Catalog

4.1. Introduction to electro dynamic fields

In our study of static fields so far, we have observed that static electric fields are produced by electric charges, static magnetic fields are produced by charges in motion or by steady current. Further, static electric field is a conservative field and has no curl, the static magnetic field is continuous and its divergence is zero.

4.2. Concept of Magnetic Circuits

The fundamental relationships for static electric fields among the field quantities can be summarized as:

$$\nabla \times \vec{E} = 0 \qquad (5.1a)$$

$$\nabla \cdot \vec{D} = \rho_v \qquad (5.1b)$$

For a linear and isotropic medium,

$$\vec{D} = \varepsilon \vec{E} \qquad (5.1c)$$

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Similarly for the magnetostatic case

$$\nabla \vec{B} = 0 \qquad (5.2a)$$

$$\nabla \times \vec{H} = \vec{J} \qquad (5.2b)$$

$$\vec{B} = \mu \vec{H} \qquad (5.2c)$$

It can be seen that for static case, the electric field vectors and and magnetic field vectors and form separate pairs.

In this chapter we will consider the time varying scenario. In the time varying case we will observe that a changing magnetic field will produce a changing electric field and vice versa.

4.3. Faraday's law

Michael Faraday, in 1831 discovered experimentally that a current was induced in a conducting loop when the magnetic flux linking the loop changed. In terms of fields, we can say that a time varying magnetic field produces an electromotive force (emf) which causes a current in a closed circuit. The quantitative relation between the induced emf (the voltage that arises from conductors moving in a magnetic field or from changing magnetic fields) and the rate of change of flux linkage developed based on experimental observation is known as Faraday's law. Mathematically, the induced emf can be written as

$$-\frac{d\phi}{dt}$$

A non zero may result due to any of the following:

(a) time changing flux linkage a stationary closed path.

(b) relative motion between a steady flux a closed path.

(c) a combination of the above two cases.

4.4. Transformer and motional EMF

The negative sign in equation was introduced by Lenz in order to comply with the polarity of the induced emf. The negative sign implies that the induced emf will cause a current flow in the closed loop in such a direction so as to oppose the change in the linking magnetic flux which produces it. (It may be noted that as far as the induced emf is concerned, the closed path forming a loop does not necessarily have to be conductive).

If the closed path is in the form of N tightly wound turns of a coil, the change in the magnetic flux linking the coil induces an emf in each turn of the coil and total emf is the sum of the induced emfs of the individual turns, i.e.,

For a circuit with a single turn (N = 1)

$$V_{emf} = -\frac{d\Psi}{dt}$$

In terms of E and B this can be written as

$$V_{emf} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$$

where ψ has been replaced by

$$\int_S \mathbf{B} \cdot d\mathbf{S}$$

and S is the surface area of the circuit bounded by a closed path L.. The equation says that in time-varying situation, both electric and magnetic fields are present and are interrelated.

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The three ways of induced EMF are,

1. By having a stationary loop in a time-varying B field.

2. By having a time-varying loop area in a static B field.

3. By having a time-varying loop area in a time-varying B field.

Stationary loop in a time-varying B field (Transformer emf)

Consider a stationary conducting loop in a time-varying magnetic B field. The equation (i) becomes

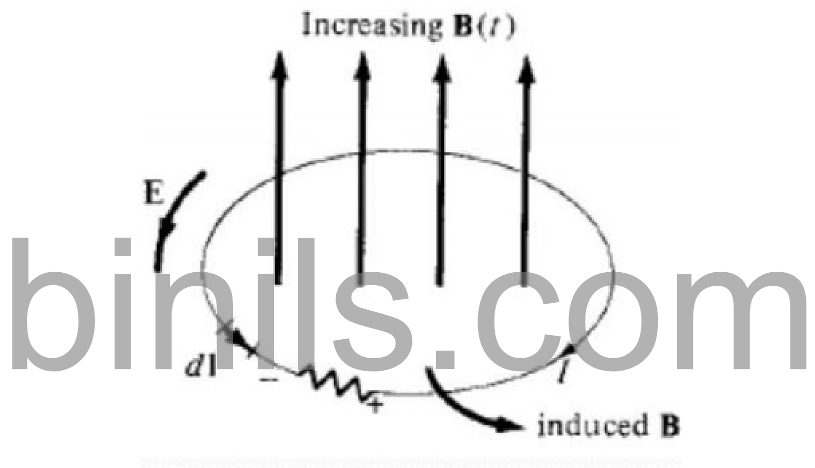

Fig 4.1 Stationary loop in a time-varying B field

$$V_{emf} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

This emf induced by the time-varying current in a stationary loop is often referred to as transformer emf in power analysis since it is due to the transformer action. By applying Stokes's theorem to the middle term, we get

$$\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

This is one of the Maxwell's equations for time-varying fields. It shows that the time-varying field is not conservative.

Moving loop in static B field (Motional emf)

When a conducting loop is moving in a static B field, an emf is introduced in the loop. The force on a charge moving with uniform velocity u in a magnetic field B is

$$\mathbf{F}_m = Q\mathbf{u} \times \mathbf{B}$$

The motional electric field Em is defined as

$$\mathbf{E}_m = \frac{\mathbf{F}_m}{Q} = \mathbf{u} \times \mathbf{B}$$

Consider a conducting loop moving with uniform velocity u, the emf induced in the loop is

$$V_{\text{emf}} = \oint_L \mathbf{E}_m \cdot d\mathbf{l} = \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

This kind of emf is called the motional emf or flux-cutting emf. Because it is due to the motional action. eg,. Motors, generators Consider a rod moving between a pair of rails Here B and perpendicular u are so the force can be given by

$$\mathbf{F}_m = I\ell \times \mathbf{B}$$
$$F_m = I\ell B$$

The equation (i) becomes

$$V_{emf} = uB\ell$$

By applying Stokes's theorem to equation (i), we get

$$\int_S (\nabla \times \mathbf{E}_m) \cdot d\mathbf{S} = \int_S \nabla \times (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{S}$$

$$\nabla \times \mathbf{E}_m = \nabla \times (\mathbf{u} \times \mathbf{B})$$

Moving loop in time-varying field

Consider a moving conducting loop in a time-varying magnetic field Then both transformer emf and motional emf are present. Thus the total emf will be the sum of transformer emf and motional emf

$$V_{emf} = \oint_L \mathbf{E} \cdot d\mathbf{l} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S} + \oint_L (\mathbf{u} \times \mathbf{B}) \cdot d\mathbf{l}$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} + \nabla \times (\mathbf{u} \times \mathbf{B})$$

4.5. Maxwell's Equation

The First Maxwell's equation (Gauss's law for electricity)

Gauss's law states that flux passing through any closed surface is equal to 1/ε0 times the total charge enclosed by that surface.

The integral form of Maxwell's 1st equation

$$\emptyset_e = \frac{q}{\epsilon 0} \text{------------} (1)$$

$$Also\ \ \emptyset_e = \int \vec{E}.d\vec{A} \text{------} (2)$$

Comparing equation (1) and (2) we have

$$\int \vec{E}.d\vec{A} = \frac{q}{\epsilon 0} \text{----------} (3)$$

It is the integral form of Maxwell's 1st equation.

Maxwell's first equation in differential form

The value of total charge in terms of volume charge density is $q = \int \rho dv$ So equation (3) becomes

$$\int \vec{E}.d\vec{A} = \frac{1}{\epsilon 0} \int \rho dv$$

Applying divergence theorem on left hand side of above equation we have

$$\int (\vec{\nabla}.\vec{E}) d.V = \frac{1}{\epsilon 0} \int \rho dv$$

$$\int (\vec{\nabla}.\vec{E}) d.V - \frac{1}{\epsilon 0} \int \rho dv = 0$$

$$\int [(\vec{\nabla}.\vec{E}) - \frac{\rho}{\epsilon 0}]\, dv = 0$$

$$(\vec{\nabla}.\vec{E}) - \frac{\rho}{\epsilon 0} = 0$$

$$(\vec{\nabla}.\vec{E}) = \frac{\rho}{\epsilon 0}$$

It is called the differential form of Maxwell's 1st equation.

The Second Maxwell's equation (Gauss's law for magnetism)

Gauss's law for magnetism states that the net flux of the magnetic field through a closed surface is zero because monopoles of a magnet do not exist.

$\int \vec{B}.\overrightarrow{dA}=0$ .... (4)

It is the integral form of Maxwell's second equation.

Applying divergence theorem

$\int(\vec{\nabla}.\vec{B})dV=0$

This implies that:

$\vec{\nabla}.\vec{B}=0$

It is called differential form of Maxwell's second equation.

The Third Maxwell's equation (Faraday's law of electromagnetic induction)

According to Faraday's law of electromagnetic induction

$\varepsilon = -N\frac{d\emptyset_m}{dt}$ .... (5)

Since emf is related to electric field by the relation

$\varepsilon = \int \vec{E}.\overrightarrow{dA}$

Also $\emptyset_m = \int \vec{B}.\overrightarrow{dA}$

Put these values in equation (5) we have

$\int \vec{E}.\overrightarrow{dA} = -N \int \vec{E}.\overrightarrow{dA} \int \vec{B}.\overrightarrow{dA}$

For N=1 ,we have

$\int \vec{E}.\overrightarrow{dA} = -\frac{d}{dt}\int \vec{B}.\overrightarrow{dA}$ .... (6)

It is the integral form of Maxwell's 3rd equation.

Applying Stokes Theorem on L.H.S of equation (6) we have

$\int(\vec{\nabla}\times\vec{E})d\vec{A} = -\frac{d}{dt}\int\vec{B}.\overrightarrow{dA}$

$\int(\vec{\nabla}\times\vec{E})d\vec{A} + \frac{d}{dt}\int\vec{B}.\overrightarrow{dA} = 0$

$(\vec{\nabla}\times\vec{E}) + \frac{d\vec{B}}{dt} = 0$

$(\vec{\nabla}\times\vec{E}) = -\frac{d\vec{B}}{dt}$

It is the differential form of Maxwell's third equation.

The Fourth Maxwell's equation (Ampere's law)

The magnitude of the magnetic field at any point is directly proportional to the strength of the current and inversely proportional to the distance of the point from the straight conductors is called Ampere's law.

$\int\vec{B}.\overrightarrow{ds} = \mu_0 i$ .... (7)

It is the integral form of Maxwell's 4 th equation.

The value of current density

$i = \int\vec{j}.\overrightarrow{dA}$

Now the equation (7) Become

$\int\vec{B}.\overrightarrow{ds} = \mu_0\int\vec{j}.\overrightarrow{dA}$

Applying Stoke's theorem on L.H.S of above equation, we have

$\int(\vec{\nabla}\times\vec{B})d\vec{A} = \mu_0\int\vec{j}.\overrightarrow{dA}$

$\int[(\vec{\nabla}\times\vec{B})d\vec{A} - \mu_0 j].\ d\mathbf{A} = 0$

$(\vec{\nabla}\times\vec{B}) = \mu_0\boldsymbol{j}$

Third Maxwell's equation says that a changing magnetic field produces an electric field. But there is no clue in the fourth Maxwell's equation whether a changing electric field produces a magnetic field? To overcome this deficiency, Maxwell's argued that if a changing magnetic flux can produce an electric field then by symmetry there must exist a relation in which a changing electric field must produce a changing magnetic flux. For more related informative topics Visit our Page: Electricity and Magnetism

4.6. Relation between field theory and circuit theory

Circuit Theory:

1) This analysis is originated by its own.

2) Applicable only for portion of radiofrequency range.

3) It is dependent and independent parameter, I and V are directly obtained from the given circuit.

4) Parameters of medium are not involved.

5) Laplace Transform is employed.

6) Z, Y and H parameters are used.

7) Low power is involved.

8) Simple to understand.

9) 2 Dimensional analysis.

10) Frequency is used for reference.

11) Lumped components are used.

Field Theory:

1) Evolved from transmission ratio.

2) Not applicable for portion of radiofrequency range.

3) Not directly obtained from E and H.

4) Parameters (Permeability and Permitiviy) are analysed in the medium.

5) Maxwell's equation is used.

6) S parameter is used.

7) High Power is involved.

8) Needs visualisation effect.

9) 3 Dimensional analysis.

10) Wavelength is used as reference.

11) Distributed components are used.