Catalog

3.1. Introduction to Magneto statics

In previous chapters we have seen that an electrostatic field is produced by static or stationary charges. The relationship of the steady magnetic field to its sources is much more complicated.

The source of steady magnetic field may be a permanent magnet, a direct current or an electric field changing with time. In this chapter we shall mainly consider the magnetic field produced by a direct current. The magnetic field produced due to time varying electric field will be discussed later. Historically, the link between the electric and magnetic field was established Oersted in 1820. Ampere and others extended the investigation of magnetic effect of electricity.

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3.2. Lorentz force

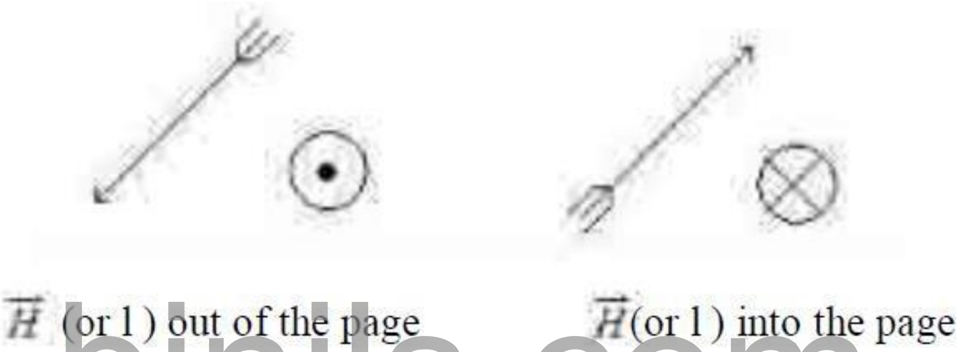
Lorentz force is the combination of electric and magnetic force on a point charge due to electromagnetic fields. A particle of charge *q* moving with velocity **v** in the presence of an electric field **E** and a magnetic field **B** experiences a force

There are two major laws governing the magnetostatic fields are:

- Biot-Savart Law
- Ampere's Law

Magnetic field intensity(H)

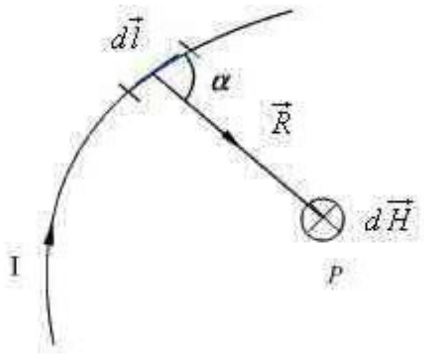
Usually, the magnetic field intensity is represented by the vector . It is customary to represent the direction of the magnetic field intensity (or current) by a small circle with a dot or cross sign depending on whether the field (or current) is out of or into the page as shown in Fig. 3.1.



Representation of magnetic field (or current)

3.3. Biot–Savart's Law

This law relates the magnetic field intensity *dH* produced at a point due to a differential current element as shown in Fig. 3.2.



The magnetic field intensity at P can be written as,

$$d\vec{H} = \frac{Id\vec{l} \times \hat{a}_R}{4\pi R^2} = \frac{Id\vec{l} \times \vec{R}}{4\pi R^3}$$

$$dH = \frac{IdlSin\alpha}{4\pi R^2}$$

$$R = |\vec{R}|$$

Where is the distance of the current element from the point P.

Similar to different charge distributions, we can have different current distribution such as line current, surface current and volume current. These different types of current densities are shown in Fig. 3.3.

By denoting the surface current density as K (in amp/m) and volume current density as J (in amp/m2) we can write:

$$Id\vec{l} = \vec{K}ds = \vec{J}dv$$

Employing Biot-Savart Law, we can now express the magnetic field intensity H. In terms of

These current distributions.

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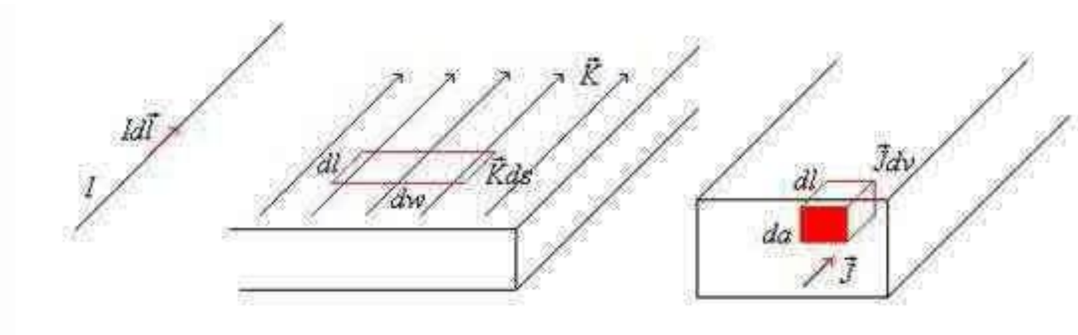


for line current

$$\vec{H} = \int_L \frac{Id\vec{l} \times \vec{R}}{4\pi R^3}$$

for surface current

$$\vec{H} = \int_S \frac{Kd\vec{s} \times \vec{R}}{4\pi R^3}$$

for volume current

$$\vec{H} = \int_V \frac{\vec{J}dv \times \vec{R}}{4\pi R^3}$$

Ampere's Circuital Law:

Ampere's circuital law states that the line integral of the magnetic field (circulation of H) around a closed path is the net current enclosed by this path. Mathematically,

$$\oint \vec{H}.d\vec{l} = I_{enc}$$

The total current I $_{enc}$ can be written as,

$$I_{enc} = \int_S \vec{J}.d\vec{s}$$

By applying Stoke's theorem, we can write\

$$\oint \vec{H}.d\vec{l} = \int_S \nabla \times \vec{H}.d\vec{s}$$

$$\therefore \int_S \nabla \times \vec{H}.d\vec{s} = \int_S \vec{J}.d\vec{s}$$

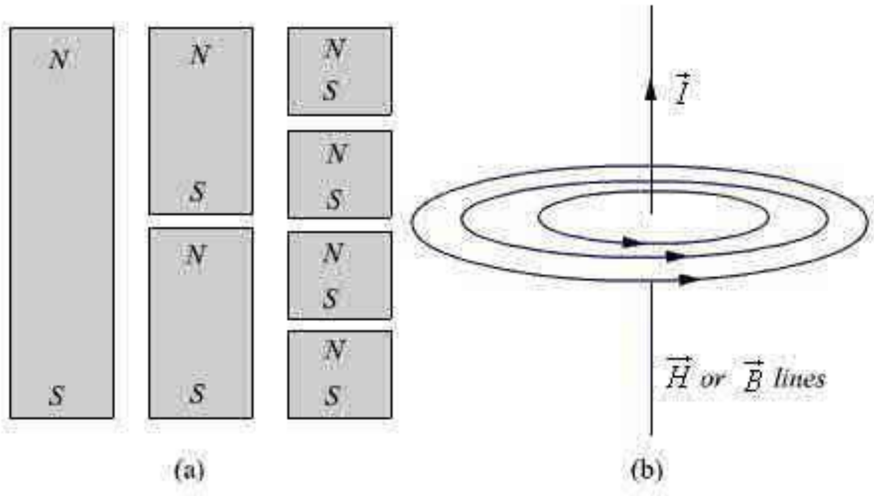
$$\therefore \nabla \times \vec{H} = \vec{J}$$

3.4. H due to straight conductors

In simple matter, the magnetic flux density related to the magnetic field intensity as where called the permeability. In particular when we consider the free space where H/m is the permeability of the free space. Magnetic flux density is measured in terms of Wb/m 2 . The magnetic flux density through a surface is given by:

$$\psi = \int_S \vec{B} \cdot d\vec{s}$$

In the case of electrostatic field, we have seen that if the surface is a closed surface, the net flux passing through the surface is equal to the charge enclosed by the surface. In case of magnetic field isolated magnetic charge (i. e. pole) does not exist. Magnetic poles always occur in pair (as N-S). For example, if we desire to have an isolated magnetic pole by dividing the magnetic bar successively into two, we end up with pieces each having north (N) and south (S) pole as shown in Fig. 4.7 (a). This process could be continued until the magnets are of atomic dimensions; still we will have N-S pair occurring together. This means that the magnetic poles cannot be isolated.



Similarly if we consider the field/flux lines of a current carrying conductor as shown in Fig. 3.5, we find that these lines are closed lines, that is, if we consider a closed surface, the number of flux lines that would leave the surface would be same as the number of flux lines that would enter the surface.

From our discussions above, it is evident that for magnetic field,

$$\oint_S \vec{B}.d\vec{s} = 0$$

which is the Gauss's law for the magnetic field. By applying divergence theorem, we can write:

$$\oint_S \vec{B}.d\vec{s} = \int_V \nabla.\vec{B}dv = 0$$

Hence,

$$\nabla.\vec{B} = 0$$

which is the Gauss's law for the magnetic field in point form.

3.5 Magnetic materials

The origin of magnetism lies in the orbital and spin motions of electrons and how the electrons interact with one another. The best way to introduce the different types of magnetism is to describe how materials respond to magnetic fields. This may be surprising to some, but all matter is magnetic. It's just that some materials are much more magnetic than others. The main distinction is that in some materials there is no collective interaction of atomic magnetic moments, whereas in other materials there is a very strong interaction between atomic moments

Types and properties

1. Diamagnetism

Diamagnetism is a fundamental property of all matter, although it is usually very weak. It is due to the non-cooperative

behavior of orbiting electrons when exposed to an applied magnetic field. Diamagnetic substances are composed of atoms

which have no net magnetic moments (ie., all the orbital shells are filled and there are no unpaired electrons). However,

when exposed to a field, a negative magnetization is produced and thus the susceptibility is negative. If we plot M vs H,

2. Paramagnetism

This class of materials, some of the atoms or ions in the material have a net magnetic moment due to unpaired electrons in partially filled orbitals. One of the most important atoms with unpaired electrons is iron. However, the individual magnetic moments do not interact magnetically, and like diamagnetism, the magnetization is zero when the field is removed. In the presence of a field, there is now a partial alignment of the atomic magnetic moments in the direction of the field, resulting

in a net positive magnetization and positive susceptibility.

3. Ferromagnetism

When you think of magnetic materials, you probably think of iron, nickel or magnetite. Unlike paramagnetic materials, the atomic moments in these materials exhibit very strong interactions. These interactions are produced by electronic exchange forces and result in a parallel or antiparallel alignment of atomic moments. Exchange forces are very large, equivalent to a field on the order of 1000 Tesla, or approximately a 100 million times the strength of the earth's field. The exchange force is a quantum mechanical phenomenon due to the relative orientation of the spins of two electron. Ferromagnetic materials exhibit parallel alignment of moments resulting in large net magnetization even in the absence of a magnetic field.

4. Ferrimagnetism

In ionic compounds, such as oxides, more complex forms of magnetic ordering can occur as a result of the crystal structure. One type of magnetic ordering is call ferrimagnetism.

3.6 Magnetization

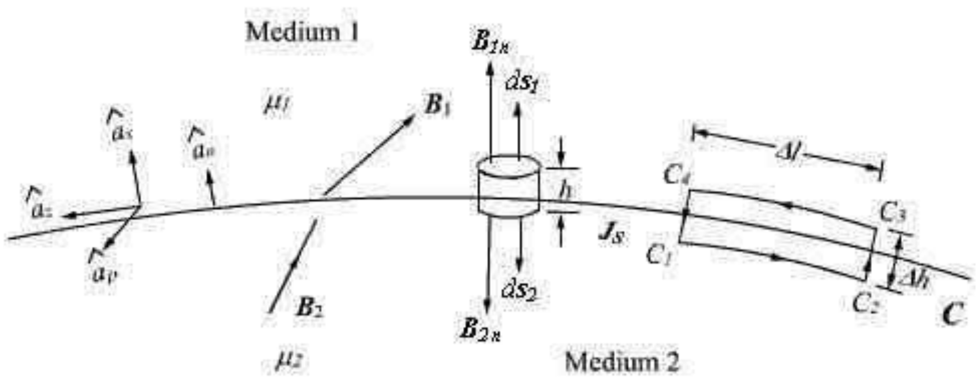
Magnetic polarization is the vector field that expresses the density of permanent or induced magnetic dipole moments in a magnetic material. The origin of the magnetic moments responsible for magnetization can be either microscopic electric currents resulting from the motion of electrons in atoms, or the spin of the electrons or the nuclei. Net magnetization results from the response of a material to an external magnetic field, together with any unbalanced magnetic dipole moments that may be inherent in the material itself.

Magnetic field in multiple media

Similar to the boundary conditions in the electro static fields, here we will consider the behavior of and at the interface of two different media. In particular, we determine how the tangential and normal components of magnetic fields behave at the boundary of two regions having different permeabilities.

The figure 4.9 shows the interface between two media permeabities and , being the normal having vector from medium 2 to medium 1. Interface between two magnetic media

To determine the condition for the normal component of the flux density vector , we consider a small pill box P with vanishingly small thickness *h* and having an elementary area for the faces. Over the pill box, we can write



To determine the condition for the normal component of the flux density vector , we consider a small pill box P with vanishingly small thickness *h* and having an elementary area for the faces. Over the pill box, we can write

$$\oint_s \vec{B}.d\vec{s} = 0$$

$$\therefore \int_{\Delta S} \vec{B}_1.d\vec{S}_1 + \int_{\Delta S} \vec{B}_2.d\vec{S}_2 = 0$$

$$d\vec{S}_1 = dS\hat{a}_n$$

$$d\vec{S}_2 = dS\left(-\hat{a}_n\right)$$

$$\therefore \int_{\Delta S} B_{1n} dS - \int_{\Delta S} B_{2n} dS = 0$$

$$B_{2n} = \vec{B}_2 \cdot \hat{a}_n$$

$$B_{1n} = \vec{B}_1 \cdot \hat{a}_n \qquad B_{2n} = \vec{B}_2 \cdot \hat{a}_n$$

$$\left(B_{1n} - B_{2n}\right)\Delta S = 0$$

$$B_{1n} = B_{2n}$$

That is, the normal component of the magnetic flux density vector is continuous across the interface.

$$\hat{a}_n \cdot \left(\vec{B}_1 - \vec{B}_2\right) = 0$$

To determine the condition for the tangential component for the magnetic field, we consider a closed path C as shown in figure 4.8. By applying Ampere's law we can write

$$\oint \vec{H}.d\vec{l} = I$$

$$\int_{C_1 - C_2} \vec{H}.d\vec{l} + \int_{C_3 - C_4} \vec{H}.d\vec{l} = I$$

We have shown in figure 4.8, a set of three unit vectors , and such that they satisfy (R.H. rule). Here is tangential to the interface and is the vector perpendicular to the surface enclosed by C at the interface

3.7 Scalar and Vector Potentials

In studying electric field problems, we introduced the concept of electric potential that simplified the computation Divergence theorem n of electric fields for certain types of problems. In the same manner let us relate the magnetic field intensity to a scalar magnetic potential and write:

$$\vec{H} = -\nabla V_m$$

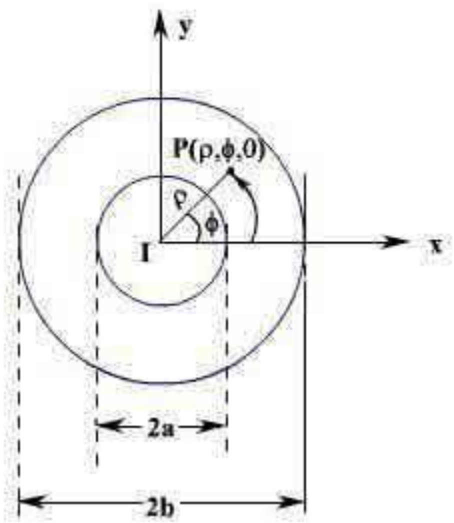
From Ampere's law , we know that

$$\nabla \times \vec{H} = \vec{J}$$

$$\nabla \times (-\nabla V_m) = \vec{J}$$

But using vector identity, we find that is valid only where . Thus the scalar magnetic potential is defined only in the region where . Moreover, Vm in general is not a single valued function of position.This point can be illustrated as follows. Let us consider the cross section of a coaxial line as shown in fig

$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$$



If V_m is the magnetic potential then,

$$-\nabla V_m = -\frac{1}{\rho}\frac{\partial V_m}{\partial \phi}$$
$$= \frac{I}{2\pi\rho}$$

$$\therefore V_m = -\frac{I}{2\pi}\phi + c$$

$$V_m = -\frac{I}{2\pi}\phi$$

$$\therefore \text{At } \phi = \phi_0 \quad V_m = -\frac{I}{2\pi}\phi_0$$

We observe that as we make a complete lap around the current carrying conductor , we reach again but V_m this time becomes

$$V_m = -\frac{I}{2\pi}(\phi_0 + 2\pi)$$

We observe that value of V_m keeps changing as we complete additional laps to pass through the same point. We introduced V_m analogous to electostatic potential V. But for static electric.

We now introduce the **vector magnetic potential** which can be used in regions where current density may be zero or nonzero and the same can be easily extended to time varying cases. The use of vector magnetic potential provides elegant ways of solving EM field problems.

Since and we have the vector identity that for any vector , , we can write Here, the vector field is called the vector magnetic potential. Its SI unit is Wb/m. Thus if can find of a given current distribution, can be found from through a curl operation.