$$

$$T = T_1 = 0.07\, KN$$

Problem 3

A wire rope is wrapped three and a half times around a cylinder as shown in below. Determine the force T_1 exerted on the free end of rope that is required to support a 1KN weight the coefficient of friction b/w the rope and the cylinder is 2.5



Given:

Coefficient of friction $\mu = 0.25$

No of turns=3.5

$T_2 = w = 1KN$

To find:

Force exerted T_1

Soln:

Tension ratio

Tension Ratio

$$\frac{T_2}{T_1} = e^{MQ}$$

$$\theta = 360 \times [{}^{\pi}/_{180}] \times 3.5$$

$$\theta = 21.99\ radian$$

$$\frac{T_2}{T_1} = e^{0.25 \times 21.99}$$

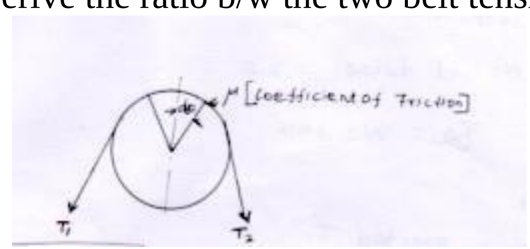
$$\frac{1}{T_1} = e^{0.25 \times 21.99}$$

$$T_1 = \frac{1}{e^{0.25 \times 21.99}}$$

$$T_1 = 4.09 \times 10^3\ KN$$

$$T = T_1 = 4.09\ KN$$

Derive the ratio b/w the two belt tension forces



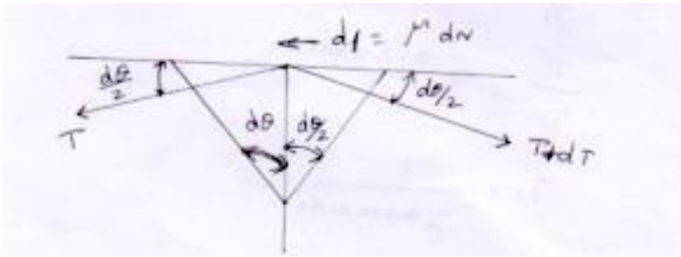
$$\frac{T_2}{T_1} = e^{M\theta}$$

T_1 = Tension in tight side

T_2 = Tension in slack side

θ = Angle of contact

μ = Coefficient of friction



$\Sigma F_X = 0$

$-T\cos\frac{d\theta}{2} + (T + dT)cos\frac{d\theta}{2} - df = 0$

$-T\cos\frac{d\theta}{2} + T\cos\frac{d\theta}{2} + (dT)cos\frac{d\theta}{2} - df = 0$

$df = \mu dN$ &$\cos\frac{d\theta}{2} = 1$

$dT = -\mu dN = 0$

$dt = \mu dN$----------- (1)

$\Sigma F_Y = 0$

dN $+([T + dT])sin\frac{d\theta}{2} - Tsin\ \frac{d\theta}{2} = 0$

$dN - T\sin\frac{d\theta}{2} - dT\sin\frac{d\theta}{2} - Tsin\ \frac{d\theta}{2} = 0$

$dN = 2T\ \frac{d\theta}{2} - dT\ do = 0$

$dN - Td\theta = 0$

$dN - Td\theta$-------- (2)

dN value sub in Eqn (1)

$dT = \mu dN$--------- (1)

$dT = \mu \times dN$

$\frac{dT}{T} = \mu d\theta$

$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\theta} \mu d\theta$

$[\log T]_{T_1}^{T2} = [\mu]_0^{\theta}$

$logT_2 - logT_1 = \mu\theta = log\frac{T_2}{T_1} = \mu\theta$

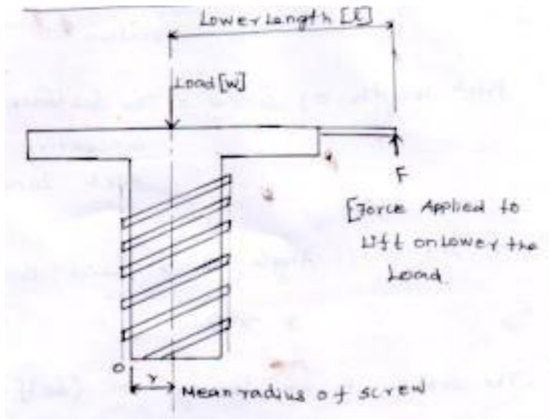
Flat for belt $\frac{T1}{T_2} = e^{\mu\theta}$

$\frac{T_2}{T_1} = e^{\mu\theta\, cosec\frac{B}{2}}$

Problem 4:

A three turns of cables is used to hold the 100N weight with a tensile force of 50 N at the other end of shown in fig. find out the coefficient of friction b/w the cable and shaft surface

Screw friction



w=load active $w\ tan[\theta + \emptyset]$

It is used to raise and lower the load

Ex, screw jack

Taking moment about the axis of the screw

$W \tan(\emptyset + \theta) \times r - F \times l = 0$

$W \times r \times \tan(\emptyset + \theta) = Fl$

Force applied to raise (or) Lower the load F

$$F = \frac{wrtan(\emptyset + \theta)}{l}$$

θ =Lead angle

$\emptyset$ =Friction angle

$\tan \emptyset = \mu$

$\theta = tan^{-1}(\mu)$

Lead of screw = the height lifted for one full from rotation

Pitch length of screw = The distance b/w the two connective thread head screw pitch length

If friction angle $\emptyset > lead\ angle \theta$

$$\emptyset > \theta$$

The screw is in locking or (self-locking) of the screw

$$\mu = \frac{tan\theta}{\tan(\emptyset + \theta)} \times 100$$

Mean radius $= r = \frac{pitch\ length}{2}$

$$r = \frac{dia\ meter}{2}$$

Problem: 1

A screw jack has a square thread of mean radius 5cm and pitch length of 1.5 cm. length of lever 50 cm. it is used to raise and lower the load of 25 KN. $\mu = 0.2$ find

(i) Force applied to raise the Load

(ii) Force applied to the lower the load

(iii) Threaded efficiency

Given

Load $w = 25KN = 25 \times 10^3 N$

Mean radius$r = 5\ cm$

Pith length= $1.5cm$

Length of lever (l) =50cm $\mu = 0.2$

Soln

(i) Force applied to raise the road

$$F = \frac{Wrtan[\emptyset + \theta]}{l}$$

$tan\emptyset = \mu$

$\emptyset = tan^{-1}(\mu)$

$\emptyset = tan^{-1}(0.2)$

$\theta = Lead\ angle$

$$tan\theta = \frac{1.5}{2 \times \pi \times 5}$$

$$\theta = tan^{-1}\left(\frac{1.5}{2 \times \pi \times 5}\right)$$

$$\theta = 2°4'$$

$$F = \frac{25 \times 10^3 \times 5}{50} \tan[11°55' + 2°4']$$

$$F = 653.54\ N$$

(ii) Force applied to lower the load

$$F = \frac{wrtan(\emptyset - \theta)}{l}$$

$$F = \frac{25 \times 10^3 \times 5 \times \tan[11°55' - 2°4']}{50}$$

$$F = 0.104\ KN$$

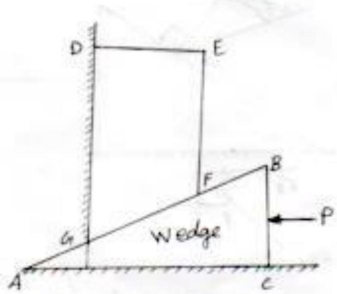
(iii) Efficiency: η

$$\eta = \frac{tan\theta}{\tan(\theta + \emptyset)} \times 100$$

$$\eta = \frac{\tan 2°4'}{\tan(2°4' + 11°55')}$$

$$\eta = 36$$

Wedge Friction

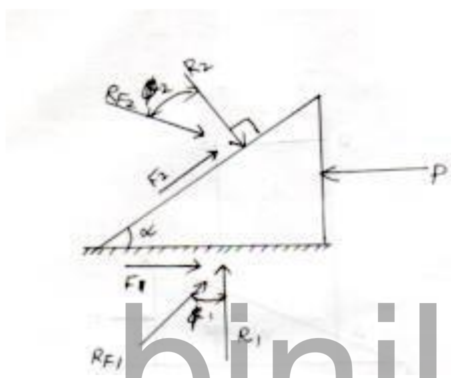


A wedge is a piece of wood or metal, usually of a triangular (or) trapezoidal shape in cross section used for Lifting Loads (or) for slight adjustment like tightening keys for shift

When the force P is applied, sliding take place on the edge AC, DG, and Gf. Hence the reactive force components, normal reactions and the frictional forces are also developed on these sliding surface

Now Let as see the direction of these reactive forces, and draw the free body diagram of wedge and body.

Equilibrium of wedge



Normal force and Frictional force are combined to a single resultant force 'R_F'

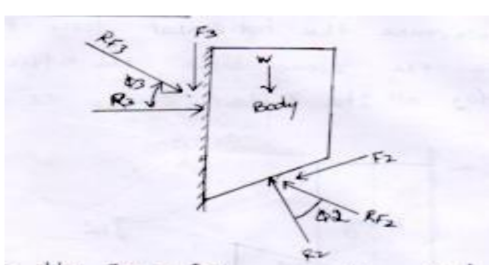
$R_{F1} \& R_{F2}$ *are drawn on wedge*

Where $R_{F1} = \sqrt{{F_1}^2 + {R_1}^2}$

$R_{F2} = \sqrt{{F_2}^2 + {R_2}^2}$

$F_1 \& F_2$ are Limiting friction $\emptyset_1, \emptyset_2$means of angle of friction

It is defined as the in b/w the angle of [Line of action of the normal reaction$N_1 \& N_2$] and Resultant force $R_{F_1} \& R_{F_2}$



Equilibrium of body

When the force 'P' push the wedge, the body tends to move upwards, hence the frictional force F_3 on the surface AD is acting download.

Then the normal reaction R_3 and F_3 are replaced by a single resultant R_{F3}, which makes on angle of $\emptyset_3$with line of action of normal reactionR_3

Concurrent forces$R_1 R_2$, self weight(w)

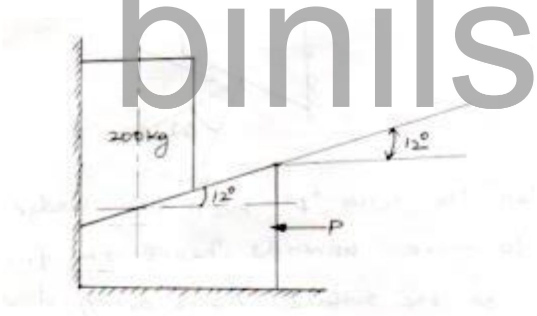
Note (i) always draw the free body diagram of wedge first then draw the force body diagram of the below

(ii) while solving, if the Load is given, solve free body of block first and if the force 'P' is given.solve the free body of wedge

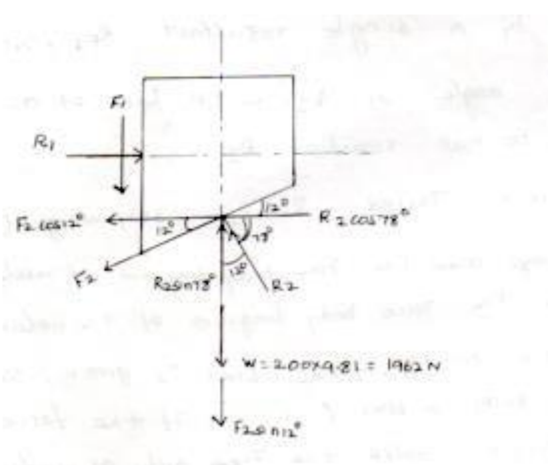
(iii) self-weight of the wedge is neglected

Problem: 1

Determine the horizontal force P required to raise the 200 kg block the efficient of friction for all the contact surface is 0.25



Free body Diagram of block



Sum of all X direction force

$\sum F_X = 0$ + → ← -

$R_1 - R_2 \cos 78 - F_2 \, cos12° = 0$

$R_1 - 0.207R_2 - \mu R_2 cos12° = 0$

$R_1 - 0.207R_2 - 0.25 \times R_2 \times cos12° = 0$

$R_1 - 0.207R_2 - 0.24R_2 = 0$

$R_1 = 0.207R_2 + 0.24R_2$

$R_1 = 0.45R_2$

Sum of all Y direction force

$\sum F_Y = 0$ ↓ - ↑ +

$-1962 - F_1 - F_2 sin12 + R_2 sin78 = 0$

$-1962 - 0.25R_1 - 0.25R_2 sin12° + R_2 sin78° = 0$

$-1962 - 0.25R_1 - 0.051R_2 + 0.97R_2 = 0$

$-1962 - 0.25 \times [0.45R_2] - 0.051R_2 + 0.97R_2 = 0$

$-1962 - 0.112R_2 - 0.051R_2 + 0.97R_2 = 0$

$-1962 + 0.807R_2 = 0$

$0.807R_2 = 1962$

$$R_2 = \frac{1962}{0.867}$$

$R_2 = 2431.22 \, N$

R_2 Value sub in eqn (1)

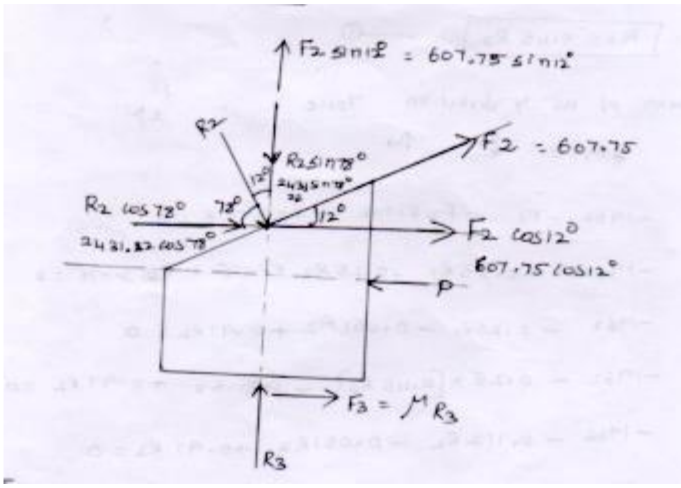
$R_1 = 0.45 \, R_2$ ---→ 0.45×2431.22

$R_1 = 1094 \, N$

$F_1 = \mu R_1 = 0.25 \times 1094 = 273.51 \, N$

$F_2 = \mu R_2 = 0.25 \times 2431 = 607.75 \, N$

Free body diagram of wedge



$\sum F_X = 0$

$-P + F_3 + 2431.22\ cos78° + 607.75\ cos12° = 0$

$-P + \mu R_3 + 278 + 594.46 = 0$

$-P = -\mu R_3 - 2378 - 594.46$

$-P = -[\mu R_3 + 2378 + 594.46]$

$P = 0.25 \times R_3 + 2378 + 594.46$-------------- (2)

$\sum F_Y = 0$

$R_3 - R_2 sin78° + 607.75\ sin12° = 0$

$R_3 - 2431.51 sin78° + 607.75\ sin12° = 0$

$R_3 - 505.47 + 126.35 = 0$

$R_3 = 505.47 - 126.35$

$R_3 = 379.11\ N$

R_3 vlaue sub in Eqn (2)

$P = 0.25 \times R_3 + 2378 + 594.46$

$P = 0.25 \times 379.11 + 2378 + 594.46$

$P = 3067.23\ N$