# UNIT-IV

## DYNAMIC OF PARTICLES

Newton's Law Of Motion

Newton"s Law

The rate of change of momentum is directly proportional to the resultant force.

The Resultant Force acting in the direction of equal to the product of mass and the acceleration in the direction of resultant Force.

$\sum F = ma$

m= mass

a= acceleration



D' Alembert' Principle:

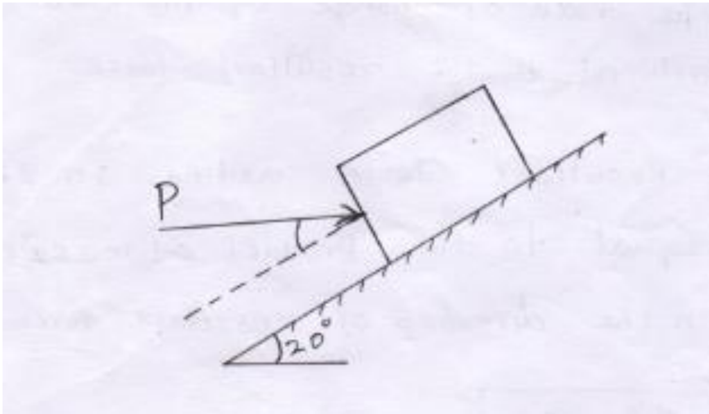
States that the inertia forces and couples, and the external forces and torques on a body together give statical equilibrium.

Inertia is a property of mater by virture of which a body resists ay change in velocity

$$F_I = \quad -mg$$

Problem:1

What horizontal force is needed to give the 50 kg block shown in fig. With an acceleration of $3 m/s^2$ up the 20° plane. Assume the coefficient of friction b/w the block and plane is 0.25.




<u>Given:</u>

Weight of block W $= 50\ kg\ = 50 \times 9.81 = 490.5N$

Acceleration a=3 $^{m}/_{s^2}$

Coefficient of friction $= 0.25$

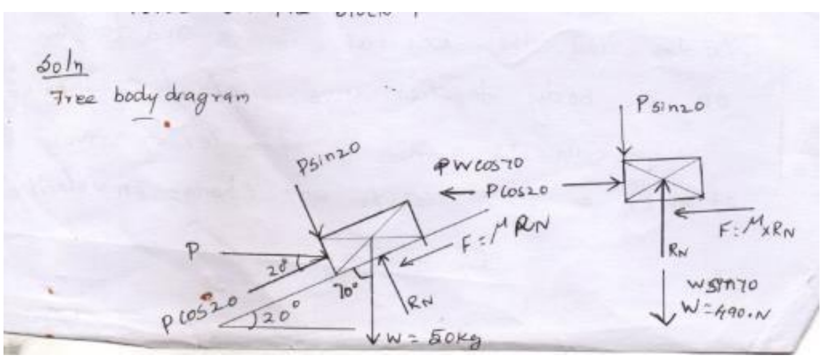
<u>To find:</u>

Force on the block P



<u>Soln:</u>

Free body diagram




$\sum FX = m\ a$

P $\cos 20 - M_{XRN} -$ w $\cos 70 = 50 \times 3$

$$P\ \ cos\ 20 - 0.25 \times R_N - \ 490.5 \times cos\ 70 = 150 \quad ------> (1)$$

$\sum FY = 0$

$R_N - p\ \sin 20 - w\ \sin 70 = 0$

$R_N - p\ \sin 20 - 490.5\ \sin 70 = 0$

$R\_N - psin20 - 490.5\ sin\ 70 = 0$

$RN = 0.34P - 460.91$

$R_N$ value in Eqn (1)

$P\ cos20 - 0.25\ [0.34 \times p - 490.91] - 490.5 \times cos70 = 150$

$0.93p - 0.085p + 122.72 - 167.76 = 150$

$0.845P - 45.04 = 150$

$0.845P = 150 + 45.04$

$0.845P = 195.04$

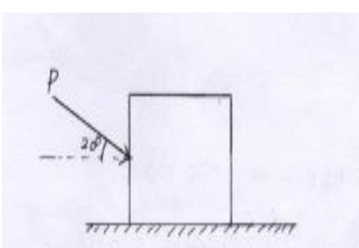


$$P = \frac{195.04}{0.845}$$

$P = 230.81N$

<u>Problem:2</u>

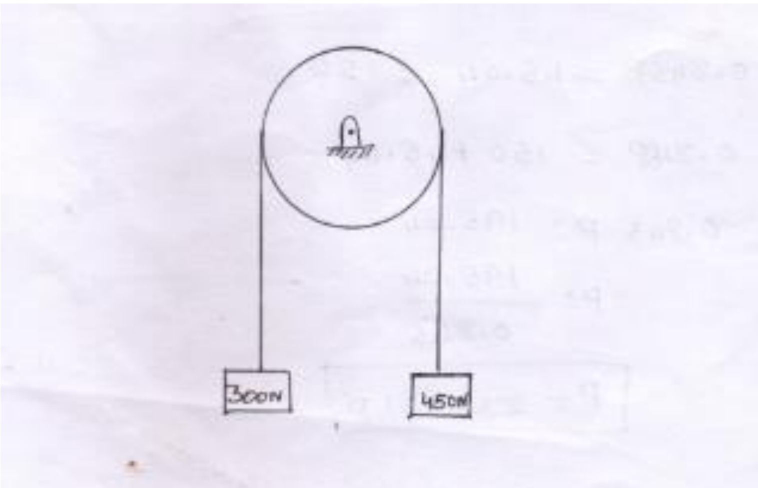
A block weighting 1KN, rest on a horizontal plane as shown in fig. Find the force P required to give an acceleration of 3 m/s$^2$ to right. Take the coefficient of friction $M_K$=0.25.




$P = 750.056N$

Problem:3

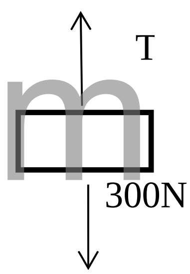
Two blocks weighting 300N and 450N are connected by a rope as shown fig. With what acceleration the heavier block comes down, and what is the tension of the rope. Pulley is frictionless and weight less.

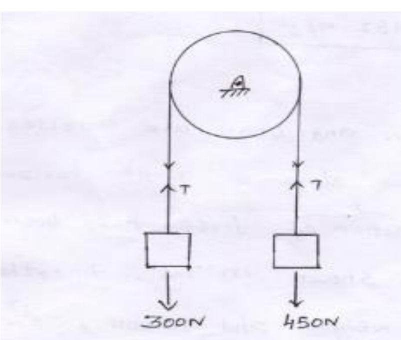


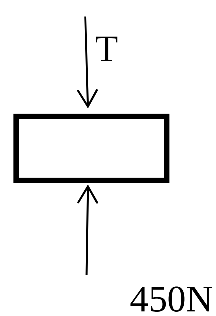

soln :

Free body diagram






$\sum F_X = ma$

T$-$300$=\frac{300}{9.81}\times a$

$T - 300 = 30.58 \times a$-----(1)

$\sum F_Y$= m a

$450 - T = \frac{450}{9.81} \times a$-----(1)

Solving Eqn (1) & (2)

$T - 300 = 30.58 \times a$

$\underline{450 - T = 45.87 \times a}$

$150 \quad = 76.45 \times a$

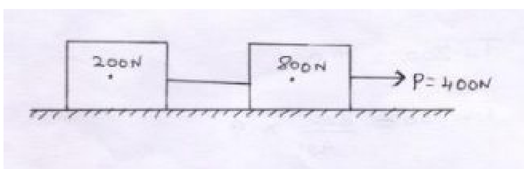
$a = \frac{150}{76.45}$

$a = 1.962\ m/s2$

Problem 4:

Two weight 800N and 400N are connected by a thread and they move along a rough horizontal plane under the action of force P of 400N applied to 800N block, as shown in Fig. Find the acceleration of the weight and tension in the thread.





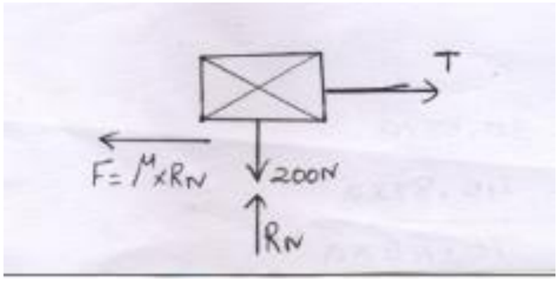

To find

Acceleration a

Tension T

Soln:

Consider block '200N'




$\sum F_Y$=0

$R_N - 200 = 0$

$R_N = 200N$

$\sum F_X = \text{m a}$

$T = F = m \times a$

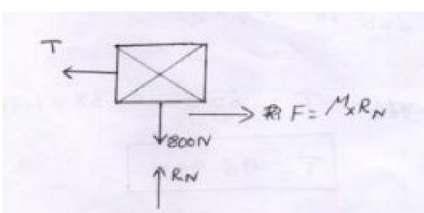
$T - \mu \times RN = \frac{200}{9.81} \times a$ μ = 0.3 assume

T−0.3 × 200 = 20.38 × $a$

T−60 = 20.38 × $a$---------->(1)



<u>Consider 800N block</u>




$\sum F_Y$=0

$R_N - 800 = 0$

$R_N = 800N$

$\sum F_X = \text{m a}$

$-T + F_N = \frac{800}{9.81} \times \text{a}$

$-T + \mu \times R_N = 81.54\times a$

$-T + 0.3 \times 800 = 81.54 \times a$

$-T + 240 = 81.54\times a$

$-T + 240 = 81.54\times a$ ------->(2)

Solving Eqn 1&2

$T - 60 = 20.38\times a$

$-T + 240 = 81.54\times a$

$180 = 101.54\times a$

$a = \frac{180}{101.92}$

$a = 1.766 \ m/s^2$

'a' Value sub in Eqn 1

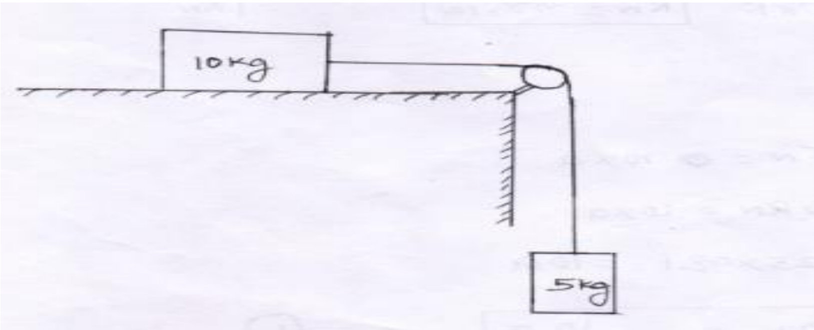
$T - 60 = 20.38 \times 1.766$

$T = 95.99N$



Problem:5

Two blocks of mass 10kg and 5kg are connected as shown in fig. Assume $M_k = 0.25$. Find the acceleration and the tension in the string if pulley is weightless and frictionless.



Given:

$$Block\ A = 10kg$$

$$Block\ B = 5kg$$

$$M_k = 0.25$$

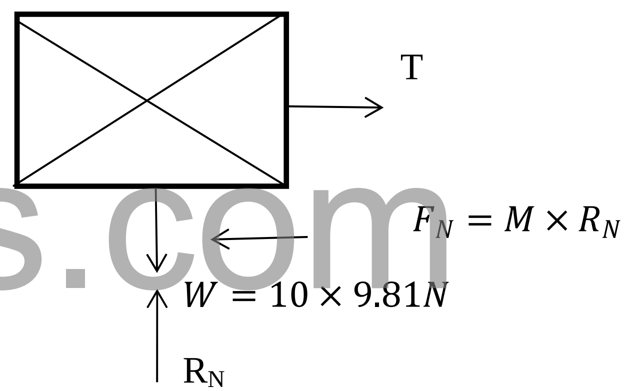
To Find:

1. Acceleration a

2. Tension T

Soln:

Consider block A (10kg)




$\sum F_Y = 0$

$$R_N - w = 0$$

$$R_N = w$$

$$R_N = 98.1\ N$$

$\sum F_X = ma$

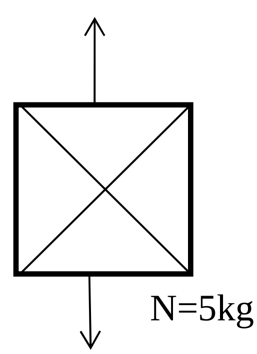
$$T - F_N = N10 \times a$$

$$T - M \times R_N = 10 \times a$$

$$T - 0.25 \times 98.1 = 10a$$

$$T - 24.52 = 10a \text{ ------> } (1)$$

Consider 5 kg block




$\sum F_Y =$ m a

$$T - w = m\,a$$

$T - 5 \times 9.81 = 5 \times a$

$W = 5 \times 9.81 = 49.05N$

$+T - 49.05 = 5a$ ----->(2)

Solving Eqn 1&2

$$T - 24.52 = 10\ a$$

$$T - 49.05 = 5\ a$$

$$24.52 = 5\ a$$

$$a = \frac{24.52}{5}$$

$$a = 4.905\ {}^{m}/{}_{s}2$$

'a' Value sub in Eqn 1



$$T - 24.52 = 10\ a$$

$$T = 10\ a\ + 24.52$$

$$T = 10 \times 4.905 + 24.52$$

$$T = 73.57\ N$$

<u>Problem 6</u>

A block of 1200 N rest on a rough inclined plane at 12° to the horizontal. It is pulled up the plane by means of a light flexible rope running parallel to the plant and passing over a light frictionless pulley at the top of the plane. The portion of the rope beyond the pulley hangs vertically down and carries a weight of 800N at its end.

If Coefficient of friction = 0.2, find a) tension in the rope (b) acceleration with which the body moves up the plane (c) distance moved is after 3sec after starts from rest.

Given:

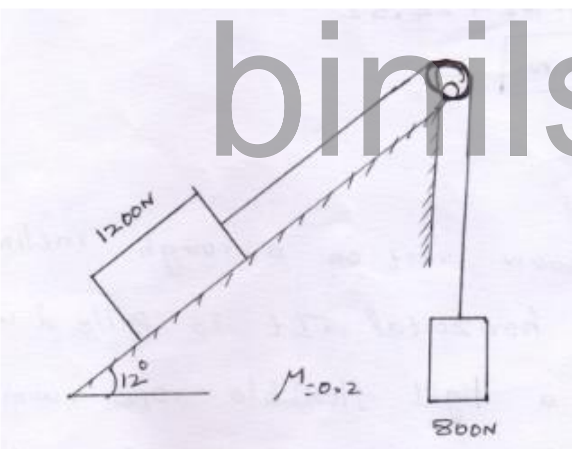
$Coefficient\ of\ friction\ \mu\ = 0.2$

$Weight\ of\ block\ w = 1200\ N$

To Find:

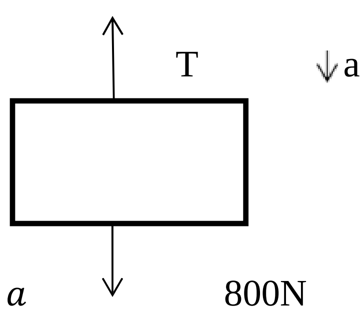
1. Tension 'T'
2. Acceleration 'a'
3. Distance moved 3sec after starts from rest

Soln:






Consider 800 N block



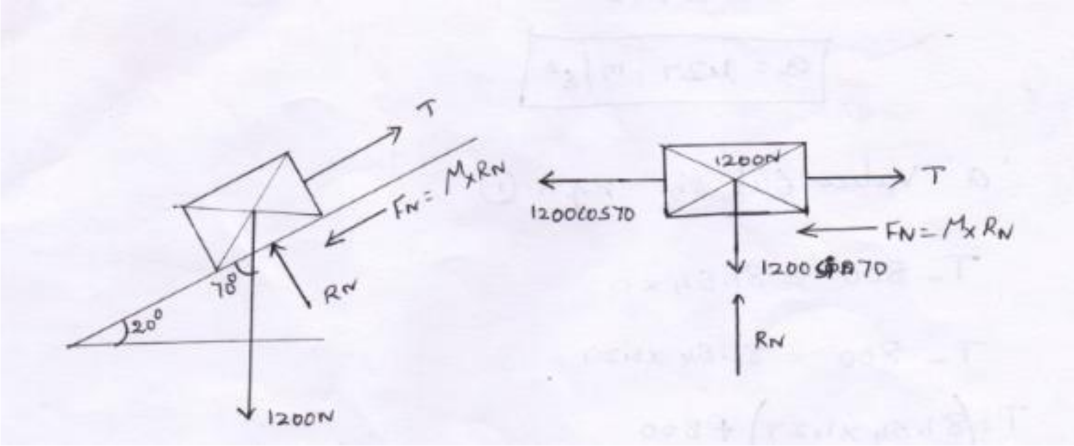

$\sum F_Y =$ m a

T−800= m a

$T - 800 = \ 800/9.81 \times \ a$

$T - 800 = 81.54\ a$ -----> (1)

Consider 1200 N block:




$\sum F_Y = 0$

$R_N - 1200\ 70 = 0$

$R_N = 1127.63 = \ 0$



$R_N = 1127.63\ N$

$\sum F_Y = \text{m a}$

$$T - \ 1200\ cos\ 70 - F_N = \ m\ a$$

$$T - 1200\ cos\ - \ M \times R_N \ = \frac{1200}{9.81} \times a$$

$$T - 410.42\ - \ 0.3 \times 1127.63 = 122.32 \times a$$

$$T - 410.42 - \ 338.28 = 122.3 \times a$$

$$T - \ 748.70\ = \ 122.3 \times a \text{ -------> } \quad (2)$$

Solve Eqn (1) & (2)

$$T\ - 800\ = \ 81.54\ a$$

$$T - \ 748\ = \ 122.3\ a$$

$$- 51.8 = \ - 40.76\ a$$

$$a = \frac{-51.8}{-40.76}$$

$$a = 1.27 \ {}^{m}/_{s}2$$

'a' Value sub in Eqn (1)

$T - 800 = 81.54 \times a$

$T - 800 = 81.54 \times 1.27$

$T = (81.54 \times 1.27) + 800$

$T = 903.55 \ N$

Consider kinetic Eqn

To Find Distance

$S = u\,t + \frac{1}{2}\,a\,t2$

$S = 0 \times 3 + 1/2(1.27) \times (3)2$

$S = 5.71 \ m$

Initial condition

$u = 0$

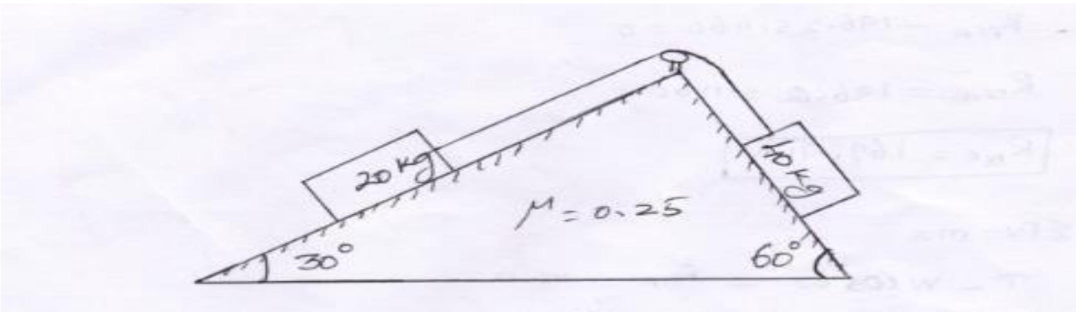
$a = 1.27 \ m/s2$

$t = 3sec$



Problem 7

Two blocks of mass 20 kg and 40 kg are connected by a rope passing over a frictionless pulley as shown in fig. (a) Assuming the coefficient of friction as 0.3 for all contact surfaces. Find the tension in the string and the aueleration of the system. Also compute the velocity of the system after 4 sec starting from rest.



Given:

$Mass\ of\ block\ A\ mA = 20\ kg$

$Mass\ of\ block\ B\ mB = 40\ kg$

$Coefficient\ of\ friction\ \mu = 0.3$

To Find:

Tension in the string T

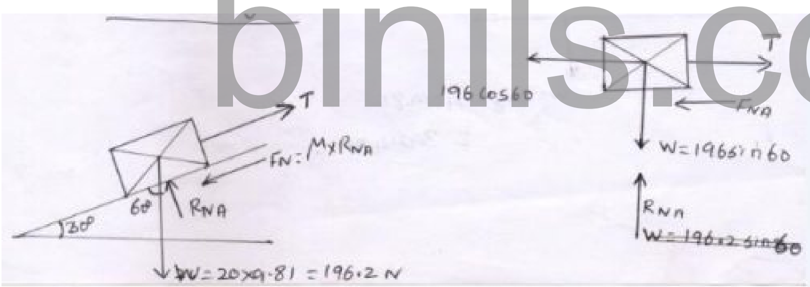
Acceleration ‘a’

Velocity of the system after 4 sec

Solution:

Consider 20 kg block






$\sum F_Y = 0$

$R_{NA} - 196.2\ sin\ 60 = 0$

$R_{NA} - 196.2\ sin\ 60$

$R_{NA} - 169.91\ N$

$\sum F_X =$ m a

$T - w\ cos\ 60 - F_{NA} = m\ a$

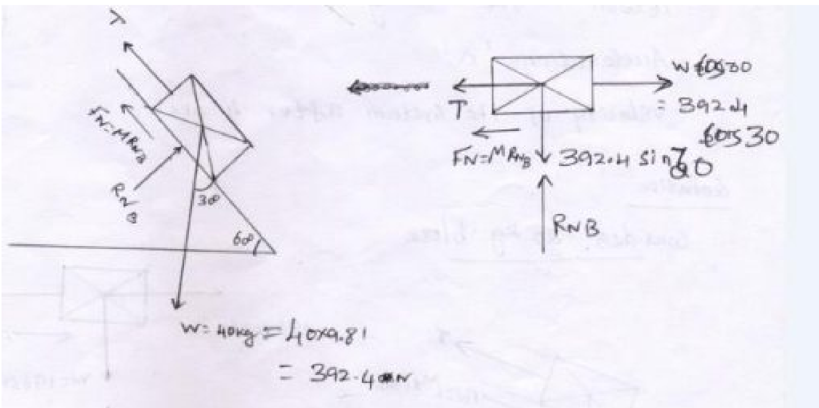
$T - 196.2\ cos\ 60 - \mu \times R_{NA} = 20 \times a$

$T - 196.2\,cos\,60 - \;0.3 \times 169.91 = 20 \times a$

$T - 98.1 - \;50.973 = 20\;a$

$T - \;149.07 = 20\;a$ -----> (1)

<u>Consider 40 kg block</u>




$\sum F_Y$=0

$R_{NB} = -392.4\,sin\,30 = 0$



$R_{NB} = 392.4 sin\,30$

$R_{NB} = 196.2N$

$\sum F_x$ =ma

$392.4\;\;cos\,30 - T\; = ma$

$339.82 - 0.3 \times R_{NB}\; - T\; = 40 \times a$

$339.82 - 0.3 \times 19.2 - T\; = 40a$

$280.96 - T\; = 40a$------->(2)

solve Eqn 1 & 2

T−149.07 = 20a

280.96−$T$ =40a

$131.89 = 60a$

$a = 131.89/60$

$a = 2.19\, m/s^2$

'a' value sub in Eqn (1)

$T - 149.07 = 20 \times 2.19$

$T = 193.03\, N$

Using kinetic Eqn

$V = u + at$ u=0 Initial stage

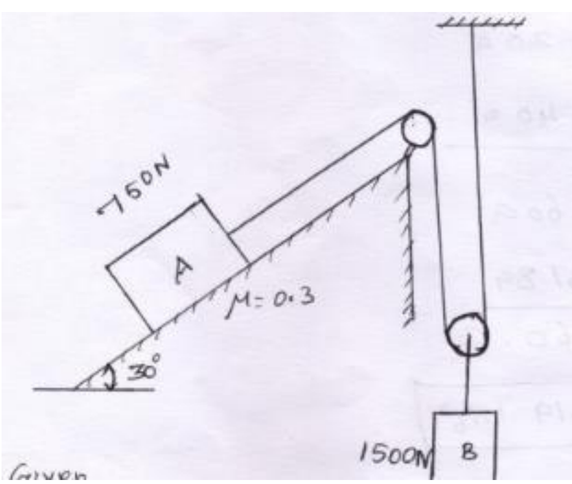
$V = 0 + 2.198 \times 4$ t=4sec

$V = 8.79\, m/sec$

Problem:



Two blocks of weight 750 N and 1500 N start from shown in fig. Find the acceleration of each block and the distance travelled by the 750 N block in 2 sec Also find the tension in the string.




Given:

$Weight\ of\ block\ A\ W_A = 750\ N$

$Weight\ of\ block\ B\ W_B = 1500\ N$

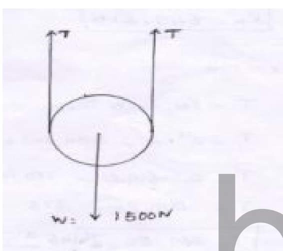
$Coefficient\ of\ friction\ \mu\ =\ 0.3$

To Find:

1. Acceleration
2. Tension
3. Distance travelled by the 750 N in 2 Sec.

Soln

Consider1500N block





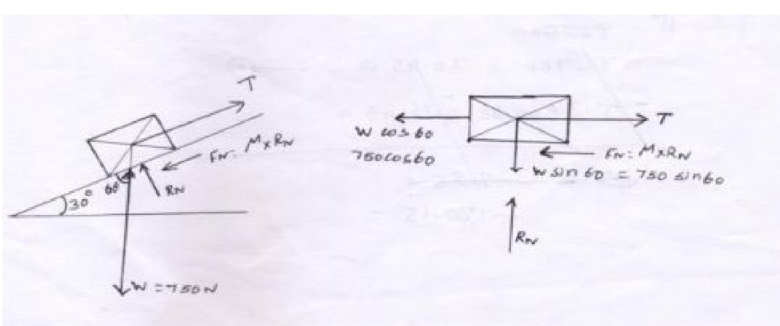

$\sum F_Y = ma$

$2\,T - 1500 = m \times a$

$2\,T - 1500 = \frac{1500}{9.81} \times a$

$2\,T - 1500 =\ 152.90 \times 2a$ --------> (1)

Consider 750 N block



$\sum F_Y = 0$

$R_N - w \sin 60 = 0$

$R_N = w \sin 60$

$R_N = 750 \sin 60$

$R_N = 649.51N$

$\sum F_X = \text{ma}$

$\text{T} - F_N - \text{w} \cos 60 = ma$

$\text{T} - \mu R_N - 750 \cos 60 = \frac{750}{9.81} \times a$

$\text{T} - 0.3 \times 649.51 - 750 \cos 60 = 76.45 \times a$

$\text{T} - 569 - 85 = 76.45a$-------- (2)

Solve eqn (1) &(2)

$2T - 1500 = 152.90 \times 2\ a$------------(1)

$2\ T - 1500 = 305.8\ a$

$\div 2 \qquad T - 750 = 152.9\ a$ --------> (1)

$T - 750 = 152.90 \times a$ -------> (1)

$T - 569.85 = 76.45 \times a$ ----------> (1)

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$- 180.15 = 76.45\ a$

$$a = \frac{-180.15}{76.45}$$

$\text{a} = -2.35 {}^{m}/{}_{s^2}$

a value sub in Eqn (1)

$T - 750 = 152.9 \times (-2.35)$

$T - 750 = 152.9 \times (-359.31)$

$T = 750 = -359.31$

$T = -359.3 + 750$

$T = 390.68\ N$

Distance travelled

$s = ut + {}^{1}/_{2}\, at2$ u=0 t=2

$s = 0 \times 2 + {}^{1}/_{2} \times (-2.35) \times (2)2$

$s = -4.7\ m$

Impact of Elastic Bodies:

A collision between two bodies to be an impact, if the bodies are in contact for short interval of a time and exert very large force on a short period of time.

On impact bodies deform first and then recover due to elastic properties and start moving with different velocities.



Types of Impact:

- Line of impact
- Direct impact
- Oblique impact
- Central impact
- Eccentric impact

Perfectly Elastic impact: [e=1]

If both of bodies regain to their original shape and size after the impact. Both momentum and energy is conserved.

In elastic impact [e<1]

The collision do not return to their original shape and size completely after the collection. Only the momentum remains conserved, but there is a loss energy.

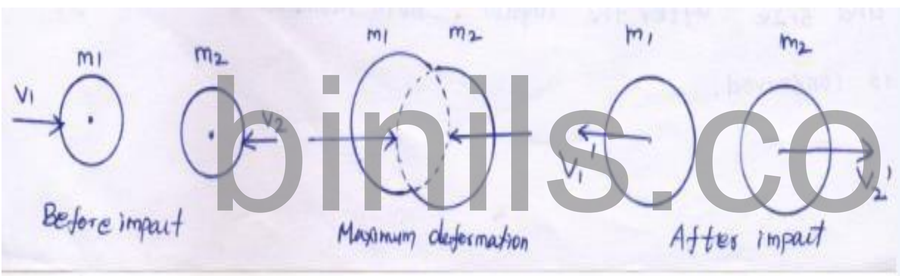
Period of collision:

During the collection, the bodies undergo a deformation for a small time interval and then recover the deformation in a further small interval.

Time elapse b/w initial contact and maximum deformation is called the period of deformation. And the instant of separation is called time of restitution or period of recovery.

Principal of collision:

Consider 2 bodies approach each other with the velocity $v_1$ and $v_2$ masses $m_1$ and $m_2$ are shown in fig.




Let 'F' be force entered due to collection at a small time. Apply conservation of momentum principal for both bodies

$$m_1 v_1 + m_2 v_2 = m_1 v_1{}^1 + m_2 v_2{}^1$$

Newton's impact Eqn:

Coefficient of restitution, $e = \frac{relative\ velocity\ of\ separation}{relative\ velocity\ of\ approach}$

$$e = \frac{v_2^1 \; v_1^1}{v_1 - v_2}$$

Total kinetic energy at before impact

$$= \frac{1}{2}\, m_1 v + \frac{1}{2} m_2 v_2^{2}$$

Total kinetic energy at after impact

$$= \frac{1}{2} m_1 v_1^{1\,2} + \frac{1}{2} m_2 v_2^{1\,2}$$

Loss of K.E=Intial K.E − Final K.E

<u>Oblique:</u>

$V_1 \sin \propto_1 = V_1^1 \sin \theta_1$

$V_2 \sin \propto_1 = V_2 \sin \theta_1$

$m_1 v_1 \cos \propto_1 + m_1 v_1^1 \cos \theta_1 + m_2 v_2^1 \cos \theta_2$ $\qquad m_1 = m_2$

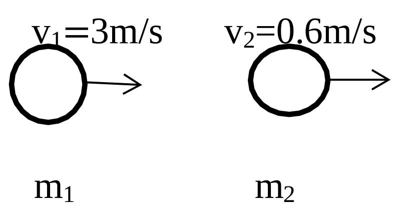
$$e = \frac{V_2^1 \cos \theta_2 - V_1^1 \cos \theta_1}{V_1 \cos \propto_1 - V_2 \cos \propto_2}$$



<u>Problem based on impact of elastic body:</u>

1. A sphere of 1 kg moving at 3 m/s, collides with another sphere of weight of 5 kg in the same Direction at 0.6 m/s. If the collision is perfectly elastic, find the velocity after impact.

<u>Given:</u>




$m_1 = 1$ kg

$m_2 = 5$ kg

$v_1 = 3$ m/s

$v_2 = 0.6$ m/s

Perfectly elastic impact e= 1

<u>To find:</u>

Velocity at after the impact $V_1^1$ & $V_2^1$

Soln:l

Law of conservation of momentum

$m_1v_1 + m_2v_2 = m_1v_1^1 + m_2v_2$

$1\times3+5\times0.6=1\ v_1^1+5\ v_2^1$

$V_1^1+5\ V_2^1 = 6$-------> (1)

The coefficient of restitution, $e = \frac{V_2^1 - V_1^1}{V_1 - V_2}$

$e = 1$[perfectly Elastic Impact]

$1 = \frac{V_2^1 - V_1^1}{3 - 0.6}$

$V_2^1 - V_2^1 = 1\times[3-0.6]$

$V_2^1 - V_1^1 = 2.4$ -----> (2)



Solve Eqn (1) & (2)

$V_1^1+5\ V_2^1 = 6$

$V_2^1 - V_1^1 = 2.4$

$6\ V_2^1 = 8.4$

$V_2^1 = 8.4/6$

$V_2^1 = 1.4$ m/s

$V_2^1$ value sub in Eqn (1)

$V_1^1+5\ V_2^1 = 6$ --------> $V_1^1 = 6 - [5\times V_2^1]$

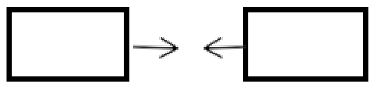
$V_1^1 += 6 - [5\times1.4]$

$V_1^1 = -1$ m/s

$V_1^1 = 1$ m/s

2. A car weighting 5 KN is moving east with a velocity of 54 k m p h and collide with a second car weighting 12 KN is moving west with a velocity of 72 k m p h If the impact is perfectly plastic, what will be the velocities of the cars. $V_1$=54km/h $v_2$=-72km/h

<u>Given:</u>



$W_1 = 5KN$ $W_2 = 12KN$

$M_1 = {}^5/_{9.81}$ $M_2 = {}^{12}/_{981}$

$W_1 = 5 \text{ KN} = {}^5/_{9.81} = 0.509 \text{ kg} = m_1$

$W_2 = 12 \text{ KN} = {}^{12}/_{9.81} = 1.22 \text{ kg} = m_2$

$V_1 = 54$ km/hr

$V_2 = -72$ km/hr



<u>To Find:</u>

Velocity of car

<u>Soln:</u>

Law of conservation momentum

$m_1 v_1 + m_2 v_2 = m_1 v_1^1 + m_2 v_2^1$

Perfectly plastic means e=0

$\therefore v_1^1 = v_2^2 = e$

$0.509 \times 54 + 1.22 \times [-72] = 0.509 \times v_1^1 + 1.22 \times v_2^1$

$27.486 - 87.84 = [0.509\, V\, c + 1.22]$

$-60.354 = Vc \times 1.729$

$$V_c = \frac{-60.354}{1.729}$$

$$Vc = -34.90\ km/hr$$

3. Direct central impact occurs between 300N body moving to right with a velocity of 6 m/s and 150N body moving to the left with a velocity of 10 m/s. Find the velocity of each body after the impact if the coefficient of restitution is 0.8.

Same as problem No:1

Ans: $V_2^1$=9.2 m/s

$V_1^1 = -3.6$ m/s

4. Two bodies, one of which 20N and velocity 10 m/s and the other of 100N with a velocity of m/s downward,each other and implinges centerlly. Find the velocity of each body of the impact if the coefficient of restitution is 0.6. Find also the loss in kinetic energy due to impact.



Given data:

$W_1$=20N $\quad m_1 = \frac{20}{9.81} \quad m_1$=2.038 kg

$V_1$=10 m/s

$W_2$=100N $\quad m_1 = \frac{100}{9.81} \quad m_2$=10.19 kg

$V_2 = -10$ m/s

Coefficient of restitution, e= 0.6

To find:

Final velocity After impact $V_1^1$ & $V_2^1$

Loss of kinetic Energy.

Soln:

Law of conservation of Energy

$m_1 v_1 + m_2 v_2 = m_1 v_1^1 + m_2 v_2^1$

$(2.038 \times 10) + (10.19 \times -10) = 2.038\, v_1^1 + 10.19 \times v_2^1$

$20.38 - 101.9 = 2.038 v_1^1 + 10.19\, v_2^1$

$-81.52 = 2.038\, v_1^1 + 10.19\, v_2^1$

$2.038\, v_1^1 + 10.19\, v_2^1 = -81.52$ --------> (1)

If coefficient of restitution Eg 'e' is given

$$e = \frac{V_2^1 - V_1^1}{V_1 - V_2}$$

$$0.6 = \frac{V_2^1 - V_1^1}{10 - (-10)} \quad \frac{V_2^1 - V_1^1}{20}$$



$V_2^1 - V_1^1 = 20 \times 0.6$

$V_2^1 - V_1^1 = 12$ ------- > (2)

Solve Eqn (1) & (2)

| | | | |
|---|---|---|---|
| | $2.038\, V_1^1 + 10.19\, V_2^1$ | = | $-$ 81.52------> (1) |
| Eqn (2) × 2.037 | $2.038\, V_2^1 - 2.038\, V_1^1$ | = | 24.456 |
| | $12.228\, V_2^1$ = | | $-$ 57.06 |

$$V_2^1 = \frac{-57.06}{12.228}$$

$V_2^1 = -4.66$ m/s

$V_2^1$ value sub in Eqn (1)

$2.038\, V_1^1 + 10.19\, V_2^1 = -81.52$ -------> (1)

$2.038\ V_1^1 + 10.19 \times (-4.66) = -81.52$

$2.038\ V_1^1 + [-47.55] = -81.52$

$V_1^1 = \frac{-81.52+47.55}{2.038}$

$V_1^1 = \frac{-33.96}{2.038}$

$V_1^1 = 16.66$ m/s

Loss of kinetic Energy:

= Initial kinetic Energy – Final kinetic Energy

[before Impact] [after impact]

Total kinetic Energy before impact

$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2}(10.19)(-10)^2$

$= \frac{1}{2}(20038)(10)^2 + \frac{1}{2}(10.19)(-10)^2$



Before K.E = 611.4 N.m

Total kinetic at after impact [find K.E]

$= \frac{1}{2} m_1 v_1^{1\,2} + \frac{1}{2} m_2^{1\,2}$

$= \frac{1}{2} \times (2.038)(-16.66)^2 + \frac{1}{2} \times (10.19)(-4.66)^2$

After K.E= 394.26 N.m

Loss of kinetic energy during impact

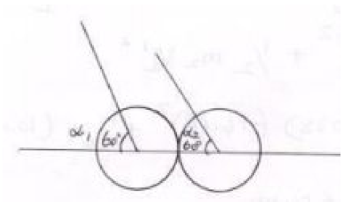
$=$ After K.E $=$ Before K.E

$=611.4-394.26$

Loss= 217.11 N.m

<u>Problem 5</u>

A ball of weight 500g moving with velocity of im/sec impings on a bar of mass 1kg moving with velocity 0.75m/s at the time of impact the velocity of the body are parallel and inclined at $60^0$to the line joining there centers. Determine the velocity direction of the ball after the impact where e= 0.6 also find the loss of kinetic energy due to impact.



$\propto_1=\propto_2= 60°$

$m_1$=500g= $\frac{500}{1000} = 0.5kg$

$m_2$= 1kg

$v_1 = 1m/s$

$v_2 = 0.75$



Coefficient of restitution e= 0.6

<u>To find:</u>

1. final velocity $v1^1$&$v\ 2^1$

2. Direction $\theta_1$& $\theta_2$

3. loss of kinetic energy

<u>Soln:</u>

Law of conservation of momentum

$m_1v_1 \cos \propto_1 + m_2v_2 \cos \propto_2 = m_1v1^1 cos\theta_1 + 1\ v2^1 cos\theta_2$

$0.635 = 0.5v1^1 cos\theta_1 + v2^1 cos\theta_2$

If coefficient of restitution is given

$e = \frac{v2^1 cos\theta_2 - v1^1 cos\theta_1}{v_1 \cos\propto_1 - v_2 \cos\propto_2}$

$0.6 = \frac{v2^1 cos\theta_2 - v1^1 cos\theta_1}{1 \cos 60 - 0.75\ os\ 60}$

$0.6[\cos - 0.7 \times \cos 60] = v2^1 cos\theta_2 - v1^1 cos\theta_1$ ---------------- (2)

Solve Eqn (1) &(2)

$0.625 = 0.5 v1^1 cos\theta_1 + v2^1 cos\theta_2$

$0.075 = v2^1 \cos\theta_2 \mp v1^1 \cos\theta_1$

$0.55 = 1.5\ v1^1 \cos\theta_1$

$v1^1 \cos\theta_1 = \frac{0.55}{1.5}$

$\therefore v1^1 \cos\theta_1 = 0.366$

$v1^1 \sin\theta_1 = v_1 \sin \propto 1$

$= 1 \sin 60$

$= 0.866$

$\frac{V_1^1 \sin \theta_1}{V_1^1 \cos \theta_1} = \frac{0.866}{0.366}$

$\tan \theta_1 = 2.366$

$\theta_1 = \tan^{-1}(2.366)$

$\theta = 67°$

$V_1^1 \cos \theta_1 = 0.366$

$V_1^1 \cos 67° = 0.366$

$V_1^1 = \frac{0.366}{\cos 67°}$

$V_1^1 = 0.94 m/s$

------> (2) $0.075 = V_2^1 \cos\theta_2 - V_1^1 \cos\theta_1$

$0.075 = V_2^1 \cos\theta_2 - 0.94 \cos 67°$

$0.075 + 0.94 \cos 67° = V_2^1 \cos\theta_2$

$0.442 = V_2^1 \cos\theta_2$

$V_2^1 \sin\theta_2 = 0.6495$

$\dfrac{V_1^1 \sin\theta_1}{V_1^1 \cos\theta_1} = \dfrac{0.6495}{0.442}$

$\tan\theta_2 = 1.469$

$\theta_2 = 55°$

$V_2^1 \cos\theta_2 = 0.442$

$V_2^1 \cos 55° = 0.442$

$V_2^1 = \dfrac{0.442}{\cos 55°}$



$V_2^1 = 0.785\ m/s$

Loss of kinetic Energy = Before K.E − After K.E

Before K.E $= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$= \frac{1}{2} \times 0.5 \times 1^2 + \frac{1}{2} \times 1 \times (0.75)^2$

Before K.E $= 0.25 + 0.281$

Before K.E $= 0.531$ N.m

After kinetic Energy $= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$= \frac{1}{2} \times 0.5 \times (0.94)^2 + \frac{1}{2} \times 1 \times (0.785)^2$

$= 0.2209 + 0.308$

After K.E= 0.528 N.m

Loss of K.E=before K.E−After K.E

=0.531−0.528

Loss K.E=$2.1\times10^{-3}$N.m

**DYNAMIC OF PARTICLES**

Dynamics

It is the branch of science which deals with the study of a body in motion.

Dynamic is further classified into two branches 1. Kinematics 2. Kinetics

Kinematics:

Kinematics is the study of motion of a moving body without considering the force.



Kinetics:

Kinetics is the study of motion of a moving body with considering external force.

Types of plane motion:

1. Rectilinear motion
2. Curvilinear motion

Rectilinear motion:

The motion of particle along a straight line.

Ex: A car moving straight road.

Ex: A stone vertically downward.

Curvilinear motion:

The motion of a particle along a curved path

Characteristic of Kinematics:

1. Displacement: 's'

   The displacement of a moving particle is the change in its position, during which the particle remains in motion. It is denoted by 's'

2. Speed:

   It is distance travelled by the particle (or) body along the path per unit time.

   $$\text{Speed} = \frac{\textbf{Distance dravelled}}{\textbf{time taken}}$$

3. Velocity 'v'

   It is the rate of change displacement.

   $$\text{Velocity} = \frac{\text{Distance travelled in a particular direction}}{\text{Time taken}} \quad m/s$$



4. Acceleration 'a'

   It is the rate of change of velocity acceleration

   $$a = \frac{\textbf{change of velocity}}{\textbf{time taken}}$$

   $$a = \frac{\textbf{final velocity} \sim \textbf{Intial velocity}}{\textbf{time taken}}$$

Negative acceleration is called retardation [When final velocity < Initial velocity]

5. Average velocity

   $$\text{Average velocity} = \frac{\textbf{Change in position}}{\textbf{Change in time}} = \frac{\Delta x}{\Delta t}$$

6. $$\frac{\text{Average speed}}{\text{Average speed}} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Mathematically Expression for Velocity and Acceleration:

Let s=Distance travelled by a particle in a straight line

t=time taken by the particle travelled this distance

Velocity= $\frac{ds}{dt}$

Acceleration= $\frac{dv}{dt}$= $\frac{d}{dt}(\frac{d}{dt})$ =$d^2s/dt^2$

Types of Rectilinear Motion:

1. Uniform acceleration
2. Variable acceleration

Rectilinear motion with uniform acceleration:

Eqn of motion in a straight line:

Consider the particle moving the uniform acceleration is a straight line.

Let u = Initial velocity ($m/s$)

v = final velocity ($m/s$)

s=Distance travelled (m)

t=time taken by the particle by the change from the u to v

a=acceleration of particle $m/s^2$

change o velocity=final velocity−Intial velocity

=v−u

Acceleration=$\frac{\text{change of velocity}}{\text{time taken}}$

a= $\frac{v-u}{t}$

a t = v−u

v=u+at---------------- > (1)

Average velocity= $\frac{\text{Initial velocity+final velocity}}{2}$

$= \frac{u+v}{2}$

Distance traveled by the particle in +sec

s=Average velocity ×time

$s=(\frac{u+v}{2})t$---------------- >(2)

velocity= $\frac{\text{distance}}{\text{time}}$

$v= \frac{s}{t}$

$s = v\ t$

$$s = \frac{u+v}{2} \times t$$

$2\ s = u + v + t$

$2s/t = u + v$

$u + v = \frac{2s}{t}$ v= u+at

$u+u + at = \frac{2s}{t}$

$$s = \frac{(2ut + at)t}{2}$$

$$s = \frac{2ut + at^2}{2}$$

$$s = \frac{2ut + at^2}{2}$$

$$s = \frac{2ut}{2} \neq \frac{at^2}{2}$$

$s = ut + {}^1/_2\, at$

s= $ut + {}^{1}/_{2}\, at^2$

from (1) v= $u + at$ t= $\frac{v-u}{a}$

$$s = u\left(\frac{v-u}{a}\right) + \frac{1}{2}\, a \times \left(\frac{v-u}{a}\right)2$$

$$s = \frac{uv-u^2}{a} + \frac{1}{2}\, a\left(\frac{v-u}{a^2}\right)$$

$$s = \frac{uv}{a} - \frac{u^2}{a} + \frac{1}{2}\;\frac{v^2+u^2-2vu}{a}$$

$$s = \frac{uv}{a} - \frac{u^2}{a} + \frac{v^2}{2a} + \frac{u^2}{2a} - \frac{2uv}{2a}$$

$$s = \frac{1}{2a}[uv \times 2 - u^2 \times 2 + v^2 + u^2 - 2uv]$$

$$s = \frac{1}{2a}[2uv - 2u^2 + v^2 + u^2 - 2uv]$$



$$s = \frac{1}{2a}[v^2 - u^2]$$

$$2\,as = v^2 - u^2$$

$v^2 = u^2 + 2a$

Problem 1:

An automobile travels 600m in 40s when it is accelerated at a constant rate of $0.6 m/s^2$. Determine the initial and final velocity and the distance travelled for the first 12s.

Given:

Total travels distance=600m

Total time=40s

Acceleration a=$0.6 m/s^2$

To find

Intitial and final velocity u &v

Distance travelled for the first 12s

Soln

Now

Distance travelled at 60 m

$s = ut + \frac{1}{2}at2$

$600 = u \times 40 + \frac{1}{2} \times 0.6 \times (40)^2$

Intial velocity.u=3$^m/_s$

Velocity v=u+at

$V = 3 + 0.6 \times 40$



Final velocity v=27 $^m/_s$

The distance travelled for the first 12s,1-2[1]

$a = 0.6\, m/m^2 \quad u = 3\, m/s$

s=ut+$\frac{1}{2}$at$^2$

s=3$\times 12 + \frac{1}{2} \times 0.6 \times (12)^2$

s=79.2 m

2. The motion of a p[article is defined by the relation $x = 3t^3 - 18t^2 + 26t + 8$

Where is the position expressed in metres and t is the time in seconds Determine (i)When the velocity is zero and (ii)The position and the total distance travelled when the acceleration becomes zero.

Given:

$x = 3t^3 - 18t^2 + 26t + 8$

x=position

t=seconds.

<u>Soln:</u>

Velocity=v= $\frac{dx}{dt}$

$v = \frac{d}{dt}(3t^2 - 18t^2 + 26 \times 1 + 0$

$v = 9t^2 - 36t + 26$

(ii)When velocity v= 0

0= $9t^2 - 36t + 26$

$9t^2 - 36t + 26$ =0

a=9 b= $-36$ c=26



$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(-36) \pm \sqrt{(-36)^2 - 4 \times 9 \times 26}}{2 \times 9}$$

$$t = \frac{-36 \pm 18.97}{18}$$

$t = \frac{36 \pm 18.97}{18}$ t= $3.094s$

$t = \frac{36 - 18.97}{18}$ t=0.946s

the velocity becomes zero t=0.946s and t=3.054s

Acceleration a= $\frac{dv}{dt}$

a= $\frac{d}{dt}[9t^2 - 36t + 26]$

$a = 9 \times 2t - 36 \times 1 + 0$

$a = 18t - 36$

Acceleration $a = 0$

$0 = 18t - 36$

$18t = 36$

$t = \frac{36}{18}$

$t = 2s$

Distance travelled from t=0 to t=2s

$t = 2s$ $\quad x = 3t^3 - 18t^2 + 26 + 8$

$x = 3 \times (2)^3 - 18 \times (2)^2 + 26 \times 2 + 8$

$x = 12\, m$



$t = 0s$ $\quad x = 3 \times (0)^3 - 18 \times (0)^2 + 26 \times 0 + 8$

$x = 8\, m$

When $t = 0.946\, s$ 'v' becomes zero

$x = 3(0.946)^3 - 18 \times (0.946)^2 + 26 \times 0.946 + 8$

$x = 19\, m$

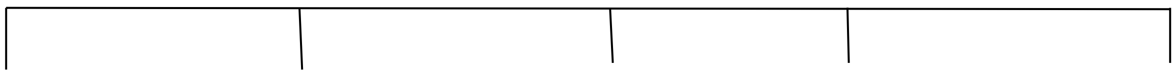
Total distance travelled $= (19 - 8) + (19 - 12)$

$= 18\, m$

3. A particle under constant deceleration is moving on a straight line and covers a distance of 25 m in the first 3s and 40 m in nest 6s. Calculate the distance it covers in subsequent 2s and the total distance covered before it come to rest.

Given:

A B C D E



S AB=25$m$ S BC= 40 $m$

t= 3$s$ t= 6$s$ t= 2$s$

(A-B) (UDRM)

S= ut+$\frac{1}{2}at^2$

$S_{A\text{-}B}= u_a\, t_{A-B} + a\, t_{AB^2}$

$25 \pm u + 3 + \frac{1}{2}(a) \times (3)^2$

$25 = 3\,u + \frac{1}{2}\, a \times 9$

25=3 u+4.5 $a$

Both side÷by 3

$\frac{25}{3} = \frac{34}{3} + \frac{4.5}{3}$ a



8.33=u+4.5 a

u+1.5 $a$ = 8.33----------- > (1)

A−C

s= $ut + \frac{1}{2}at^2$

$65 \pm u \times 9 + \frac{1}{2}at^2$

s=65 t= 3 + 6 = 9

65= $9u + 40.s\, u$

÷9

7.22=u+4.5 a

u±4.5 $a$ = 7.22

u= $7.22 - 4.5\ a$--------->(2)

sub (ii) in (i)

$7.22 - 4.5\ a + 1.5\ a = 8.33$

$-3a = 8.33 - 7.22$

$-3a = 1.108$

$a = 1.108/-3$

a= $-0.369\ m/s^2$ --------- >(3)

sub (iii) in (2)

$u = 7.22 - 4.5 \times (0.369)$

u= $8.88\ m/s$

To find velocity at point c



v=u+at

$v_c = u_A + at_{A-C}$

$=8.88+(-0.369)(9)$

$v_c = 5.56\ m/s$

For the motion from C to D (UDRM)

$v_c = 5.56 m/s$ $\quad t_{C-D} = 2s$ $\quad a = -0.369\ m/s^2$

s= $ut + \frac{1}{2} at^2$

$S_{C-D} = u_c t_{C-D} + \frac{1}{2} a\ t_{CD}^2$

$=5.56 \times 2 + \frac{1}{2} \times (-0.369) 2^2$

$S_{C-D} = 10.38$ m

Distance travelled in subsequent t=2s

s=10.38 m

For the motion from C-E (UDRM)

$V_c$=5.56 $m/s$ a= $-0.369 m/s$ $V_E$= 0

We have

$v^2 - u^2 = 2as$

$v_E^2 - v_C^2 = 2as$

$0^2 - (5.56)^2 = 2 \times (-0.369) \times s_{CE}$

$s_{CE}$=41.8 m

Total distance travelled before it comes to res

$= S_{AB}\ S_{BC} + S_{CE}$

=25+40+41.8

Total distance=106.9 m



4. The position of a particle which moves along a straight line is defined as $s = t^3 - 6t^2 - 15t + 40$ where s is expressed in m and + is in sec. Determine the (a) time at which the velocity will be zero. ( b) the position and distance travelled by the particle at that time (c) acceleration of the particle at that time (d) the distance travelled by the particle when t=4 to t=6

Given:

$s = t^3 - 6t^2 - 15t + 40$

Soln:

a) t=? Velocity v=0

$v = \frac{ds}{dt}$

$v = \frac{d}{dt}(t^3 - 6t^2 - 15t + 4)$

v=$3t^2 - 6 \times 2t - 15 \times 1 + 0$

v=$3t^2 - 12t - 15$

v=0

$3t^2 - 12t - 15 = 0$

t= $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

a= 3  b= $-12$ c$-15$

t= $\frac{-b \pm \sqrt{(-12)^2 - 4 \times 3(-15)}}{2 \times 3}$

t= $\frac{12 \pm \sqrt{144+130}}{6}$

t= $\frac{12 \pm \sqrt{324}}{6}$

t= $\frac{12 \pm 18}{6}$



t= $\frac{12+18}{6} = \frac{30}{6}$

T=5 Sec

&

t= $\frac{12-18}{6} = \frac{-6}{6}$

t= $-1$ $sec$

t$\neq -1$

t= 5 $sec$

b) t=5 Sec & displacement s=?

s= $t^3 - 6t^2 - 15t + 40$

s= $5t^3 - 6(5)^2 - 15 \times 5 + 40$

s= $-60\ m$

t= 0

s= $0^3 - 6 \times 0^2 - 15 \times 0 + 40$

s= $40\ m$

Distance travelled= $[s_t = 5] - [s_t = 0]$

$= -60 - 40 \qquad = -100\ m$

Distance travelled $= 100\ m$

3) when t=6 sec displacement 's'

s= $t^3 - 6t^2 - 15t + 40$

s= $6^3 - 6 \times 6^2 - 15 \times 6 + 40$

s= $4^3 - 6 \times 4^2 - 15 \times 4 + 40$

s= $-52\ m$



Distance travelled when t=4 to 5 sec

$= s_t = 5 - s_t = 4$

$= -60 - [-52]$

$= -60 + 52$

$= -8 = 8\ m$

Distance travelled when t=5 to 6

$= s_t = 5 - s_t = 5$

$= -50 - (-60)$

=10 m

Total distance travelled=8+10 $= 18\ m$

4) Acceleration a

a= $\frac{dv}{dt} = \frac{d}{dt}[3t^2 - 12t - 15]$

a=3× $2t - 12 \times 1$---(5)

a=6t−12

t=5 sec

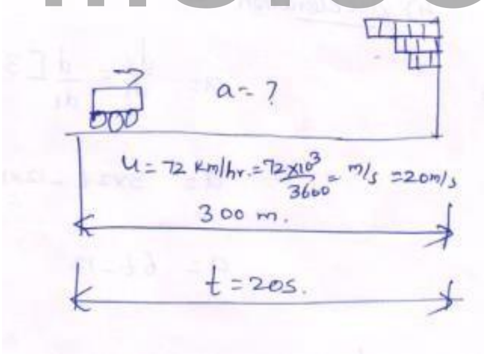
a=6× $5 - 12$

a= $30 - 12$

a=18$m/s^2$

5) A driver of a car travelling at $72km/h$ Observes the traffic light 300 m ahead of him turning red. The traffic light is timed to remain red for 20 seconds before it turns without stopping to wait for its turn green, Determine (i) the required uniform acceleration of the car (ii) the speed with which the motorist crosses the traffic light.






Soln:

Displacement

s= $ut + \frac{1}{2}at^2$

300=20$\times 20 + \frac{1}{2} \times a \times 20^2$

a= $-0.5\ m/s^2$ (Retardation)

Final velocity

v=u+at

v= $20 + (-0.5) \times 20$

v= $10\, m/s$

v= $\frac{10\times 3600}{1000} km/hr$

v= $36\ km/hr$

Problem:5

A particle starting from rest moves in a straight like and its acceleration is given by a=$50-36t^2 m/s^2$ Where t is in sec.Determine the velocity of the particle when it has travelled 52m.

Given



a= $50 - 36t^2$

s= $52m$

To find

Velocity

Soln

Acceleration a= $\frac{dv}{dt}$

$dv$=a× $dt$

$dv$=a× $dt$

$dv$=(50−$36t^2$)dt

$\int dv$=$\int(50 - 36t^2)dt$

$\int dv$=$\int(50 - 36t^2)dt$

v=$50t - 36 \times \frac{t^3}{3}$

v= $50t - 36 \times \frac{t^3}{3}$

v= $50t - 12t^3 + c_1$

when t= 0 v = 0 $c_1 = 0$

v= $50t - 12t^3$

ds= v × dt

ds= $50t - 12t^3 \times dt$

ds= $50t - 12t^3 \times dt$

$\int ds = \int (50t - 12t^3)dt$

s= $\frac{50t^2}{2} - 12 \times \frac{t^4}{4} + c_2$



s= $25t^2 - 3t^4 + c_2$

when t=0 s= 0 $c_2 = 0$

s= $25t^2 - 3t^4$

Now s=52 m finding out t

52=25× $t - 3t^4$

52=25$t - 3t^4$

Put $t^2 = t$

52=25 t−3$t^2$

$3t^2 - 25t + 52 = 0$

a=3

b=−25

c=52

$$t = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{-(25) \mp \sqrt{(-25)^2 - 4 \times 3 \times 52}}{2 \times 3}$$

t=2.0816sec & t=2sec

when t= 2sec   $v = 50 \times 2 - 12 \times 2^3$

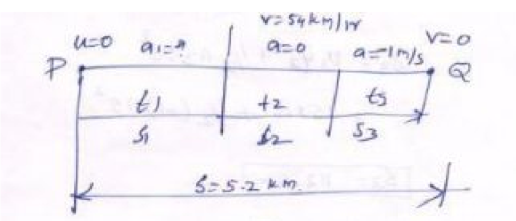
v=2m/s

when t=200816 sec   $v = 5050 \times 2.0816 - 12 \times (2.0810)^2$

$v = -4.163/m/s$

6. Two stations pand Q are 5.2km apart. A train stars from rest at the station P and accelerates uniformly to attain a speed of 54 km/hr in 30 sec. The speed is maintained until the brakes are applied. The train comes to rest at the station Q with uniform retaration of 1m/$s^2$. Determine the total time required to cover the distance b/w these two station






<u>Consider Phase I</u>

U=0

$t_1$=30 sec

$v_1$=15m/s

$v_1 = u + a_1 t_1$ $^{v=u+at}$

$15 = 0 + a_1 \times 30$

$a_1 = 0.5$ m/$s^2$

$s_1 = ut_1 + \frac{1}{2} a_1 t1^2$

$s_1 = 0 + \frac{1}{2} + a_1 t1^2$

$s_1 = 225$m

Consider Phase –III

$v_1 = 15$m/s

$a_3 = -1 m/s^2$

V=0

V=$u + at$

$0 = 15 - 1 \times t_3$

$0 = 15 - +3$



$t_3 = 15$ sec

$s_3 = u_3 v_3 + {}^1/_2\, a_3 t3^2$

$= 15 \times 15 + {}^1/_2\,(-1)15^2$

$s_3 = 112.5m$

Consider Phase-II

$s_2 = s - [s_1 + s_3]$

$s_2 = 5200 - [225 + 112.5]$

$s_2 = 4862.5$m

$s_2 = ut + {}^1/_2\, at^2$ $\quad a = 0$

$s_2$=ut

4862.5=15× $t$

$t_2=\frac{4862.5}{15}$

$t_2$=324.167sec

Total time=30+324.167+15

time=369.167 sec

multiply 2

120= $14\,u + 49\,a$

14 u+49 a=129=0 ---- (1)

÷=14

$u + 3.5a = 8.57$

$u$=8.57-3.5 a---- (2)



Sub Eqn (2) in (1)

$u + a$=10-------(1)

$3.57 - 3.5a + a = 10$

$8.57 - 2.5a = 10$

-2.5a= $10 - 8.57 = 1.43$

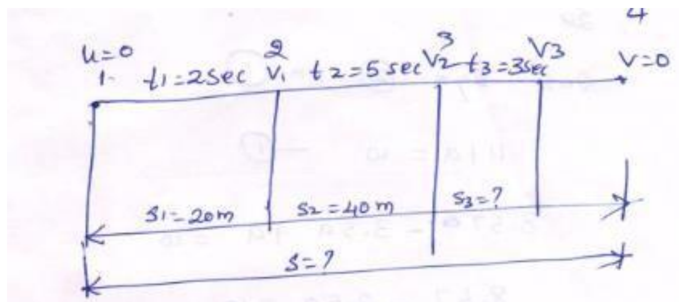
$a=\frac{1.43}{-2.5}$

$a = -0.572\ m/s^2$

u+$(-0.572) = 10$

u=10.572m/s

7. A particle under constant declaration is moving in a straight line and covers a distance of 20m in first 2seconds, and 40min the next 5sec. Calculate the distance it covers in the he subsequent 3sec and total distance travelled by the particle before it comes to rest.




Soln:

Phase (1)-2

The displacement s=$ut + {}^{1}/_{2}\, at^2$

$t$=2 sec s=20m



20=$u \times 2 + {}^{1}/_{2}\, at^2$

20=2u+2$a$

$\div$ 10 $u + a = 10$--------------(1)

Phase 1-3

s= $u + {}^{1}/_{2}\, at^2$

60=$u \times + {}^{1}/_{2} \times a \times 7^2$

60=7$u + {}^{1}/_{2} \times 49a$

S= $20 + 40 = 60$

$t$=2+5=7

Considered 3rd phase

t=2+5+3=10

$s_3$=10.572$\times 10 - {}^{1}/_{2} \times 0.572 \times 10^2$

$s_3$=17.142 m

$v_3$=$u + at$

$v_3$=10.57$-$0.572 $\times$ 10

$v_3$=4.857 m/s

Considered 4th phase

$u_4$=4.857 v=0 a=$-$0.572 $m/s^2$

V=$u + at$

0=4.857+($-$0.572)$\times t$

$t$=8.5 sec

Total time =2+5 + 3 + 8.5 = 18.5 $sec$

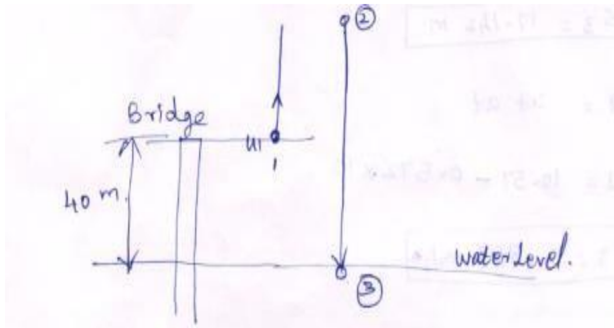


Total distance travel =s= $ut + \frac{1}{2} at^2$

S=10.57 $\times$ 18.5 $- {}^{1}/_{2} \times 0.57 \times 18.5^2$

S=97.78m

8. A stone is thrown vertically upwards at a point on a bridge located 40m above the water. If it strikes the water after 4sec, determine (i) the speed at which the stone was thrown up and (ii)The speed at which the stone strikes the water.




Soln:

For the a= $-9.81\ m/s^2$ $v_2 = 0$ $t_{1-2} = t$ $s_{1-2} = h$

$v$ =u+at

$0 = u - 9.81 \times t$

u= +9.81 t-------->(1)

Distance s=ut+$\frac{1}{2}at^2$

$s_{1-2} = 9.81t \times t - \frac{1}{2}9.81t^2$

$s_{1-2} = 9.81t^2 - 4.905t^2$

$s_{1-2} = 4.905t^2$ $s_{1-2} =$ h----- >(2)

h=4.905 $t^2$

For motion 2 to 3

$s_{2-3} = h + 40 =$ $v_2 = 0$ $t_{2-3} = 4 - t$

a= $9.81\ m/s^2$

s= $ut + \frac{1}{2}at^2$

$s_{2-3} = u_2\ t_{2-3} + \frac{1}{2}at_{2-3}{}^2$

h+40 $= 0 + \frac{1}{2} \times 9.81 \times (4 - t)^2$

$h+40 = \frac{1}{2}9.81 \times (4-t)^2 = 4.905[4-t]^2$

sub in (2)

$4.905t^2 + 40 = 4.905[16 + t^2 - 8t]$

$4.905t^2 + 40 = 78.48 - 4.905t^2 - 39.24t$

$40 = 78.48 - 39.24t$

$-39.24t = 40 - 78.48$

$-39.24t = -38.48$

$t = +0.98s$

$u = 9.81 \times t = 9.81 \times 0.98$

$u = 9.62 m/s$

$v_3 = v_2 + 9.81(4-t)$

$v_3 = 0 + 9.81(4 - 0.98)$

$v_3 = 29.62\ m/s$