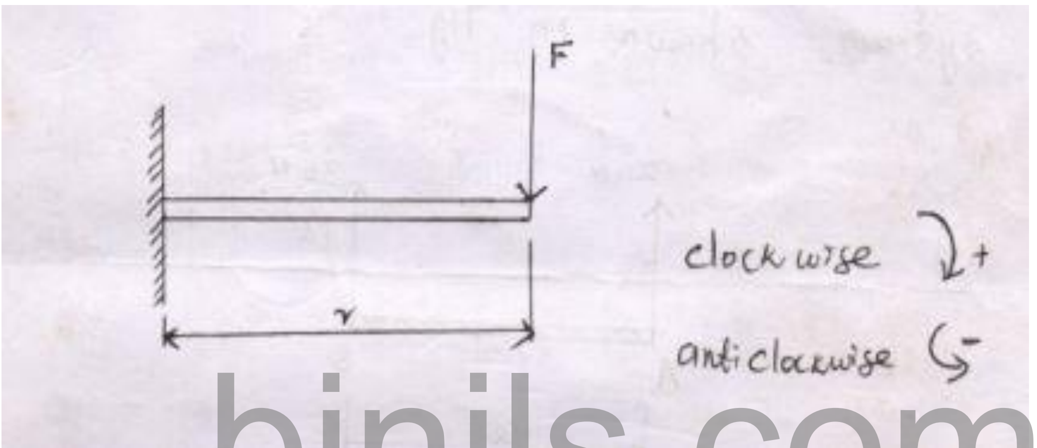
# UNIT II

## Statics of Rigid bodies in Two Dimensional

**Moment of force:**

Moment of force is defined as the product of the force and perpendicular distance of the line of the force from the point.





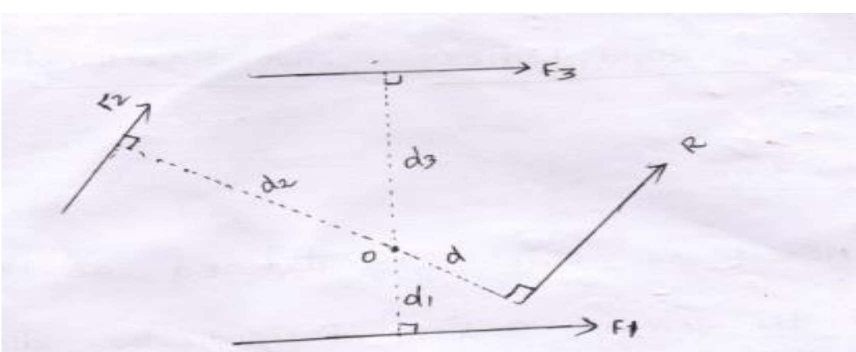

$$Moment = Force \times perpendicular\ distance.$$

$$Mo = F \times d\ N.m$$

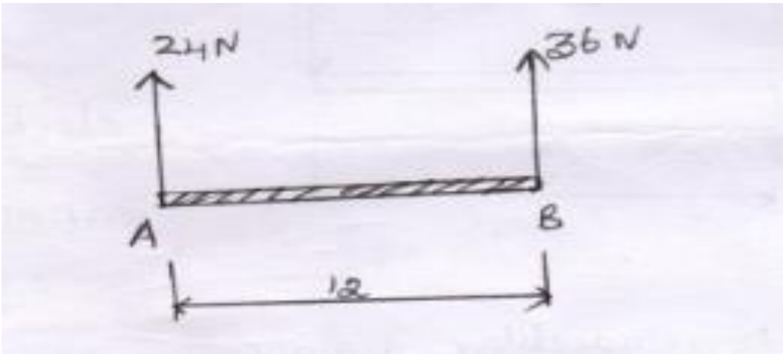
**Varignon's Theorem:**

The algebraic sum of the moment of any number of force about any point in their plane is equal to the moment of their resultant about the same point.

$$F1 \times d1 + F2 \times d2 + F3 \times d3 = R \times d$$



Find the resultant force for the parallel force System shown in fig.




Resultant force 'R'

$$R = 24 + 36$$

$$R = 60N$$



Location of resultant force:

Algebric sum of moment of all force about a

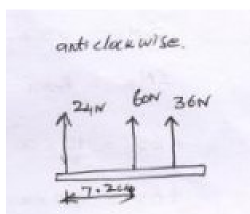
$$\sum M_A = -36 \times 12$$

$$\sum M_A = -432\, N.m$$

$$\sum M_A = 432\, N.m(clockwise)$$

By virginal theorem

$$\sum M_A = R \times x$$

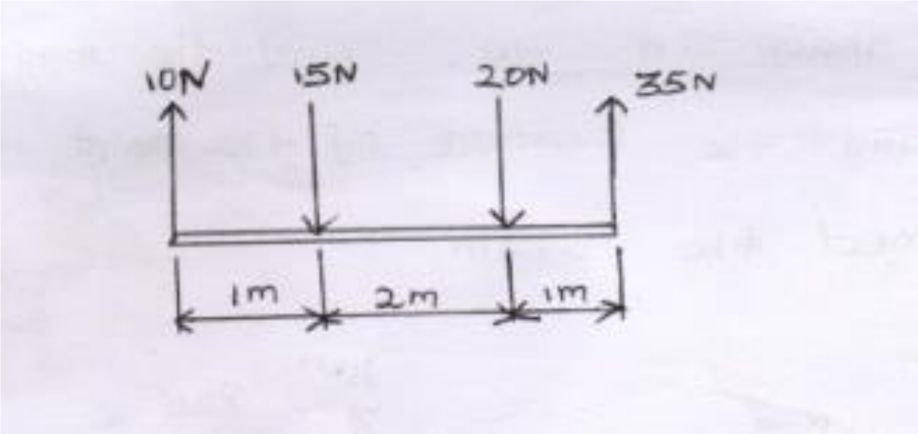



$$+432 = +60 \times x$$

$$x = \frac{+432}{+60}$$

$$x = 7.2\, cm$$

2. Four parallel forces of magnitude 10N, 50N, 20N and 35N as shown in fig. Determine the magnitude and direction of the resultant. Find the distance of the resultant from point A.




Solution:-

Magnitude of resultant:-



$R = 10 - 15 - 20 + 35$

$R = +10N$

Locating of the resultant

$\sum M_A = R \times x$

$\sum M_A = (15 \times 1) + (20 \times 3) + (-35 \times 4)$

$\sum M_A = -65\ N.m$

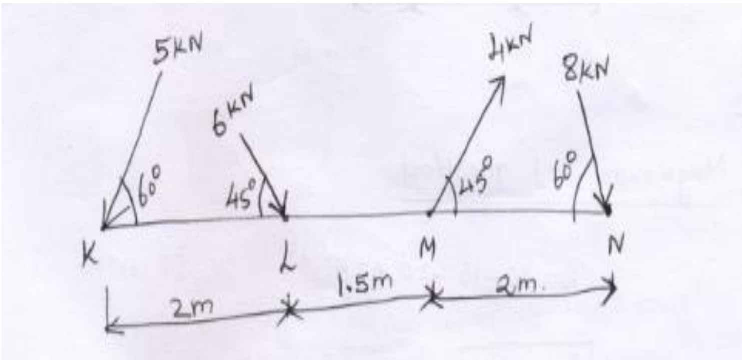
$\sum M_A = R \times x$

$+65 = 10 \times x$

$x = (+65)/(+10)$

$x = 6.5m$

1. A system of forces acts on a weightless beam as shown I fig. Find the magnitude of the resultant and the location of the point where the resultant met the beam.




Given:

$$Load\ at\ K = 5KN\ at\ 60^{\circ}$$

$$K = 6KN\ at\ 45^{\circ}$$

$$M = 4KN\ at\ 45^{\circ}$$

$$N = 8KN\ at\ 60^{\circ}$$



To find:

Resultant force & location

Soln:

Resultant R=$\sqrt{(\sum FH)^2 + (\sum FV)^2}$

$$\sum F_H = 0 \overset{+}{\rightarrow}\overset{-}{\leftarrow}$$

$$= -5\ cos\,60\ +\ 6\ cos\,45\ + 4\ cos\,45\ + 8\ cos\,60$$

$$\sum F_H = 8.57\ KN$$

$$\sum F_v = 0 \uparrow + \quad \downarrow -$$

$= -5\ sin\ 60 + 6\ sin\ 45 + 4\ sin\ 45 + 8\ sin\ 60$

$\sum F_v = -12.67\ KN$

R $= \sqrt{(\sum FH)^2 + (\sum FV)^2}$

R $= \sqrt{(8.57)^2 + \sum(12.67)^2}$

$R = 15.3\ Kn$

Inclination of the resultant $\alpha = tan^{-1}\left(\frac{\sum F_V}{\sum F_H}\right)$

$\alpha = tan^{-1}(\frac{12.67}{8.57})$

$\alpha = 55.92°$

To locate the resultant:

$\sum M_k = 0 \downarrow + \uparrow -$

$\sum M_k = 0 + [+sin45 \times 2] + [-4sin45 \times 3.5] + [+8sin\ 60 \times 5.5]$

$\sum M_k = +36.69\ KN.m\ (clockwise)$

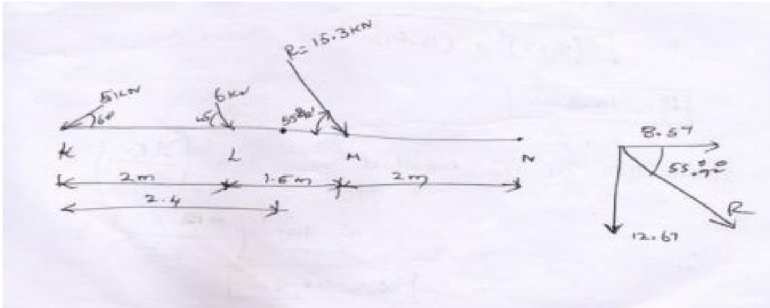
By varigon's Theorem

$\sum M_k = R \times x$

$+36.69 = 15.3 \times x$

$x = \frac{+36.69}{15.3}$

$x = 2.4m$

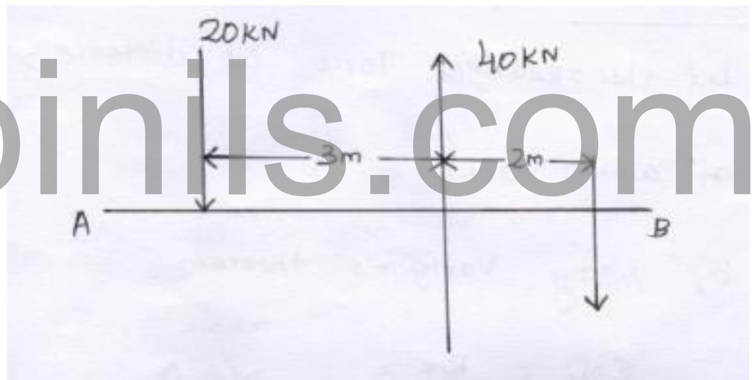


Problem:1

A coplanar parallel force system consisting of three forces acts on a rigid bar AB as shown fig. below

a) Determine the simplest equivalent action for the force system.

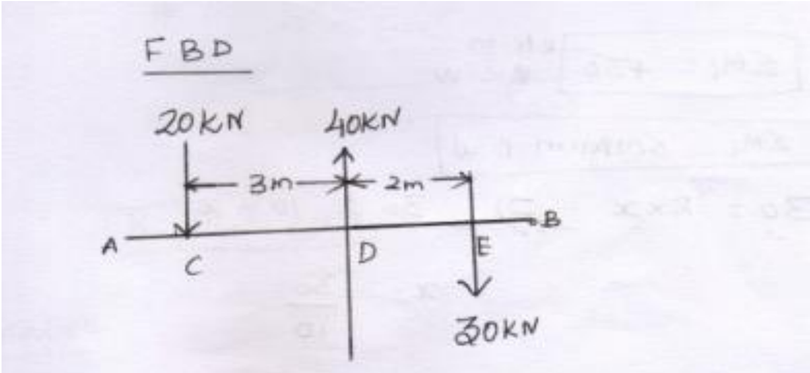
b. If an additional force of 10kn acts along the bar A to what be simplest equivalent action.






soln:

(a) simplest Equivalent force:




Sum of Horizontal force $\sum F_H = 0$

$$\sum FH = 0$$

Sum of vertical force $\sum FV = 0$

$\sum FV = 20 + 40 - 30 = -10KN$

Magnitude of Resulatant Force = R

$$R = \sqrt{(\sum(F_H)^2 + (\sum(F_V)^2}$$

$$= \sqrt{0^2 + (-10)^2}$$

$$R = \sqrt{100}$$

$$R = 10N$$

Line of Action:-

Let the resultant force at distance 'X' From the line of action 20KN

By using varigon's theorem

$$\sum M_c = R \times x$$

$$\sum M_c = (-40 \times 3) + (30 \times 5) = 120 + 150$$

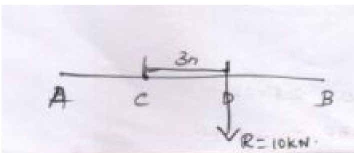
$$\sum M_c = +30\ Nm$$

$$\sum M_c = 30N.m\, c.w$$

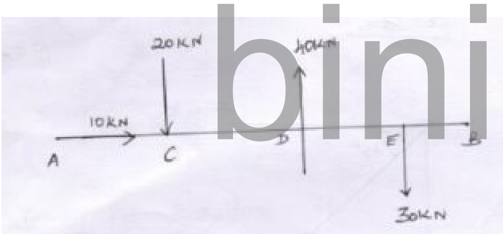
$$30 = R \times x \quad \rightarrow \quad 30 = 10 \times x$$

$$X = \frac{30}{10}$$

$$x = 3m$$




b) With additional force of 10KN from Ato B






Sum of Horizontal force $\sum F_H$=0

$$\sum F_H = 10KN$$

Sum of horizontal force $\sum F_v$=0

$$\sum F_v = -20 + 20 - 30$$

$$\sum F_v = -10KN$$

Resultant Force 'R' $R = \sqrt{(\sum(F_H)^2 + (\sum(F_V)}2$

$$R = \sqrt{(10)^2 + [-10]}^2$$

$R = \sqrt{100 + 100} = \sqrt{200}$

$R = 14.14\ KN$

Location

$\sum M_c = \sum F_v$

$\sum M_c = (-40 \times 3) + (30 \times 5) = -30kN.M$

$\sum M_c = 30\ KN.M$ clockwise

$30 = 10 \times x$

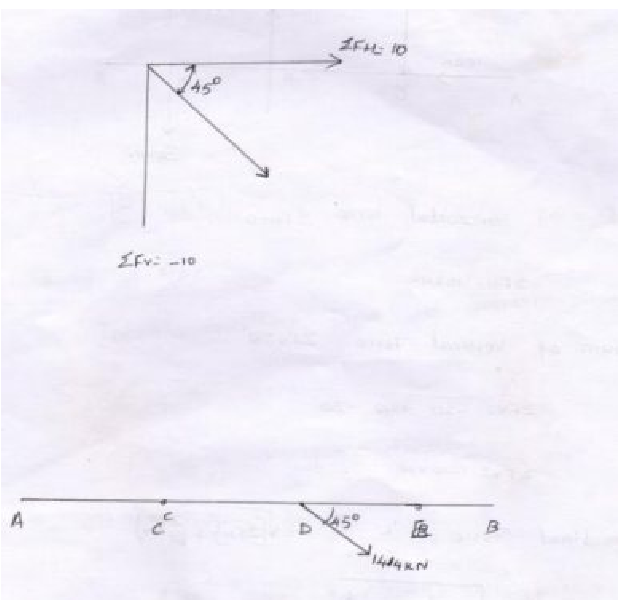
$x = 13m$

Location

$\theta = tan^{-1}\left(\frac{\sum F_v}{\sum F_v}\right) = tan^{-1}\left(\frac{10}{10}\right)$

$\theta = 45°$





$-80P = -4628.2$

$$P = \frac{4628.2}{80}$$

$P = 61.60N$

ii) Magnitude of the Resultant force:

Resultant $R = \sqrt{(\sum(F_H)^2 + (\sum(F_V)}$

$\sum F_H = -61.60 - 100\, cos60$

$\sum F_H = -111.60\ N$

$\sum F_v = 100 \sin 60$

$\sum F_v = 86.6N$



$R = \sqrt{[-111.60]^2 + [86.6]^2}$

$R = 141.26N$

iii) Point of Application

By Varigon's theorem

$\Sigma M_o = R \times x$

$\Sigma M_o = 61.60 \times 40 + [-100\, sin\ 60 \times 80\,] = 0$

$\Sigma M_o\ 2464 - 6928$

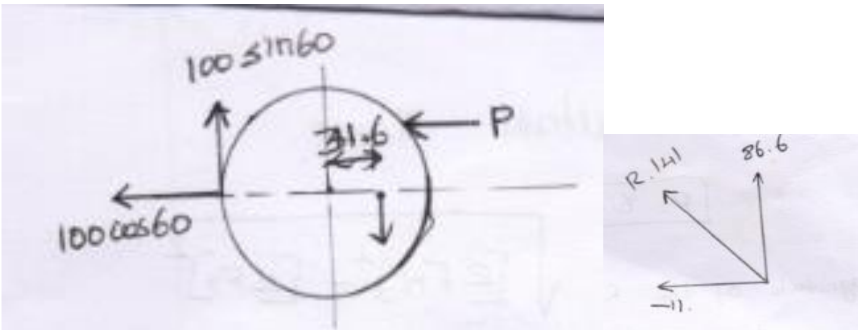
$\sum M_O = -4464.2$ Counts clocks wise

$\sum M_O = 4464$ Clockwise

$\Sigma M_o = R \times x$

$4464.2 = 141.26 \times x$

$X = 31.60\, mm$




6. A 200KN vertical force is applied to the end of a lever which attached a shaft as B as shown in Fig Below. Determine the(i) magnitude of horizontal force (ii) The smallest force applied at which creates the same moment about B(iii) How far from the end B, at 400KN Vertical force must to create the same moment about B (iv) Replace the given system of force at B.




$Vertical\ load\ at\ point\ A = 200KN$

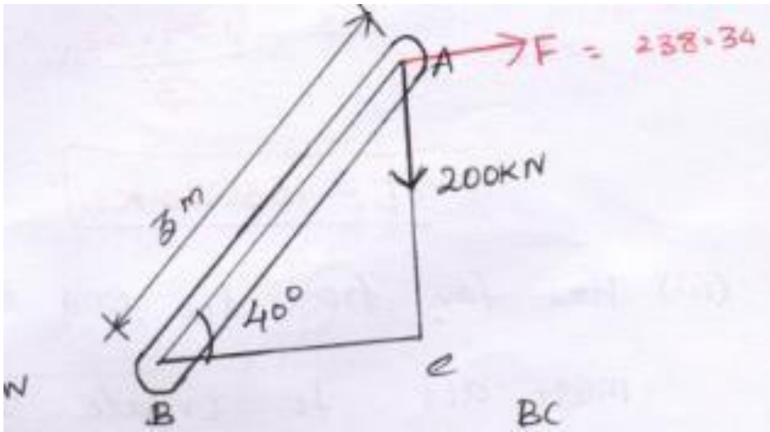
$Length\ of\ bar\ L = 3m$

$Angle = 40°$

Soln:

(i) The magnitude of horizontal force applied at 'A" which create same moment about 'B'

Take moment about 'B'




$M_0 = +200 \times BC$

$M_O = +200 \times 2.29$

$M_O = +459.62\ KN.M$

$cos\theta = \frac{BC}{3}$

$cos\theta = \frac{BC}{3}$

$BC = 2.29m$

→F

$MD = 459.62\ KN.m$



$sin\,\theta = \frac{AC}{AB}$

Take moment About 'o' horizontal force

act forwards right

$sin\,\theta = \frac{AC}{AB}$

$\sin 40 = \frac{AC}{3}$

$AC = 1.92m$

$M_D = F \times AC$

$459.62 = F \times 1.92$

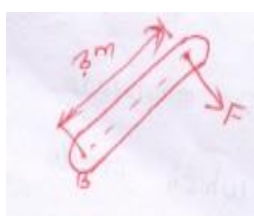
$F = \frac{459.62}{1.92} \frac{KN.m}{m}$

$F = 238.34\ KN$

ii) The smallest force applied at which create the same moment about 'B'

moment About $B = 459.62\ KN.m$

$M_B = F \times 3$



$459.62 = F \times 3$

$F = \dfrac{459.62}{3}$

$F = 153.20\, KN$

(iii) How far from the end B, a 400KN vertical force must act to create the same moment about B.

Let 400KN Vertical force act at a distance of 'x' A to have same moment −459.62 KN.m clockwise

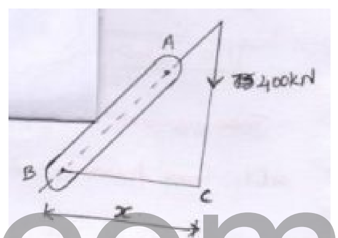
To have clockwise moment 400 N Vertical force on the right side of A

$Moment = -459.62\, KN.m$

$-400 \times x = -459.62$

$x = (-459.62)/(-400)$

$x = 1.149\, m$



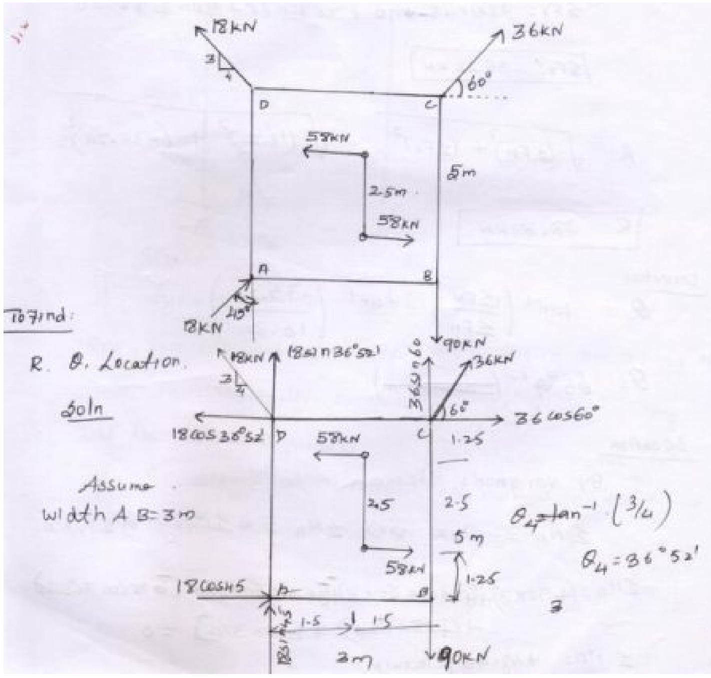



iv) Replace the given system of Force at B


$Downward\ load = 200KN$

$Moment\ at\ B = 459.63\, KN.m$

7. Determine the resultant of The calendar non concurrent force system shown in fig. below. Calculate its mangnitute and direction and locate its position with respect to the sides AB and AD




Resultant force



$R = \sqrt{(\sum(F_H)^2 + (\sum(F_V)^2}$

$\sum F_H = 0+ \longrightarrow \quad \longleftarrow$ - $F_H = F\, cos\theta$

$\Sigma F_H = 18cos\, 45 + 36\, cos\, 60 - 18cos\, 36° \, 52' - 58 + 58$

$\Sigma F_H = 16.32\, KN$

$\sum F_V = 0 \quad +\uparrow \quad \downarrow$

$\Sigma F_v = 18\, sin\, 45 - 90 + 36\, sin60° + 18\, sin36°52' = 0$

$\sum F_V = -35.26\, KN$

$R = \sqrt{(\sum(F_H)^2 + (\sum(F_V)^2} = \sqrt{(16.32)^2 + (-35.26)^2}$

$R = 38.88KN$

Direction:-

$$\theta = tan^{-1}\left(\frac{\Sigma_{FV}}{\Sigma_{FH}}\right) = tan^{-1}\left(\frac{-35.26}{16.32}\right)$$

$$\theta = 65°9'$$

Location:

By varignon's theorem

$$\Sigma M_A = R \times x \text{ (or)} \Sigma M_A = \Sigma F_H \times y \text{ or } \Sigma F_V \times x$$

$$\Sigma M_A = (+90 \times 3) + (36 \cos 60 \times 5) + (-36 \sin 60 \times 3) + (-18 \cos 36°52' \times 5) + (+58 \times 1.25) + (-58 \times 3.75) = 0$$

$\sum M_A = +49.46\ KN.M$ (clockwise)

$\sum M_A = 49.46$(clockwise)

$$\Sigma M_A = \Sigma F_V \times x$$

$$49.46 = 35.26 \times x$$

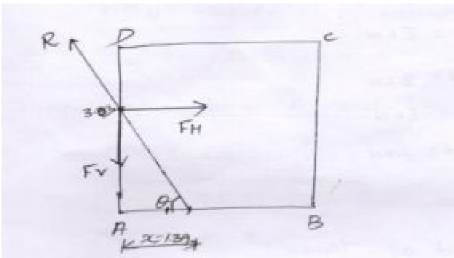
$$x = 1.39m$$



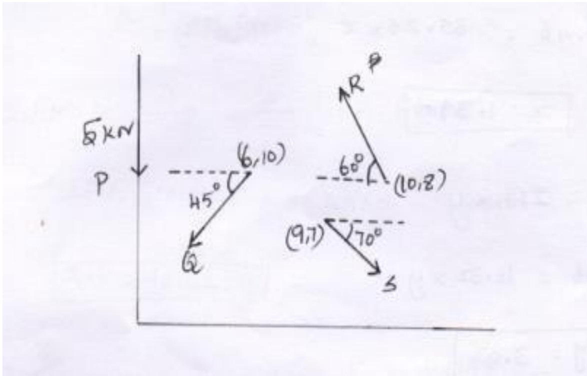
$$\sum M_A = \sum F_H \times y$$

$$49.46 = 16.32 \times y$$

$$y = 3.03$$




8. A system of four forces P, Q, R and s of magnitude 5KN, 8KN, 6KN And 4KN respectively acting on a body are shown in rectangular coordinates. As shown in fig find the moment of the forces about the origin O. also find the resultant moment of the forces about O. The distance are in meters.



Given:

$Load\ on\ P = 5KN$

$Load\ on\ Q = 3KN$

$Load\ on\ R = 6KN$

$Load\ on\ S = 4KN$



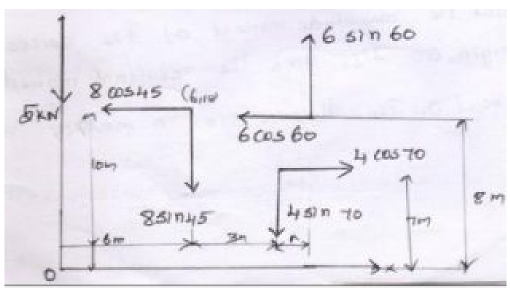
To Find:

1. moment of Forces

2. Resultant

Soln:-

Free body diagram



Moment of P

Moment of force 'P' about the orgin, $M_P$

$M_p = 5 \times 0$

$M_p = 0$

Moment of Q

Moment of force 'Q' about the orgin, $M_Q$

$M_Q = (8 sin\, 45 \times 6) + (-8 cos\, 45 \times 10)$

$M_Q = -22.64 KN.m$

$M_Q = +2.64\, KN\ \ C.W$

Moment of R

Moment of force R about the orgin $M_R$

$M_R = -75.96\, KN.m$

$M_R = 75.96\, c.w$

Moment of S



Moment of force s about the Orgin '$M_S$'

$M_s = (4\, cos \times 7) + (4 sin\, 70 \times 9)$

$M_s = 43.40\ KN.m$

**Statics of Rigid bodies Force couple system**

Moment of a Force:-

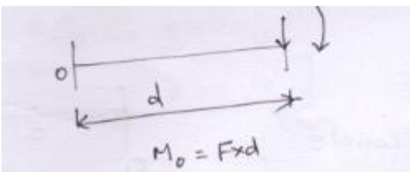
Moment of a Force about a point is defined as the product of the force and the perpendicular distance of the line of action of the force from the point

$M = F \times d$

$F = Force$

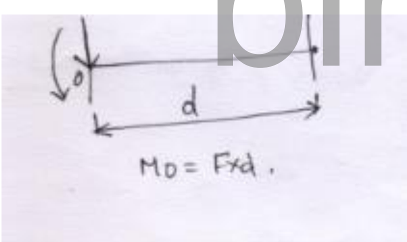
$d = perpendicular\ distance$

The clockwise direction of moment is positive direction of moment




The Anticlockwise bending moment gives the negative direction of moment

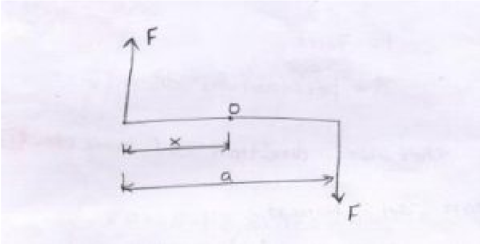





Coupled force:

It is a turning effect produce in the body of object by applying two forces having same magnitude put in opposite Direction.

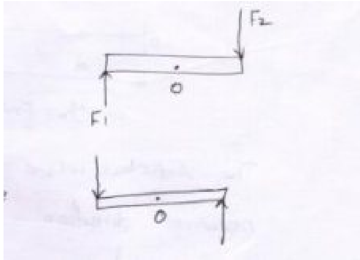
Two forces F and –F having the same magnitude, parallel lines of action and opposite sense are said to form a couple.




Types of couple:

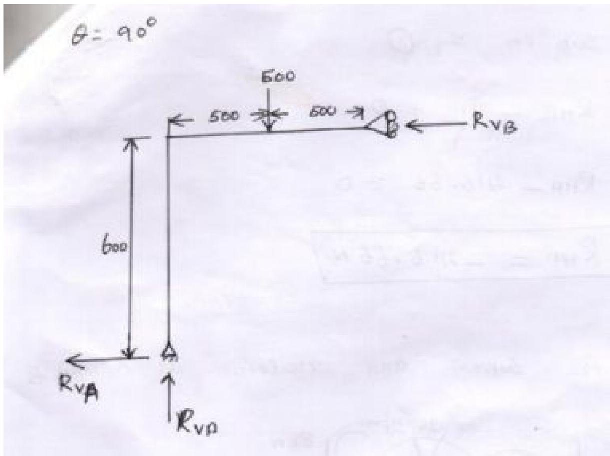
1. clock wise couple

2. Anti- clockwise couple




$\theta = 90°$






$\Sigma F_H = 0$

$-RVA - RVB = 0$--------- (1)

$\sum FV = 0$

$RV_A - 500 = 0$

$RV_A = 500N$

$\sum M_A = 0$

$[500 \times 500] + [-RV_B \times 600] = 0$

$250 \times 103 = RV_B \times 600 = 0$

$-RV_B = -250 \times 103$

$$RV_B = \frac{-250 \times 10^3}{-600}$$

$RV_B = 416.66N$



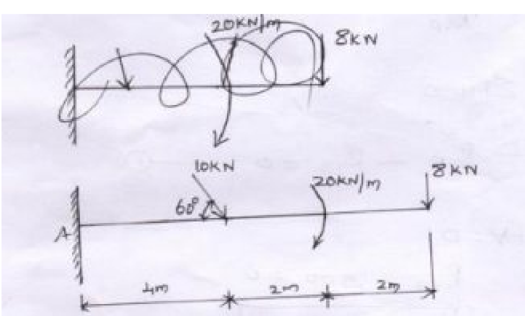
$R_{VB}$ Sub in Eqn (1)

$-RH_A - RH_B = 0$

$RH_A = -416.66N$

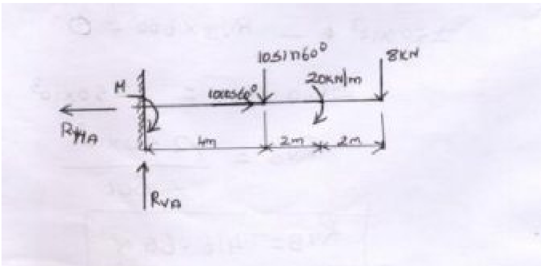
Problem:

Determine the support and reaction at A and B



Given

Free body diagram




$$-1019.61\ RV_B = -250 \times 103$$

$$RV_B = \frac{-250 \times 10^3}{-1019.61}$$

$$RV_B = 245.19\ N$$

Sub in Eqn------(1)

$$-RH_A - RV_B\ \cos 30 = 0$$

$$-RH_A = -RV_B \cos 30 = -245.19 \cos 30$$

$$RH_A = 212\ N$$

$$RV_A\ Sub\ in\ (2)$$

$$RV_A +\ \sin 30 = 500$$

$$RV_A = -RV_B\ \sin 30 +\ 500$$

$$RV_A = -245.19 \times\ \sin 30 +\ 500$$

$$RV_A = -122.5 + 500$$

$$RV_A = 377.5N$$

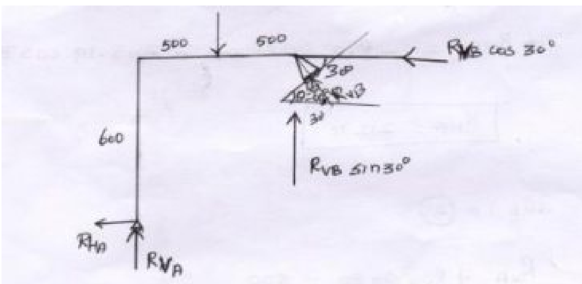
$$RV_B = \frac{-250 \times 10^3}{-1000}$$

$$RV_B = 250\ N$$

$RV_A + RV_B = 500 ---> RV_A = 500 - 250$

$RV_A = 250N$

ii) when $\theta$=60°




$\sum F_H = 0$

$-RH_A - RV_B \cos 30 = 0$

$\sum F_v$ =0

$RV_A - 500 + Sin\, 30 = 0$

$RV_A + RV_B\, Sin\, 30° = 500$

$\sum M_A$ =0

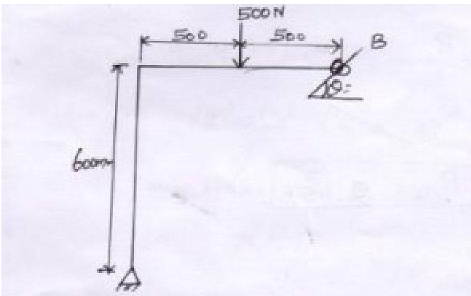
$\sum M_A$ =[500× 500] + $RV_B$ cos 30 × 600] + [$-RV_B$ sin 30° × 1000]

$$\sum M_A = 250 \times 10^3 - RV_B\, 519.61 - RV_B\, 500 = 0$$

$250 \times 10^3$ −01019.61 $RV_B$ =0

Problem:

A Frame supported at A and B is subjected to force 500N as shown in fig compute the Reaction the support for the cases i) $\theta$ =90°$\theta$ =60°




Given $\theta = 0° \theta = 90° \theta = 60°$

To find

Reaction at the support

i) $\theta = 0^0$

$\sum F_H = 0$

$RH_A = 0$



$\sum F_V = 0$

$-500 + RV_B + RV_A = 0$

$RV_A + RV_B = 500$------> (1)

$\sum M_A = 0$

$[500 \times 500] + [RV_B \times 1000] = 100$

$250 \times 10^3 - 1000 RV_B = 0$

$-1000 RV_B = -250 \times 10^3$

To find reaction 'R'

$\sum F_H = 0$

$-RH_R - F_{PQ}\cos 25^0 = 0$

$-RH_R = F_{PQ} cos25^\circ$

$RH_R = -F_{PQ} cos25^\circ$

$RH_R = - \times 3 \times cos25$

$RH_R = 2.45\,N$

$\sum F_V = 0$

$RV_R - 4 - FP_Q sin\,23 = 0$

$RV_R - 4 - 3 \times 25^\circ = 0$

$RV_R = 4 + 3sin\,25^\circ$

$RV_R = 5.26\,N$

$\sqrt{[RH_R]^2 + [RV_R]^2}$



R= $\sqrt{(2.45)^2 (5.26)^2}$

$R = 5.80N$

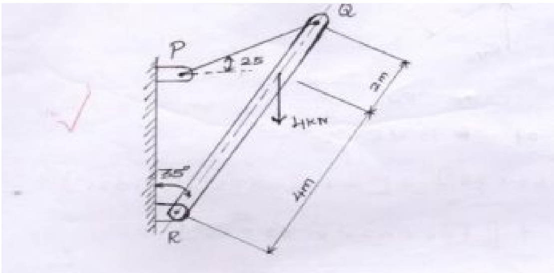
$\theta = tan^{-1}\left(\frac{\sum R_H}{\sum R_V}\right)$

$\theta = tan^{-1}\frac{5.26}{2.43}$

$\theta = 65^0$

Problem:

4000N load acts on the beam held by a cable PQ as shown in fig. The weight of the beam can be neglected. Draw the free body diagram of the beam and find tension in cable PQ. Also find the reaction force at R




To find:

1. Free body diagram

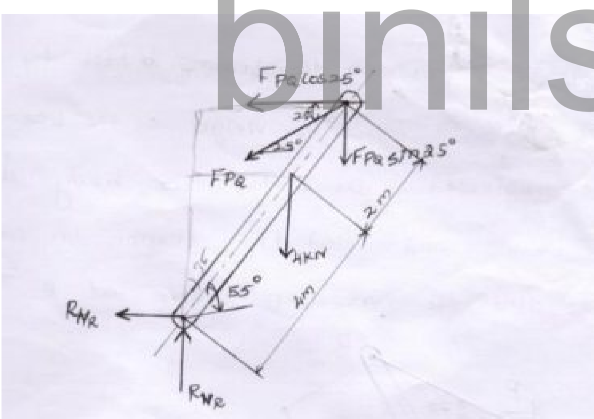
2. Tension in cable PQ

3. Reaction on Force R

Soln:

1. Free body diagram:






2. Tension in cable 'PQ'

Moment at point 'R'

$$\sum M_R = [4 \times \sin 35°] + [-F_{PQ} \cos 25° \times 6 \cos 35°] + [F_{PQ} \sin 25° \times 6\, sin 35°] = 0$$

$$\sum M_R = 9.177 - F_{PQ} \times 4.454 + 1.45 F_{PQ} = 0$$

$-4.45F_{PQ} + 1.45F_{PQ} = -9.177$

$-3F_{PQ} = -9.177$

$F_{PQ} = \frac{-9.177}{-3}$

$F_{PQ} = 3N$

**Procedure for finding out the resultant of non current coplander force system:**

1. Resolve the given forces, if they are inclined to reference x and y Axis.
2. Find the sum of horizontal component of forces $\sum FH$
3. Find the sum of vertical component of forces $\sum FV$
4. 4.Calculate the resultant force R $= \sqrt{(\sum(F_H)^2 + (\sum(F_V)^2}$
5. Angle of inclination of resultant $\theta = \tan^{-1}[\frac{\sum FV}{\sum FH}]$
6. If the force moment system is converted into a single force, coordinate position is given by

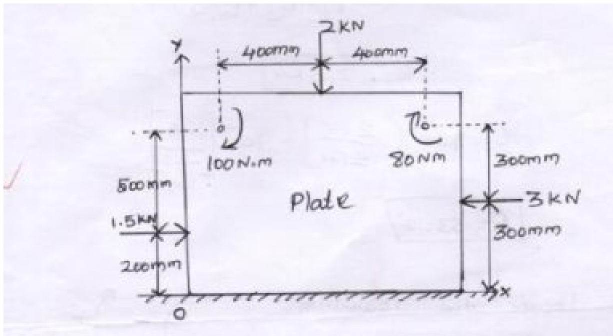


$$\sum M_o = R \times x$$

$$\sum M_o = \sum F_v \times x$$

$$\sum M_o = \sum F_H \times y$$

A plate os acted upon by three force and two couple as shown in fig. determine the resultant of these force couple system and find co-ordinate x of the point on the x axis through which the resultant is passed



Given

Three force $1.5KN, 2KN, 3KN$

Two couple $100N.m$ $80\ N.m$

To find

Resultant force, location

Soln:

Resultant force R = $\sqrt{(\sum(F_H)^2 + (\sum(F_V)}2$

Sum of horizontal

$\sum F_H = 0$

$\sum F_H = 1.5 - 3$

$\sum F_H = -1.5KN1$

Sum of vertical force $\sum Fv$ =0

$\sum F_V = -2\ KN$

Resultant $R = \sqrt{(\sum(F_H)^2 + (\sum(F_V)}$

$R = \sqrt{[-1.5]^2[-2]^2}$

$R = 2.5KN$

$\theta = tan^{-1}\left[\frac{\sum F_V}{\sum F_V}\right] = tan^{-1}\left[\frac{-2}{-1.5}\right]$

$\theta = 53.13°$

To locate the resultant

By varigon's Thorem $\downarrow + \uparrow -$

$\sum M_o = R \times x \; and \sum M_o = \sum F_y \times x$

$\sum M_o = [3 \times 0.3] + [-2 \times 0.5] + [-1.5 \times 0.2] + [-0.1] + [-0.08] = 0$

$\sum M_o = -0.58 \; KN.M$

$\sum Mo = 0.58 \; KN.M$ [clock wise]

The co-ordinate x of the point through which the resulted passes is given by

$\sum$Mo= $\sum FY \times x$ $x = \frac{0.58}{2}$

$0.58 = 2 \times x$

$x = 0.29m$

$x = 290mm$



we want to find the intersection

$\sum M_o = \sum F_H \times y$

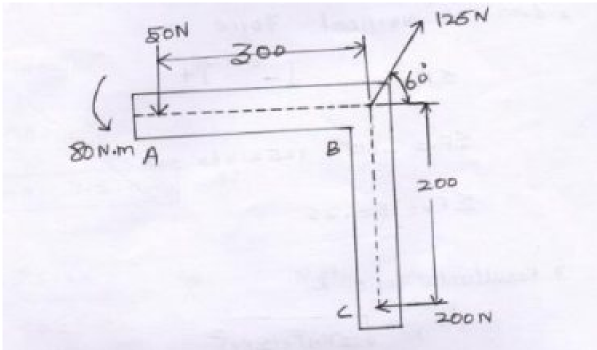
$0.581.5 \times y$

$y = 0.387 \; m$

The three forces and a couple shown below are applied to an angel bracket

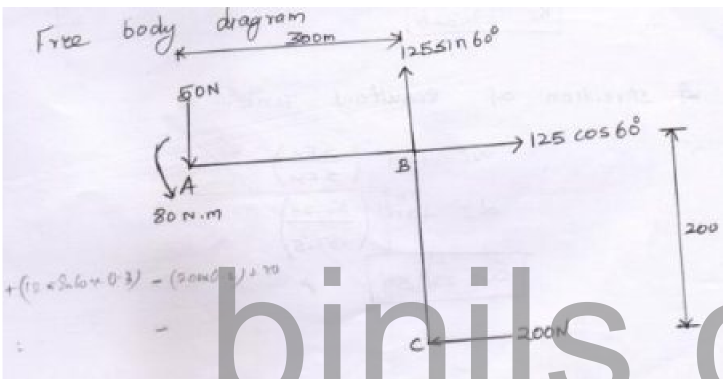
(i) Find he Resultant of this system of forces

(ii) Locate the points where the line o action of the resultant intersects line AB and the line BC




Soln

Free body diagram






1. Sum of Horizontal force

$\sum F_H = 0$ $\quad \overset{+}{\rightarrow} \underset{-}{\leftarrow}$

$$\sum F_H = +125 \cos 60 - 200 = 0$$

$$\sum F_H = -137.5N$$

2. Sum of Vertical Force

$$\sum F_V = 0 \downarrow - \uparrow +$$

$$\sum F_V = -50 + 125 \sin 60 = 0$$

$$\sum F_V = 58.25$$

3. Resultant force' R'

$$R = \sqrt{(\Sigma(F_H)^2 + (\Sigma(F_V)^2}$$

$$R = \sqrt{[-137.5]^2 + [58.23]^2}$$

$$R = 149.32N$$

4. Direction of Resultant force $\alpha$

$$\alpha = \tan^{-1}(\frac{\sum FV}{\sum FV})$$

$$\alpha = \tan^{-1}(\frac{58.25}{137.5})$$

$$\alpha = 22°57'$$



Location of Resultant Force:

By Varigon's Theorem

$$\sum M_A = \sum F_V \times x \; and \sum M_A = \sum F_H \times y$$

$$\sum M_A = (200 \times 0.2) + (-125 \, \sin 60 \times 0.3) - 80:0$$

$$\sum M_A = 40 - 32.47 - 80$$

$$\sum M_A = -7.5 \, N.m$$

$$\sum M_A = \sum F_V \times x$$

$7.5 = 58.25 \times x$

$x = 7.5/58.25 = 0.12\, m$

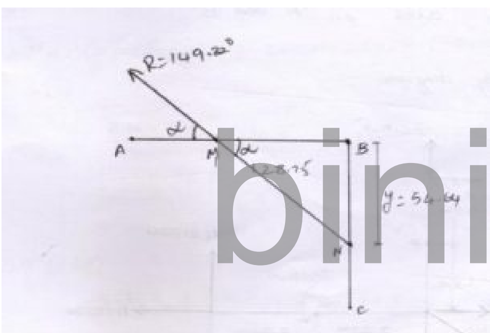
$x = 128.75 mm$

$\sum M_A = \sum F_v \times y$

$7.5 = 137.25 \times y$

$y = 7.5/137.25 = 0.05m$

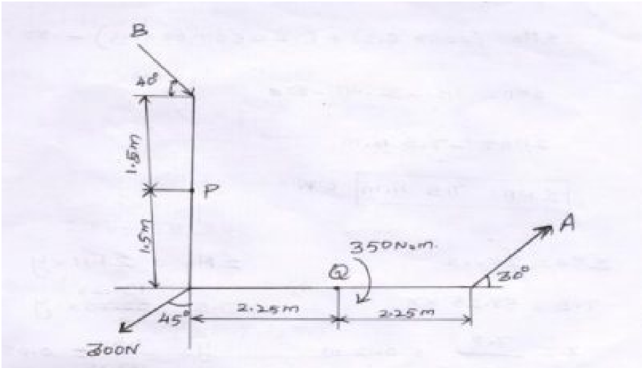
$y = 54.64\, mm$






Problem:

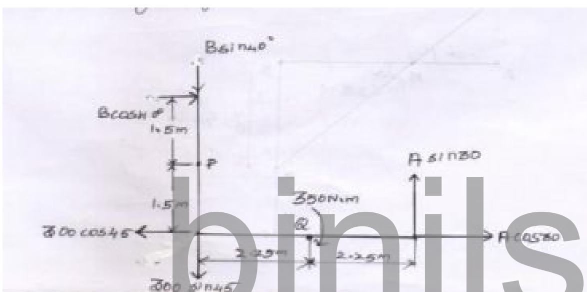
A system of forces acts as shown in fig.find the magnitude of A and B so that resultant of the force system passes through P and Q




To Find:

Forces acts on A and B

Soln: Free body diagram






The resultant forces passes through P and Q is moment About pis zero and also moment about Q=0

It only means that the algebraic sum of moment about P and Q is equal to zero

$\sum M_P = 0$ $\quad\quad$ $\downarrow + \ \uparrow -$

$$\sum M_P = (+B \cos 40 \times 1.5) + (300 \cos 45 \times 1.5) + 350 + (-A \sin 30 \times 4.5) + (-A \cos 30 \times 1.5) = 0$$

$$\sum M_P = 1.149B + 318.19350 - 2.25A - 1.29$$

$$\Sigma M_P = 1.49B + 668.19 - 3.54A = 0$$

$-3.54A + 1.149B = -668.19$-------->(1)

$\sum M_Q = 0 \quad \downarrow + \uparrow -$

$\sum M_Q = (B\,cos40 \times 3) + (-B\,sin40 \times 2.25) + (-300sin\,45 \times 2.25) + 350 + (-A\,sin$
30×2.25)=0

$2.29\,B - 1.44B - 477 + 350 - 1.125A = 0$

$0.85B - 127 - 1.125A = 0$

$-1.25A + 0.85B = 127 ---------->(2)$

Solve 1&2

$-3.54A + 1.149B = -668.19$----(1)

$-1.125A + 0.85B = 127$-----(2)

$(1) \times 1.25 \;\; => -3.982A + 1.292B = -751.7$

$(2) \times 3.54 \;\; => 3.982A + (-)\,3.009B =<->449.58$



---------------------------------------------------

$-171B = -1201.29$

$B = (-1201.29)/(-1.71)$

$B = 702.508N$

B Value substituting in Eqn (1)

$-3.54A + 1.149 \times 702.508 = -668.19$

$-3.54A + 807.182 = -668.19$

$-3.54A = -668.19 - 807.182$

$-3.54A = -1475.37$

$A = (-1475.37)/(-3.54)$

$A = 416.77\,N$

Result:-

$Force\ on\ A = 416.77N$

$Force\ on\ B\ = 702.508N$

Take moment about 'A'

$\sum M_A$=0

$\sum M_A = (500 \times 11) + (-200 \times 7) + (1200 \times 5) + (-300 \times 2)$

$\sum M_A = 5500 = 1400 + 6000 - 600$

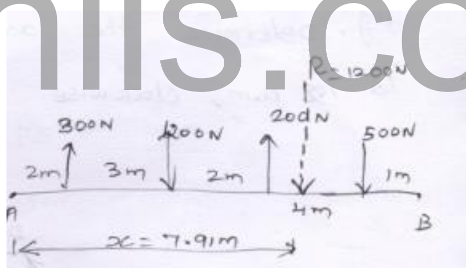
$\sum M_A = 9500\ N.m$

By varignon's theorem

$\sum M_A = R \times x$

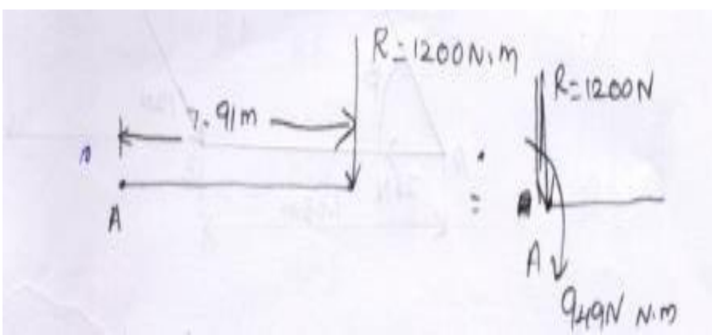
$9500 = 1200 \times x$

$x\ = 7.91m$





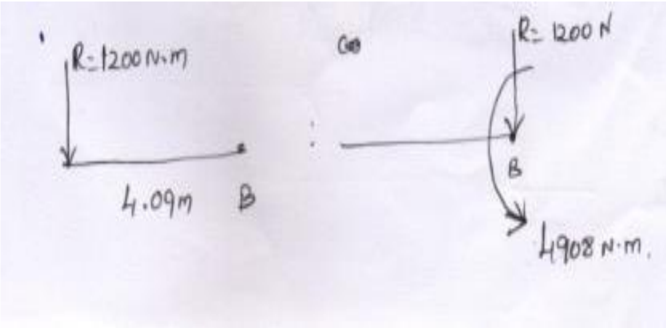

Force couple systemant 'A'



$Couple\ at\ A = 1200 \times 7.91$

$A = 9492\ N.m$

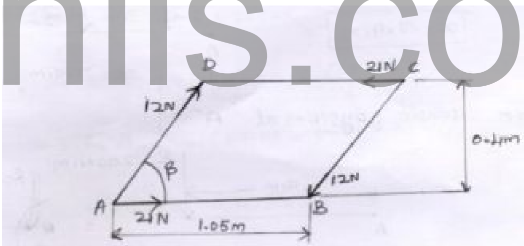
Couple system at B




Problem:

A plate ABCD in the shape of parallelogram is acted upon the two couples, as shown in the fig. Determine the angle B if the resultant couple is 1.8 N.m clockwise

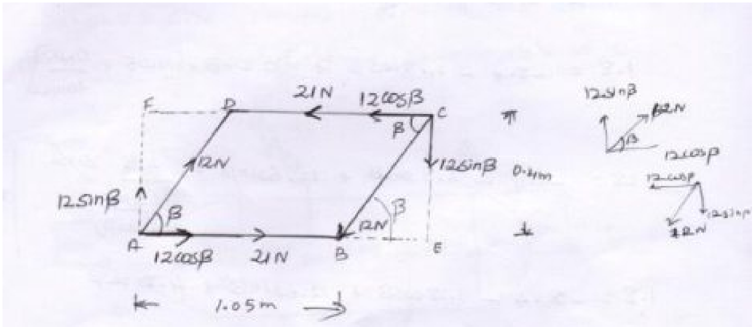





Given:

Resultant couple =1.8N. m

Free body diagram




$Distance\ of\ AE = AB + BE$

$AB = 1.05\ m$

To find BE

$tan\beta = \frac{CE}{BE} = \frac{0.4}{BE}$

$BE = 0.4/tan\ \beta$



$AE = AB + BE$

$AE = 1.05 + \frac{0.4}{tan\ \beta}$

Given the resultant couple $\sum M_A = 1.8 N.M$

Take moment about A

$\sum M_A = [-21 \times 0.4] + [-12\ cos\beta \times 0.4] + [12 sin\beta \times AE]$

$\sum M_A = 1.8\ N.M$

$1.8 = -8.4\ - 4.8cos\beta + 12\ sin\beta \times [1.05 + \frac{0.4}{tan\beta}]$

$1.8 = -8.4\ - 4.8cos\beta + 12.6\ sin\beta + \frac{4.8}{\frac{sin\beta}{cos\beta}}\ sin\ \beta$ $\qquad tan\,\theta = \frac{sin\,\theta}{cos\,\theta}$

$$1.8 = -8.4 - 4.8\,cos\beta\ + 12.6\,sin\,\beta + 4.8\,cos\,\beta$$

$$1.8 + 8.4 = -4.8\,cos\beta + \ 12.6\,sin\beta\ + 4.8cos\,\beta$$

$$10.2\ = 12.6sin\beta$$

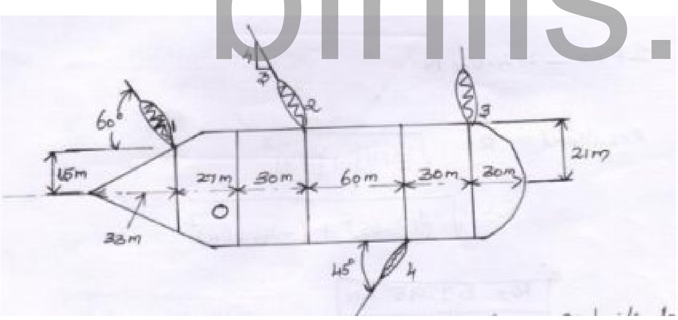
$$Sin\beta\ = \frac{10.2}{12.6}$$

$$B\ = sin - 1\left(\frac{10.2}{12.6}\right) \qquad B\ = 54°$$

Problem

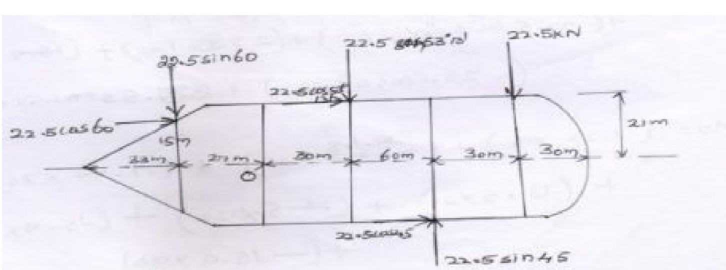
Four tugboats are used to bring an ocan large ship to us pier. Each tugboat exerts a 22.5KN force in direction as shown in fig (i) determine the equivalent force couple system at ‘o’






(ii) Determine a single equivalent force and its location along the longitudinal axis of the ship

Soln: Free body diagram




$$\Sigma F_H = 22.5\,cos\,60 + \ 22.5\,cos + \ 53° + 22$$

$\sum F_H$=40.65$KN$

$\sum F_v = -22.5\ sin\ 60 - 22.5 +\ \ sin\ 53° + 22.5\ sin\ 45 - 22.5$

$\sum F_v = -44.04N$

Resulant R= $\sqrt{(\sum(F_H)^2 + (\sum(F_V)}$2

$R = \sqrt{[40.6.5]^2 + [-44.04]^2}$

$R = 59.95KN$

Direction $\theta = tan^{-1}\left(\frac{\sum F_V}{\sum F_H}\right) = tan^{-1}\left[\frac{44.04}{40.65}\right] = 47°3'$

To find location:

$\sum M_o = R \times x$

$\sum M_o = (22.5cos\ 60\ \times 15) + (-22.5sin\ 60 \times 27) + (22.5sin\ 53° \times 30) +$

$(22.5\ cos\ 53°\ 31 \times 21) + (22.5 \times 120) + (-22.5\ cos\ 45 \times 21) + (-22.5\ \ 45 \times 90)$



$\sum M_o = (11.25 \times 15) + (-19.48 \times 27) + (17.99 \times 30) + (13.5 \times 21) +$

$(22.5 \times 120) + (-15.9 \times 21) + (-15.9 \times 90)$

$\sum M_o = 1319.5\ KN.m$

Location

$\sum M_o = 1319.5$

$\sum M_o = R \times x$

$x = 1319/59.95$

$x = 22.01m$

Magnitude of couple

$M = R \times x$

$= 59.95 \times 22.01m$

$M = 1319.55\ KN.m$

**Equilibrium of Rigidbodies – support Reactions**

Beam:

A beam is horizontal structural member which carries a load transverse (perpendicular) to its axis and transfers the load through support reactions to supporting columns or walks



Frame:

A structure made up of up of several members riveted or welded together is known as frame.

Support Reactions of Beam:-

The force of resistance exerted by the support on the beam is called support reaction.

Types of support

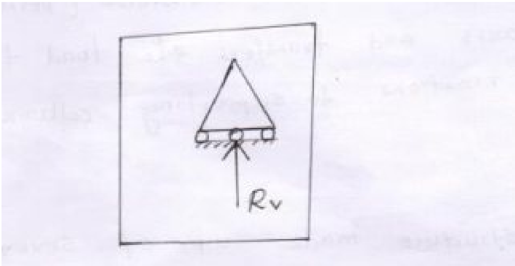
1. Roller support

2. Hinged support

3. Fixed support

1. Roller support:

It consist of the rollers as the bottom. It has only one vertical reaction.

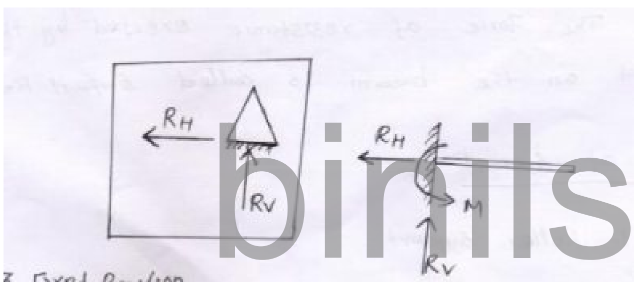



2. Hinged support:

It resists the horizontal and vertical moment. It has two Reaction

(i) Horizontal reaction

(ii) Vertical reaction






3. Fixed reaction:

It is the Stronged support. This support has following reaction

(i) Vertical reaction

(ii) Horizontal reaction

(iii) Rotational reaction (moment)

Types of load:

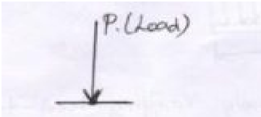
1. Point load

2. Uniformly distributed load (UDL)

3. Uniformly varying load (UVL)

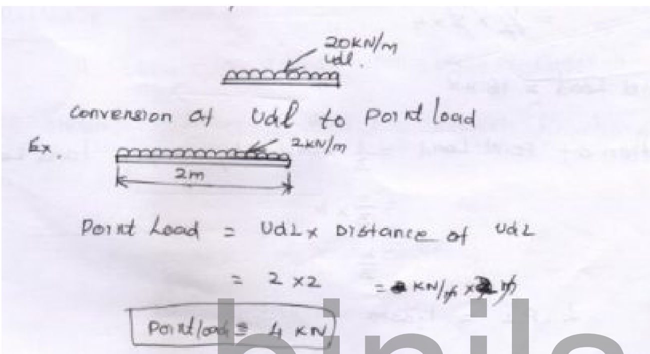
1. Point load:

Load which is acting at a particular point (i.e.) point load;




2. Uniformly distributed Load

The load which is spread over a beam in such a manner that each unit length at the beam carries same intensity of the load is called uniformly distributed load.






$$Point\ load\ = udl \times distance of\ udl$$

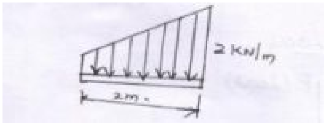
$$= 2 \times 2$$

$$Point\ = 4KN$$

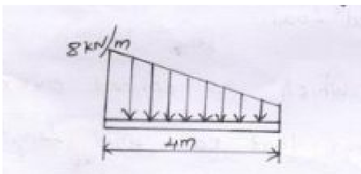
$$Location\ of\ point\ = \frac{uniformly\ disbuted\ load\ length}{2}$$

3. Uniformly varying load

A load which is varying from the particular point along particular length is called uniformly varying load




Conversion of uniformly varying load to point load




$Point\ load\ = 1/2 \times\ uniformly\ varying\ load\ \times\ \ length\ of\ uniformly\ load$

$= 1/2 \times 8 \times 4$

$Point\ load\ = 16KN$



$Location\ of\ point\ load\ = \frac{1}{3} \times\ uniformly\ varying\ load\ length$

$= \frac{1}{3} \times 4$

$= 4/3$

$L.P.L = \ 1.33m$

**Procedure for solving the support reaction problem**

1.Sum of all the horizontal force is zero $\sum F_H = 0$

To find $R_H$ ($R_{HB}$)

2.Sum of all the vertical force is zero $\sum FV$=0

To find $R_{VA}$+$R_{VB}$

3. Take moment of force about A ($\sum MA$=0 To $R_{FV}$=0 to find $R_{VA}$

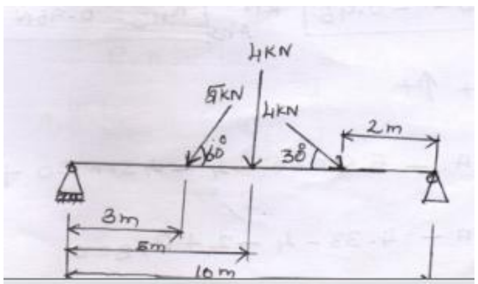
(OR)

4. Substitute $R_{VB}$ in $\sum FV$=0 Eqn

To find $R_{VA}$

Problem-I

A beam is acted upon by a system of forces shown in fig. Find the support Reactions



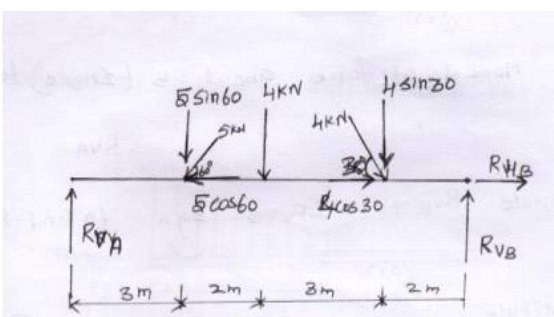



To find

Reaction at the support. $R_V$&$R_{VB}$,$R_{HB}$

Soln:

Free body diagram




$\sum F_H = 0$ $\quad \overset{+}{\rightarrow} \overset{-}{\leftarrow}$

$\sum F_H = -5\cos 60 + 4cos30 + R_{HB}$

$\sum F_H = 0.96 + R_{HB}$

$R_{HB} = -0.96KN$

$R_{HB} = 0.96N(\rightarrow)$

$\sum F_V = 0$

$\sum F_V = -R_{VA} - 5sin\,60 - 4 - 4sin\,30 + R_{VB} = 0$

$RV_A - 4.33 - 4 - 2 + RV_B = 0$

$RV_A + RV_B - 10.33 = 0$

$RV_A + RV_B = 10.33$ ------> (1)

Take moment of force about A

$\sum M_A = 0$

$\sum M_A = (5sin\,60 \times 3) + (4 \times 2) + (4sin\,30 \times 8) + (Rvb \times 10) = 0$



$12.99 + 8 + 16 - 10\,R_{VB} = 0$

$36.99 - 10\,R_{VB} = 0$

$-10\,R_{VB} = (-36.99)/(-10)$

$$R_{VB} = \frac{-36.99}{-10}$$

$Ans: R_{VB} = 3.69\,N$

$R_{VB\ value}$ sub in Eqn ----- > (1)

$R_{VA} + R_{VB} = 10.33$

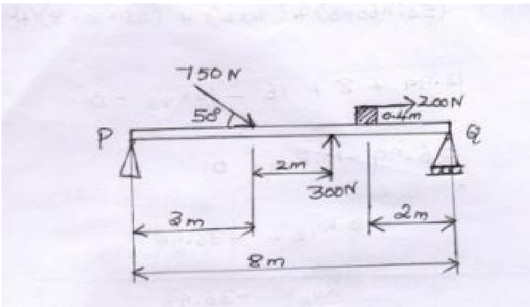
$R_{VA} + 3.69 = 10.33$

$R_{VA} = 10.33 - 3.69$

$R_{VA} = 6.64N$

Problem 2

A beam is loaded as shown in fig find the magnitude direction and the location of the resultant of the system of forces.




To find

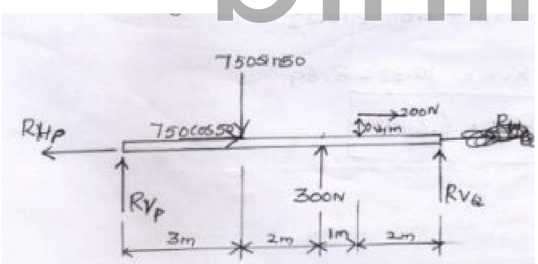
1. Resultant force &direction

2. location of resultant force

Soln

Free body diagram






$\Sigma F_V = 0$ ↑ + ↓ -

$-R_{HP} + 750\cos 50 + 200 = 0$

$-R_{HP} + 482 + 200 = 0$

$-R_{HP} + 682 = 0$

$-R_{HP} = -682N$

↑ ↓

$\sum F_V = 0 \quad + \quad -$

$R_{VP} - 750 \sin 50 + 300 + R_{VQ} = 0$

$R_{VP} + R_{VQ} - 574.53 + 300 = 0$

$R_{VP} + R_{vQ} = -300 + 574.53$

$R_{VP} + R_{VQ} = 274.53\ N$------(1)

Take moment About 'p'

$\sum M_P = 0$

$(750 \sin 50 \times 3) + (200 \times 0.4)(-R_{VQ} \times 8) = 0$

$1753.59 - 1500 + 80 - 8\ R_{VQ} = 0$

$-8\ R_{VQ} = -1753.59 + 1500 - 80$

$-8\ R_{VQ} = 333.59$



$R_{VQ} = (-333.59)/(-8)$

$R_{VQ} = 41.9N$

$R_{VQ} = 41.69N$

$R_{VQ}$ value sub in Eqn(1)

$R_{VP} + R_{VQ} = 274.53$

$R_{VP} + 41.69 = 274.53$

$R_{VP} = 274.53 + 41.69$

$R_{VP} = 232.83N$

Resultant $R = \sqrt{(\sum F_H)^2 + (\sum(F_V)^2}$

Resultant force consider $R = \sqrt{(\sum H_P)^2 + RV_P{}^2}$

Only hinged support $R = \sqrt{[682]^2 + [232.83]^2}$

$R = 720N$

Direction $\theta = tan^{-1}\left[\frac{\sum V_P}{\sum V_H}\right]$

$\theta = tan^{-1}\left(\frac{232.83}{682}\right)$

$\theta = 18°50'$

Location

By varignon's theorem

$\sum M_P = R \times x$

$\sum M_p = 0$ +↓ ↑-



$\sum M_P = (750 \times sin\, 50 \times 3) + (300 \times 5) + (200 \times 0.4) + (-R_{VQ} \times 8)$

$\sum M_P = 1753.59 - 1500 - 800 - 333.52$

$\sum M_P = 0.07\, N.m$

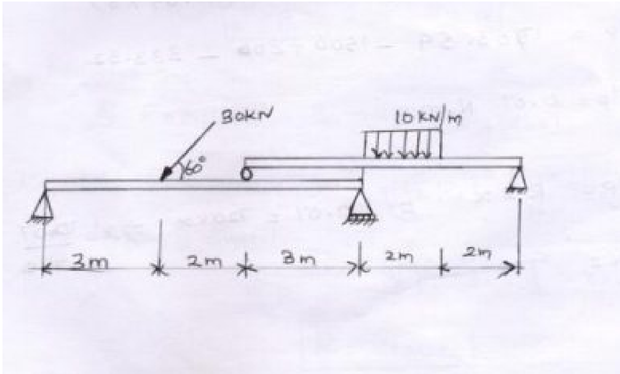
$\sum M_P = R \times x$ => $0.07 = 720 \times x$ => $x = 0.07/720$

Problem: 3

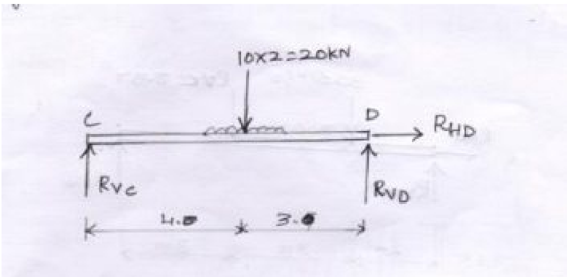
Two beams AB and CD are shown in fig Aand D are hinged supports Band C are rollers supports.

(i) Sketch the free body diagram of the beam AB and deter mine the reaction at the support A and B.

(ii) Sketch the free body diagram of the beam CD and determine the reactions at the supports C and $D_0$




Free diagram of beam CD




$\sum F_H = 0 \underset{+}{\rightarrow} \underset{-}{\leftarrow}$



$R_{HD} = 0$

$R_{VC} - 20 + R_{VD} = 0$

$R_{VC} + R_{VD} = 20$ - - - - - - - (2)

Take moment abut c=0

$\sum M_C = (20 \times 4)(-R_{VD} \times 7) = 0$

$80 - 7\, R_{VD} = 0$

$R_{VD} = \frac{-80}{-7}$

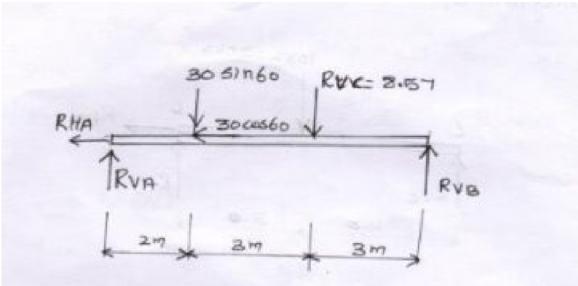
$R_{VD} = 11.42 KN$

$R_{VD}$ value sub in Eqn (1)

$R_{VC} + 11.42 = 20 \;\; => R_{VC} = 20 - 11.42$

$R_{VC} = 8.57KN$

Free body diagram of beam AB




$\sum F_H = 0 \quad \underset{+}{\rightarrow}\underset{-}{\leftarrow}$

$\sum F_H = R_{HA} - 30\cos 60 = 0$

$RH_A - 15 = 0$

$RH_A = 15KN$

$\sum F_V = 0 \quad \uparrow + \downarrow -$

$R_{VA} - 30\sin 60 - 8.57 + R_{VB} = 0$

$R_{VA} + R_{VB} - 25.98 - 8.57 = 0$

$R_{VA} + R_{VB} = 34.55$------ (1)

Take moment about A

$\sum M_A = 0$

$\sum M_A = (30\sin 60 \times 2) + (8.57 \times 5) + (R_{VB} \times 8) = 0$

$51.96 + 42.85 - 8\, R_{VB} = 0$

$-8\, R_{VB} - \times 94.81 / \times 8$

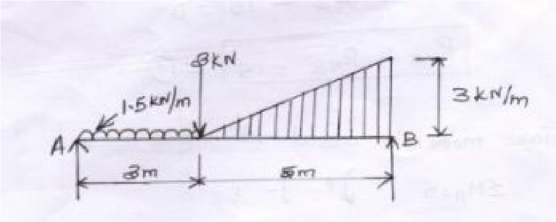
$R_{VB} = 11.85KN$

$R_{VB} + 11.85 = 34.85$

$R_{VA} = 22.69KN$

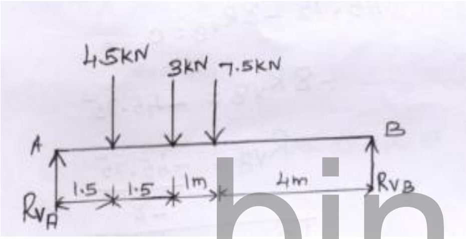
Problem:4

Determine the support reaction of the simply supported beam shown in fig




Soln:

Free body diagram






$UDL\ to\ PL = UDL\ \times dis\ of\ VDL$

$= 1.5 \times 3 = 4.5$

$Length\ of\ UDL = \frac{UDL}{2} = \frac{3}{2} = 1.5$

$UVL\ to\ PL\ = \frac{1}{2} \times UVL \times\ length\ of\ UVL$

$= \frac{1}{2} \times 3 \times 5$

$PL = 7.5\ KN$

$Length\ of\ PL = \frac{1}{3} \times 3 = 1m$

$\sum F_H = 0 \overset{+}{\rightarrow}\overset{-}{\leftarrow}$

$\sum F_V = 0 \quad \downarrow + \quad \uparrow +$

$R_{VA} \ - \ 4.5 - 3 - 7.5 + R_{VB} = \ 0$

$+ \ R_{VA} \ - \ 15 \ = \ 0$

$R_{VA} \ + \ R_{VB} \ = \ 15$ ------ (1)

Take moment about A

$\sum M_A = 0 \downarrow + \uparrow -$

$(4.5 \times 1.5) + (3 \times 3) + (7.5 \times 4) + (R_{VB}) = 0$

$45.75 - 8 \ R_{VB} = 0$

$-8 \ R_{VB} = -45.75$

$R_{VB} = \frac{-45.75}{-8}$



$R_{VB} = 5.71 KN$

$R_{VB} \ + R \quad = 15 - - - - - (1)$

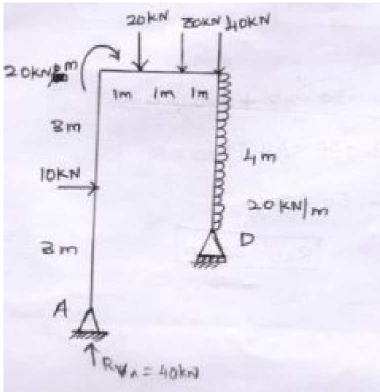
$R_{VA} \quad + \ 5.71 = 15$

$R_{VA} = 15 - 5.71$

$R_{VA} = 9.28 KN$

Problem 5

Find the reactions for the frame shown in fig. the line of action of 40k passes through the point A.

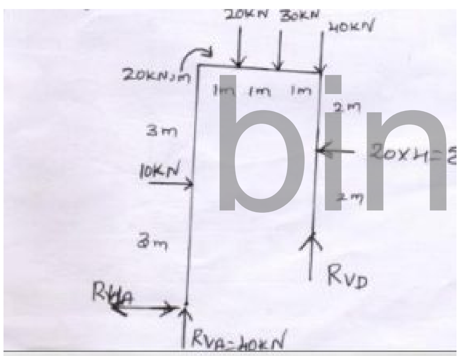



Given

Line of action at point A $\quad R_{VA}$=40KN

Soln:

Free body diagram





$\sum FH$=0 $\underset{+\ -}{\rightarrow\leftarrow}$

$-R_{HA} + 10 - 80 = 0$

$-R_{HA} - 70 = 0$

$-R_{HA} = 70$

$R_{HA} = -70KN$

$R_{HA} = -70KN$

$\sum FV$=0

$R_{VA} - 20 - 30 - 40 + R_{VD} = 0$

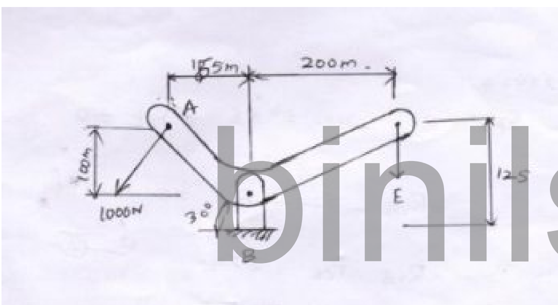
$40 - 20 - 30 - 40 + R_{VD} = 0$

$-50 + R_{VD} = 0$

$R_{VD} = 50KN$

Problem:

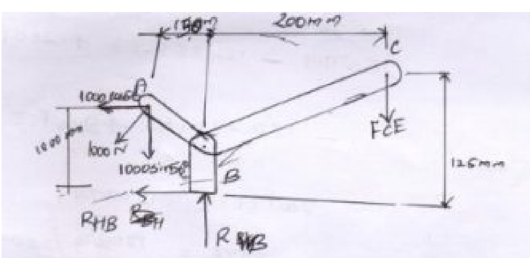
Lever ABC of machine component is subjected to a force 1000n at point A as shown in fig. compute the reaction act B and the force at CE.






Soln:

Free body diagram




$\theta = \tan^{-1}\left(\frac{100}{500}\right)$

$\theta = 33^{0}\ 41^{1}$

$\sum F_H$=0

$-R_{HB} - 1000\, cos\, 560 = 0$

$- R_{HB} = 1000\, cos\, 560$

$R_{HB} = -554.7\, N$-----(1)

$\sum F_V$=0

$R_{VB} - 1000\, Sin\, 56 - FCE = 0$

$R_{VB} = -829 - \; FCE = 829 - - - - - (2)$

$M_B = \; [-1000\, cos\, 56\; \times 100] + [-1000 sin\, 560 \times 150] + [F_{CE}] = 0$

$-55919 - 124355.63 + 200\; F_{CE} = 0$

$-180274.63200\, F_{CE} = 0$

$200\, F_{CE} = 180274$

$F_{CE} = 180274/200$

$F_{CE} = 901.373N$

Resultant $R_B=\sqrt{(RV_B)^2+(RH_B)^2}$

$=\sqrt{(1730)^2+(-554)^2}$

$R_B$=1817N

$\theta=\tan^{-1}(\frac{RV_B}{RH_B})=\tan^{-1}(\frac{1730}{554})$

$\theta=72^0 14^1$

To find

Reaction of support

Soln

Sum of horizontal forces $\sum F_H$

$\sum F_H$=0

$-RH_A - 10\ cos\ 60\ = 0$

$-\ RH_A\ = -10cos\ 60 = -5$

$RH_A\ = 5N$

Sum of vertical forces $\sum F_V$

∑FV=0

$R_{VA} = 10sin\ 60\ \ - 8 = 0$

$R_{VA} = -8.66 - 8 = 0$

$R_{VA} - 16.66 = 0$

$R_{VA} = 16.66N$



Moment About ‘o’

$Mo = 0$

$Mo = M = [10sin\ 60 \times 4] + 20 + [8 \times 8] = 0$

$M + 34.64 + 20 + 64 = 0$

$M +\ \ 118.64 = 0$

$M\ = 118.64KN.m$

$M\ = 118.64\ m$