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1.1 SOURCES AND EFFECTS OF ELECTROMAGNETIC FIELDS

Electromagnetic theory is a prerequisite for a wide spectrum of studies in the field of Electrical Sciences and Physics. Electromagnetic theory can be thought of as generalization of circuit theory. There are certain situations that can be handled exclusively in terms of field theory. In electromagnetic theory, the quantities involved can be categorized as source quantities and field quantities. Source of electromagnetic field is electric charges: either at rest or in motion. However an electromagnetic field may cause a redistribution of charges that in turn change the field and hence the separation of cause and effect is not always visible.

Electromagnetics (EM) is a branch of physics or electrical engineering in which electric and magnetic phenomena are studied.

Sources of Electromagnetic Fields:

- Current Carrying Conductors
- Mobile Phones
- Microwave oven
- Computer and Television
- High Voltage Power lines
- Transformer
- Electric relays
- Television /Radio
- Electric Motors
- Wave guides
- Antennas
- Optical Fibers
- Radars and lasers

Effect of Electromagnetic Fields

- Plants and Animals
- Humans
- Electrical Components

Fields are classified as

- Scalar Field
- Vector Field

Electric charge is a fundamental property of matter. Charge exists only in positive or negative integral multiple of electronic charge,e^{-}, e= 1.60 × 10-19 coulombs. It may be noted here that in 1962, Murray Gell-Mann hypothesized Quarks as the basic building blocks of matters. Quarks were predicted to carry a fraction of electronic charge and the existence of Quarks has been experimentally verified.] Principle of conservation of charge states that the total charge (algebraic sum of positive and negative charges) of an isolated system remains unchanged, though the charges may redistribute under the influence of electric field. Kirchhoff's Current Law (KCL) is an assertion of the for conservative property of charges under the implicit assumption that there is no accumulation of charge at the junction.

Electromagnetic theory deals directly with the electric and magnetic field vectors where as circuit theory deals with the voltages and currents. Voltages and currents are integrated effects of electric and magnetic fields respectively. Electromagnetic field problems involve three space variables along with the time variable and hence the solution tends to become correspondingly complex. Vector analysis is a mathematical tool with which electromagnetic concepts are more conveniently expressed and best comprehended. Since use of vector analysis in the study of electromagnetic field theory results in real economy of time and thought, we first introduce the concept of vector analysis

VECTOR FIELDS

Scalars and Vectors

Vector analysis is a mathematical tool with which electromagnetic concepts are most convenient expressed and best comprehended.

A **Scalar** is a quantity that has only magnitude.

Quantities such as time, mass, distance, temperature, entropy, electric potential and population are scalar

A **Vector** is a quantity that has both magnitude and direction.

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Vector quantities include velocity, force, displacement and electric field intensity. Another class of physical quantities is called **tensors**, of which scalars and vectors are special cases.

A Scalar quantity is represented by a letter e.g., A, B, U and V.

A Vector quantity is represented by a letter with an arrow on top of it, such as $\vec{A}$ and $\vec{B}$

A field is a function that specifies a particular quantity everywhere in a region.

Unit Vector

A vector **A** that has both magnitude and direction. The magnitude of A is a scalar written as A or $|A|$.A unit vector $\mathbf{a_A}$ along **A** is defined as a vector whose magnitude is unity (i.e., 1) and its direction is along **A**, that is

$$\boldsymbol{a_A} = \frac{\boldsymbol{A}}{|\boldsymbol{A}|} = \frac{\boldsymbol{A}}{\boldsymbol{A}}$$

Notes that $|\boldsymbol{a_A}| = \mathbf{1}$

We may write **A** as

$$\boldsymbol{A} = \boldsymbol{A a_A}$$

Which completely specifies **A** in terms of its magnitude **A** and its direction $\boldsymbol{a_A}$

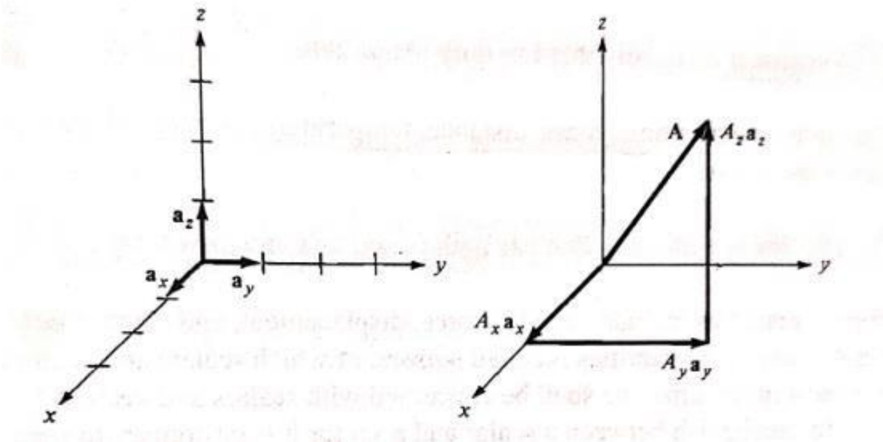

Figure 1.1.1 Unit Vectors

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-06]

A Vector **A** in Cartesian or rectangular coordinates represented by

$$(\boldsymbol{A_x}, \boldsymbol{A_y}, \boldsymbol{A_z}) \quad \textbf{or} \quad \boldsymbol{A_x a_x} + \boldsymbol{A_y a_y} + \boldsymbol{A_z a_z}$$

A_x, A_y, A_z are called the components of **A** in the ***x, y, z*** directions respectively; a_x, a_y, a_z are unit vector in the ***x, y, z*** directions respectively

The magnitude of vector **A** is given by

$$A = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

and the unit vector along **A** is given by

$$a_A = \frac{A_x a_x + A_y a_y + A_z a_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}}$$

Sum and Difference of two vectors

The sum of two vectors is the resultant of two vectors.

Two vectors **A** and **B** can be added together to give to another vector **C**

$$C = A + B$$

The vector addition is carried out components by component. Thus if

$$\vec{A} = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$

$$\vec{B} = B_x\vec{a}_x + B_y\vec{a}_y + B_z\vec{a}_z$$

Sum of two vectors $\quad C = A + B$

$$\vec{A} + \vec{B} = (A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z) + (B_x\vec{a}_x + B_y\vec{a}_y + B_z\vec{a}_z)$$

$$\vec{A} + \vec{B} = (A_x + B_x)\vec{a}_x + (A_y + B_y)\vec{a}_y + (A_z + B_z)\vec{a}_z$$

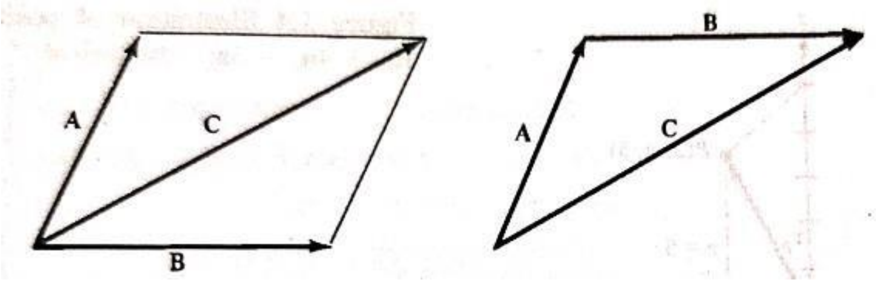

Figure 1.1.2 Vector Addition

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[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-07]

Difference of two vectors $\boldsymbol{C} = \boldsymbol{A} - \boldsymbol{B}$

$$\vec{A} - \vec{B} = (A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z) - (B_x\vec{a}_x + B_y\vec{a}_y + B_z\vec{a}_z)$$

$$\vec{A} - \vec{B} = (A_x - B_x)\vec{a}_x + (A_y - B_y)\vec{a}_y + (A_z - B_z)\vec{a}_z$$

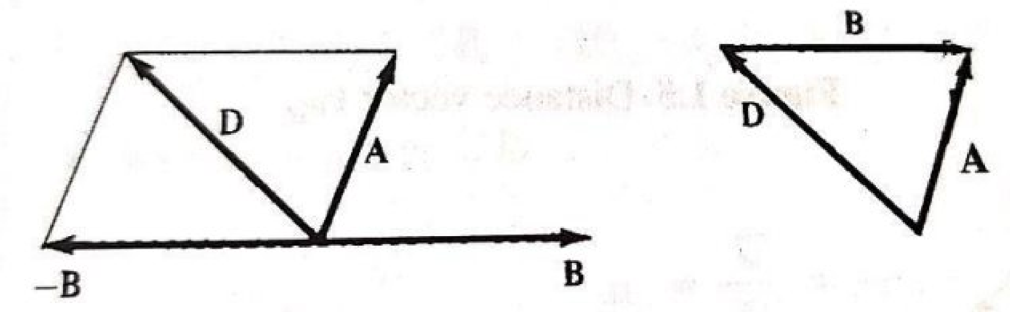

Figure 1.1.2 Vector Subtraction

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-07]

Multiplication of a Scalar and a Vector

When two vectors **A** and **B** are multiplied, the result is either a scalar or a vector depending on they are multiplied. Thus there are two types of vector multiplication:

- Scalar or dot product : $\boldsymbol{A}.\boldsymbol{B}$
- Vector or Cross product: $\boldsymbol{A} \times \boldsymbol{B}$

Multiplication of three vectors **A**, **B** and **C** can result in either

- Scalar triple product:$\boldsymbol{A}.(\boldsymbol{B} \times \boldsymbol{C})$
- Vector triple product: $\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C})$

Dot Product or Scalar Product

The **dot product** of two vectors **A** and **B** written as **A.B** is called either the scalar product because it is scalar or the dot product due to the dot sign. If

$$\vec{A} = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$

$$\vec{B} = B_x\vec{a}_x + B_y\vec{a}_y + B_z\vec{a}_z$$

$$\vec{A}.\vec{B} = (A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z).(B_x\vec{a}_x + B_y\vec{a}_y + B_z\vec{a}_z)$$

$$\vec{A}.\vec{B} = (A_xB_x + A_yB_y + A_zB_z)$$

$$\vec{a}_x.\vec{a}_x = \vec{a}_y.\vec{a}_y = \vec{a}_z.\vec{a}_z = 1$$

$$\vec{a}_x.\vec{a}_y = \vec{a}_y.\vec{a}_z = \vec{a}_z.\vec{a}_{xz} = 0$$

Cross Product or Vector Product

The **Cross Product** of two vectors **A** and **B** written **A***XB***,** is a vector quantity whose magnitude is the area of the parallelogram formed by **A** and **B** and is in the direction of advance of a right handed screw as **A** is turned into **B**

The vector product is given by

$$A.B = AB\cos\theta$$

Let **A** and **B** are two vectors

$$\vec{A} = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$

$$\vec{B} = B_x\vec{a}_x + B_y\vec{a}_y + B_z\vec{a}_z$$

Where $\vec{a}_x$, $\vec{a}_y$ and $\vec{a}_z$ are unit vectors in the direction of **x, y, z**

$$|A \times B| = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Note:

$$\vec{a}_x.\vec{a}_y = \vec{a}_z$$

$$\vec{\vec{a}}_y \,.\, \vec{\vec{a}}_z = \vec{\vec{a}}_x$$

$$\vec{\vec{a}}_z \,.\, \vec{\vec{a}}_x = \vec{\vec{a}}_y$$

The vector $\boldsymbol{B} \times \boldsymbol{A}$ has the same magnitude but the opposite direction

$$\boldsymbol{B} \times \boldsymbol{A} = -\boldsymbol{A} \times \boldsymbol{B}$$

1.2 CO- ORDINATE SYSTEMS

A point or vector can be represented in any curvilinear coordinate system, which may be orthogonal or non orthogonal.

An **orthogonal system** is one in which the coordinates are mutually perpendicular.

To describe a vector accurately and to express a vector in terms of its components, it is necessary to have some reference directions. Such directions are represented in terms of various co-ordinate systems. There are various co-ordinates systems available in mathematics, the co-ordinate systems are

- **Cartesian or Rectangular co-ordinate system**
- **Cylindrical co-ordinate system**
- **Spherical co-ordinate system**

CARTESIAN OR RECTANGULAR CO-ORDINATE SYSTEM

There are three simple methods to describe a vector accurately such as specific lengths, directions, angles, projections or components. The simplest methods of these are Cartesian or Rectangular co-ordinate system.

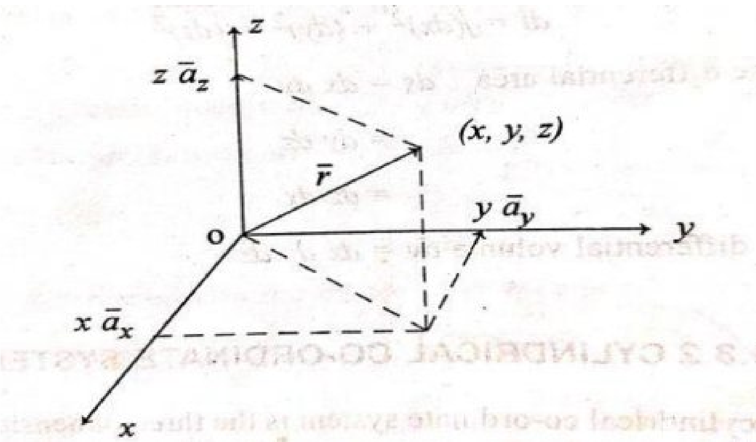

Figure 1.2.1 Cartesian co-ordinate system

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-30]

In Cartesian co-ordinate system three co-ordinate axes ***x, y, z*** are mutually right angles to each other. Considered a point **P(*x, y, z*)** in space at a distance **r** from the orgin. The vector **r** can be represented as

$$\vec{r} = x\vec{a_x} + y\vec{a_y} + z\vec{a_z}$$

$\vec{a_x}, \vec{a_y}, \vec{a_z}$ are Unit Vector

x, y, z are the components vectors. Components vectors have a magnitude and direction. Unit vectors have unit magnitude and directed along the co-ordinate axis.

A Unit Vector in a given direction is a vector in that direction divided by its magnitude. It is given by

$$\vec{a_r} = \frac{\vec{r}}{|r|}$$

$$|r| = \sqrt{x^2 + y^2 + z^2}$$

$$\vec{a_r} = \frac{x\vec{a}_x + y\vec{a}_y + z\vec{a}_z}{\sqrt{x^2 + y^2 + z^2}}$$

The ranges of the co-ordinate variables ***x, y, z*** are

$$-\infty < x < \infty$$

$$-\infty < y < \infty$$

$$-\infty < z < \infty$$

Considered the points **P(*x, y, z*)** and **Q(*x*+*dx*, *y*+*dy*, *z*+*dz*)** in a rectangular co-ordinate system.

The differential length ***dl*** from **P** to **Q** is the diagonal of the parallel piped is given by

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The differential length $dl = \sqrt{(d_x)^2 + (d_y)^2 + (d_z)^2}$

The differential area $d_s = d_x d_y$

$$d_s = d_y d_z$$

$$d_s = d_z d_x$$

The differential Volume $d_v = d_x d_y d_z$

CYLINDRICAL CO-ORDINATE SYSTEM

The circular cylindrical co-ordinate system is very convenient whenever we are dealing with problems having cylindrical symmetry.

A point **P** is cylindrical co-ordinates is represented as (ρ, φ, z) and is shown in figure 1.2.2.

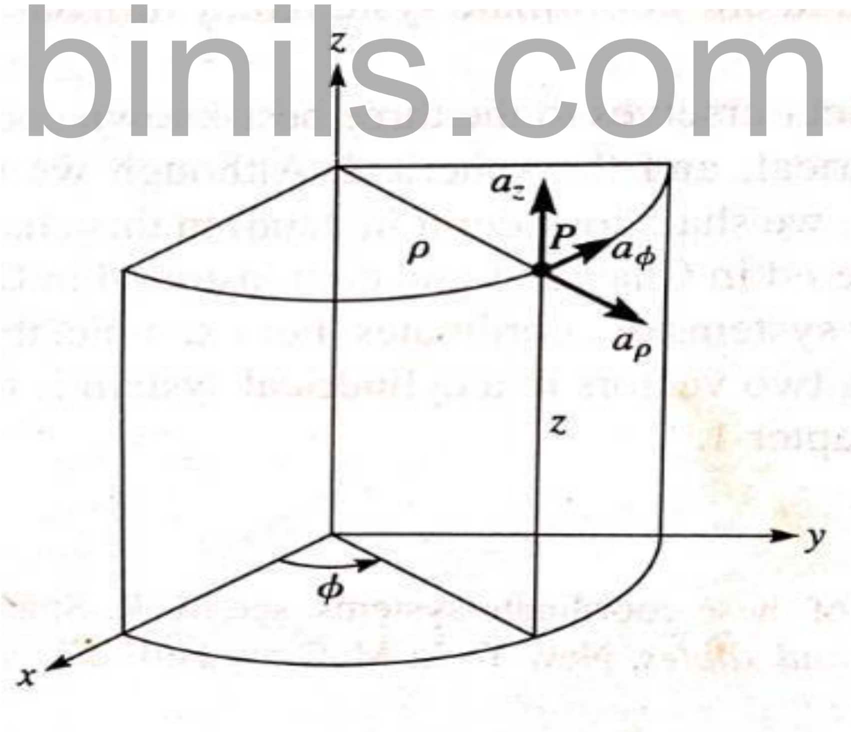

Figure 1.2.2 Point P and unit vector in the cylindrical co-ordinate system

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-30]

- $\boldsymbol{\rho}$ is the radius of cylinder passing through **P** or the radial distance from the $\boldsymbol{z}$ - axis
- $\boldsymbol{\varphi}$ called the azimuthal angle, is measured from the $\boldsymbol{x}$ – axis in the xy – plane
- $\boldsymbol{z}$ is the same as in the Cartesian system .

Considered any point as the intersection of three mutually perpendicular surfaces. They is a circular cylinder **($\boldsymbol{\rho}$ =constant),** a place **($\boldsymbol{\varphi}$ =constant),** and another place **($\boldsymbol{z}$ =constant)**

A differential volume element in cylindrical co-ordinate may obtained by increasing $\boldsymbol{\rho}$, $\boldsymbol{\varphi}$ and $\boldsymbol{z}$ by the differential increments $\boldsymbol{d\rho}$, $\boldsymbol{d\varphi}$ and $\boldsymbol{dz}$ .The shape of this small volume is truncated wedge .As the volume element becomes very small.its shape approaches that of a rectangular parallel piped. It has sides of the length $\boldsymbol{d\rho}, \boldsymbol{\rho d\varphi}$ and $\boldsymbol{dz}$

The differential length $\quad \boldsymbol{dl} = \sqrt{(\boldsymbol{d\rho})^2 + (\boldsymbol{\rho d\varphi})^2 + (\boldsymbol{dz})^2}$

The differential area $\quad \boldsymbol{d_s = d\rho . \rho d\varphi = \rho d\rho d\varphi}$

$$\boldsymbol{d_s = \rho d\varphi . dz}$$

$$\boldsymbol{d_s = dz . d\rho = d\rho dz}$$

The differential Volume $\quad \boldsymbol{d_v = d\rho . \rho d\varphi . dz}$

$$\boldsymbol{d_v = \rho d\rho d\varphi dz}$$

The ranges of the co-ordinate variables are

$$\boldsymbol{0} \leq \boldsymbol{\rho} < \infty$$

$$\boldsymbol{0} \leq \boldsymbol{\varphi} < 2\boldsymbol{\pi}$$

$$-\infty < z < \infty$$

Now the unit vector $\boldsymbol{a_\rho}, \boldsymbol{a_\varphi}$ and $\boldsymbol{a_z}$ are mutually perpendicular because our co-ordinate system is orthogonal: $\boldsymbol{a_\rho}$ points in the direction of increasing $\boldsymbol{\rho}$, $\boldsymbol{a_\varphi}$ in the direction of increasing $\boldsymbol{\varphi}$ **and** $\boldsymbol{a_z}$ in the positive $\boldsymbol{z}$ - direction

$$\boldsymbol{a_\rho}.\boldsymbol{a_\rho}=\boldsymbol{a_\varphi}.\boldsymbol{a_\varphi} = \boldsymbol{a_z}.\boldsymbol{a_z} = 1$$

$$\boldsymbol{a_\rho}.\boldsymbol{a_\varphi}=\boldsymbol{a_\varphi}.\boldsymbol{a_z} = \boldsymbol{a_z}.\boldsymbol{a_\rho} = 0$$

$$\boldsymbol{a_\rho} \times \boldsymbol{a_\varphi} = \boldsymbol{a_z}$$

$$\boldsymbol{a_\varphi} \times \boldsymbol{a_z} = \boldsymbol{a_\rho}$$

$$\boldsymbol{a_Z} \times \boldsymbol{a_\varphi} = \boldsymbol{a_\varphi}$$

SPHERICAL CO-ORDINATE SYSTEM

The spherical co-ordinate system is most appropriate when one is dealing with problems having a degree of spherical symmetry. A point **P** can be represented as $(\boldsymbol{r}, \boldsymbol{\theta}, \boldsymbol{\varphi})$ and it is illustrated in figure 1.2.3.

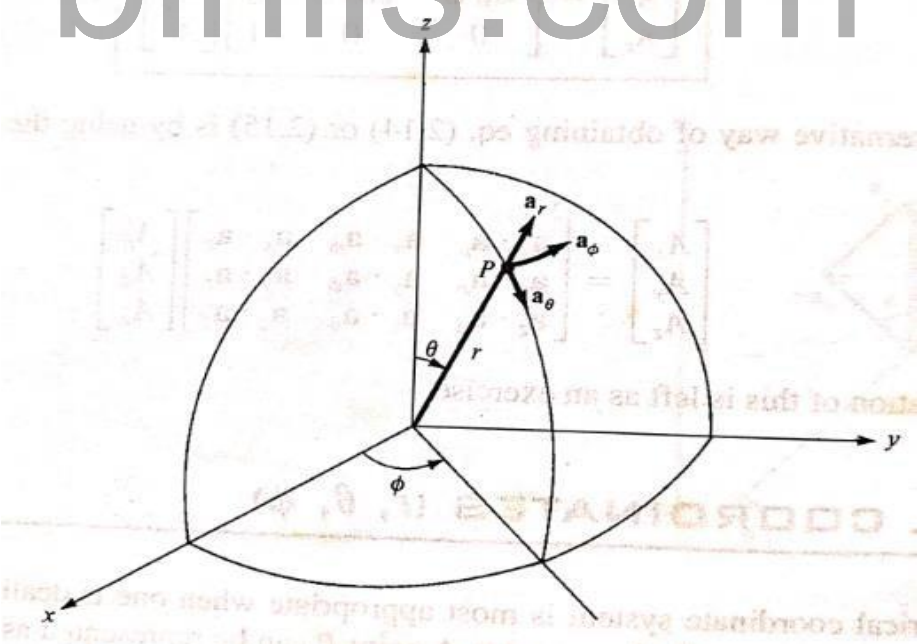

Figure 1.2.3 Point P and unit vector in the spherical co-ordinate

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-34]

- **r** is defines as the distance from the origin to **P** or the radius of a sphere entered at the origin and passing through **P .**
- $\boldsymbol{\theta}$ (Called the colatitudes) is the angle between the $\boldsymbol{z}$ – axis and position vector of **P.**
- $\boldsymbol{\varphi}$ is measured from the $\boldsymbol{x}$ **-** axis (the same azimuthal angle in cylindrical co-ordinates)

Considered any point as the intersection of the spherical surfaces **(**$\boldsymbol{radius\ r}$ **=constant),** conical surface($\boldsymbol{\theta}$**,** angle between $\boldsymbol{r\ and\ z}$**=constant),** and plane surface **(**$\boldsymbol{\varphi}$ **=constant).**The co-ordinates of spherical system are $\boldsymbol{r, \theta, \varphi}$

A differential volume element may be obtained in spherical co-ordinate by increasing $\boldsymbol{r, \theta}$ and $\boldsymbol{\varphi}$ by $\boldsymbol{dr, d\theta\ and\ d\varphi}$ the sides of this volume element are $\boldsymbol{dr, rd\theta\ and\ r\sin\theta d\varphi}$**.**

The differential length $\boldsymbol{dl} = \sqrt{(\boldsymbol{dr})^2 + (\boldsymbol{rd\theta})^2 + (\boldsymbol{r\sin\theta d\varphi})^2}$

The differential area $\boldsymbol{d_s = dr.rd\theta = r\ dr\ d\theta}$

$$\boldsymbol{d_s = rd\theta . r\sin\theta d\varphi = r^2 \sin\theta\ d\theta\ d\varphi}$$

$$\boldsymbol{d_s = r\sin\theta d\varphi . dr = r\sin\theta d\varphi\ dr}$$

The differential Volume $\boldsymbol{d_v = dr.rd\theta.r\sin\theta d\varphi}$

$$\boldsymbol{d_v = r^2 \sin\theta\ d\theta\ d\varphi\ dr}$$

The ranges of the co-ordinate variables are

$$\boldsymbol{0 \le r < \infty}$$

$$\boldsymbol{0 \le \theta} < \pi$$

$$\boldsymbol{0} < \varphi < 2\boldsymbol{\pi}$$

Now the unit vector$\boldsymbol{a_r}$, $\boldsymbol{a_\theta}$ and $\boldsymbol{a_\varphi}$ are mutually orthogonal:$\boldsymbol{a_r}$ being directed along the radius or in the direction of increasing $\boldsymbol{r}$, $\boldsymbol{a_\theta}$ in the direction of increasing θ and $\boldsymbol{a_\varphi}$ in the direction of creasing φ.

$$\boldsymbol{a_r}.\boldsymbol{a_r}=\boldsymbol{a_\theta}.\boldsymbol{a_\theta}=\boldsymbol{a_\varphi}.\boldsymbol{a_\varphi}=1$$

$$\boldsymbol{a_r}.\boldsymbol{a_\theta}=\boldsymbol{a_\theta}.\boldsymbol{a_\varphi}=\boldsymbol{a_\varphi}.\boldsymbol{a_r}=0$$

$$\boldsymbol{a_r}\times\boldsymbol{a_\theta}=\boldsymbol{a_\varphi}$$

$$\boldsymbol{a_\theta}\times\boldsymbol{a_\varphi}=\boldsymbol{a_r}$$

$$\boldsymbol{a_\varphi}\times\boldsymbol{a_r}=\boldsymbol{a_\theta}$$

TRANSFORMATION OF CO-ORDINATE SYSTEM

It is necessary to transform a vector from one co-ordinate system to another co-ordinate system. Transformation of a vector between Cartesian and cylindrical co-ordinate system and Cartesian and spherical system are carried out.

(A) Transformation between Cartesian and cylindrical systems

A vector in Cartesian co-ordinate system can be converted into cylindrical co-ordinates system.

(i) Conversion of Cartesian to Cylindrical system

The Cartesian co-ordinate system $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$ can be converted into cylindrical co-ordinates system. $(\boldsymbol{\rho}, \boldsymbol{\varphi}, \boldsymbol{z})$

Given	**Transform**
$\boldsymbol{x}$	$\boldsymbol{\rho}=\sqrt{\boldsymbol{x}^2+\boldsymbol{y}^2}$
$\boldsymbol{y}$	$\boldsymbol{\varphi}=\tan^{-1}\left(\frac{\boldsymbol{y}}{\boldsymbol{x}}\right)$
$\boldsymbol{z}$	$\boldsymbol{z}=\boldsymbol{z}$

(ii) Conversion of Cylindrical to Cartesian system

The cylindrical co-ordinates system. (ρ, φ, z) can be converted into Cartesian co-ordinate system x, y, z

Given	Transform
ρ	$x = \rho \cos \varphi$
φ	$y = \rho \sin \varphi$
z	$z = z$

(B) Transformation between Cartesian and Spherical systems

A vector in Cartesian co-ordinate system can be converted into spherical co-ordinates system.

(i) Conversion of Cartesian to Spherical system

The Cartesian co-ordinate system x, y, z can be converted into cylindrical co-ordinates system. (r, θ, φ)

Given	Transform
x	$r = \sqrt{x^2 + y^2 + z^2}$
y	$\theta = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$
	$= \cos^{-1}\left(\frac{z}{r}\right)$
z	$\varphi = \tan^{-1}\left(\frac{y}{x}\right)$

(ii) Conversion of Spherical to Cartesian co-ordinate system

The Cartesian co-ordinate system $\boldsymbol{x, y, z}$ can be converted into cylindrical co-ordinates system. $(\boldsymbol{r}, \boldsymbol{\theta}, \boldsymbol{\varphi})$

Given	Transform
$\boldsymbol{r}$	$\boldsymbol{x = r \sin \theta \cos \varphi}$
$\boldsymbol{\theta}$	$\boldsymbol{y = r \sin \theta \sin \varphi}$
$\boldsymbol{\varphi}$	$\boldsymbol{z = r \cos \theta}$

1.3 GRADIENT, DIVERGENCE, CURL

Del Operator:

The del operator(∇), is the vector differential operator. In Cartesian co-ordinates,

$$\nabla = \frac{\partial}{\partial_x}\vec{a_x} + \frac{\partial}{\partial_y}\vec{a_y} + \frac{\partial}{\partial_z}\vec{a_z}$$

This vector differential operator, otherwise known as the **gradient operator.**

- The gradient of a scalar V, written as ∇V
- The divergence of a vector A, written as $\nabla . A$
- The curl of a vector A, written as $\nabla \times A$
- The Laplacian of a scalar V, written as $\nabla^2 V$

GRADIENT OF A SCALAR

The gradient of a scalar field V is a vector that represents both the magnitude and the direction of the maximum space rate of increase of V.

The gradient of any scalar function is the maximum space rate of change of that function. If the s scalar V represents electric potential, ∇V represents potential gradient.

$$\nabla = \frac{\partial}{\partial_x}\vec{a_x} + \frac{\partial}{\partial_y}\vec{a_y} + \frac{\partial}{\partial_z}\vec{a_z}$$

$$\nabla V = \frac{\partial V}{\partial_x}\vec{a_x} + \frac{\partial V}{\partial_y}\vec{a_y} + \frac{\partial V}{\partial_z}\vec{a_z}$$

This operation is called the gradient

$$\nabla V = grad\, V$$

DIVERGENCE OF A VECTOR

The divergence of A at a given point P is the outward flux per unit volume as the volume shrinks about P.

The divergence of a vector A at any point is defined as the limit of its surface integrated per unit volume as the volume enclosed by the surface shrinks to zero

$$\nabla . A = \lim_{v \to 0} \frac{1}{v} \oint\!\!\oint_s A.\vec{n}\, ds$$

It can be expressed as

$$\nabla = \frac{\partial}{\partial_x}\vec{a}_x + \frac{\partial}{\partial_y}\vec{a}_y + \frac{\partial}{\partial_z}\vec{a}_z$$

$$A = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$

$$\nabla . A = \left(\frac{\partial}{\partial_x}\vec{a}_x + \frac{\partial}{\partial_y}\vec{a}_y + \frac{\partial}{\partial_z}\vec{a}_z\right) + (A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z)$$

$$\nabla . \mathrm{A} = \frac{\partial A_x}{\partial_x} + \frac{\partial A_y}{\partial_y} + \frac{\partial A_z}{\partial_z}$$

This operation is called divergence

$$\nabla . A = div\ A$$

Divergence of a vector is a scalar quantity.

CURL OF A VECTOR

The curl of **A** is an axial (or) rotational vector whose magnitude is the maximum circulation of **A** per unit area as the area tends to zero and whose direction is the normal direction of the area when the area is oriented to make the circulation maximum.

The curl of vector **A** at any point is defined as the limit of its surface integral of its cross product with normal over a closed surface per unit volume as the volume shrinks to zero.

$$|\mathrm{Curl}\ A| = \lim_{v \to 0} \frac{1}{v} \oiint_s \vec{n} \times A\ ds$$

It can expressed as

$$\nabla = \frac{\partial}{\partial_x}\vec{a}_x + \frac{\partial}{\partial_y}\vec{a}_y + \frac{\partial}{\partial_z}\vec{a}_z$$

$$A = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial_x} & \frac{\partial}{\partial_y} & \frac{\partial}{\partial_z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla \times A = \left(\frac{\partial A_z}{\partial_y} - \frac{\partial A_y}{\partial_z}\right)\vec{a}_x - \left(\frac{\partial A_z}{\partial_x} - \frac{\partial A_x}{\partial_z}\right)\vec{a}_y + \left(\frac{\partial A_y}{\partial_x} - \frac{\partial A_x}{\partial_z}\right)\vec{a}_z$$

$$\nabla \times A = \left(\frac{\partial A_z}{\partial_y} - \frac{\partial A_y}{\partial_z}\right)\vec{a}_x + \left(\frac{\partial A_x}{\partial_z} - \frac{\partial A_z}{\partial_x}\right)\vec{a}_y + \left(\frac{\partial A_y}{\partial_x} - \frac{\partial A_x}{\partial_z}\right)\vec{a}_z$$

This operation is called curl.

$$\nabla \times A = Curl\ A$$

SOLENOIDAL AND IRROTATIONAL VECTORS

- A vector $\vec{A}$ is said to be solenoidal if its divergence is zero.

  i.e $\nabla . \vec{A} = 0$, then $\vec{A}$ is said to be solenoidal.

- A vector $\vec{A}$ is said to be irrotational if its curl is zero.

  i.e $\nabla \times \vec{A} = 0$, then $\vec{A}$ is said to be irrotational.

1.4 THEOREMS AND APPLICATIONS

DIVERGENCE THEOREM:

Divergence of a Vector

The divergence of ***A*** at a given point ***P*** is the outward flux per unit volume as the volume shrinks about ***P*.**

The divergence of a vector ***A*** at any point is defined as the limit of its surface integrated per unit volume as the volume enclosed by the surface shrinks to zero

$$\nabla . A = \lim_{v \to 0} \frac{1}{v} \oiint_s A.\vec{n}\, ds$$

It can be expressed as

$$\nabla = \frac{\partial}{\partial_x}\vec{a_x} + \frac{\partial}{\partial_y}\vec{a_y} + \frac{\partial}{\partial_z}\vec{a_z}$$

$$A = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$

$$\nabla . A = \left(\frac{\partial}{\partial_x}\vec{a_x} + \frac{\partial}{\partial_y}\vec{a_y} + \frac{\partial}{\partial_z}\vec{a_z}\right) + \left(A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z\right)$$

$$\nabla . \mathrm{A} = \frac{\partial A_x}{\partial_x} + \frac{\partial A_y}{\partial_y} + \frac{\partial A_z}{\partial_z}$$

This operation is called divergence

$$\nabla . A = div\, A$$

Divergence of a vector is a scalar quantity.

DIVERGENCE THEOREM

The volume integral of the divergence of a vector field over a volume is equal to the surface integral of the normal component of this vector over the surface bounding the volume.

$$\iiint_v \nabla . A\, dv = \oiint_s A.ds$$

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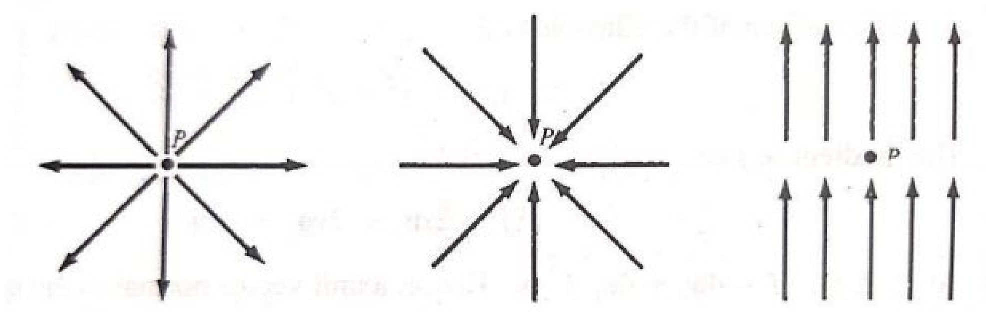

Figure 1.4.1 Illustration of the divergence of a vector at P

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-72]

Proof:

The divergence of any vector $\boldsymbol{A}$ is given by

$$\nabla = \frac{\partial}{\partial_x}\vec{a}_x + \frac{\partial}{\partial_y}\vec{a}_y + \frac{\partial}{\partial_z}\vec{a}_z$$

$$\boldsymbol{A} = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$

$$\nabla.\boldsymbol{A} = \left(\frac{\partial}{\partial_x}\vec{a}_x + \frac{\partial}{\partial_y}\vec{a}_y + \frac{\partial}{\partial_z}\vec{a}_z\right) + \left(A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z\right)$$

$$\nabla.\mathrm{A} = \frac{\partial A_x}{\partial_x} + \frac{\partial A_y}{\partial_y} + \frac{\partial A_z}{\partial_z}$$

Divergence Theorem

$$\iiint_v \nabla.\boldsymbol{A}\, dv = \oint\!\!\oint_s \boldsymbol{A}.\boldsymbol{ds}$$

Take the volume integral on both sides

$$\iiint_v \nabla.\boldsymbol{A}\, dv = \iiint_v \left[\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial_y} + \frac{\partial A_z}{\partial_z}\right] d_x d_y d_z$$

That is $d_v = d_x d_y d_z$

$$\iiint_v \nabla.\boldsymbol{A}\, dv = \iiint_v \left[\frac{\partial A_x}{\partial_x} d_x d_y d_z + \frac{\partial A_y}{\partial_y} d_x d_y d_z + \frac{\partial A_z}{\partial_z} d_x d_y d_z\right]$$

$$\iiint_v \nabla . A\, dv = \iiint_v [\frac{\partial A_x}{\partial_x} d_x d_y d_z] + \iiint_v [\frac{\partial A_y}{\partial_y} d_x d_y d_z] + \iiint_v [\frac{\partial A_z}{\partial_z} d_x d_y d_z]$$

Considered the first portion of equation

$$\iiint_v \frac{\partial A_x}{\partial_x} d_x d_y d_z$$

Assume the limit as $x_1\ to\ x_2$

$$\iiint_v \frac{\partial A_x}{\partial_x} d_x d_y d_z = \iiint_{x1}^{x_2} \frac{\partial A_x}{\partial_x} d_x d_y d_z$$

$$= \iint [A_x]_{x1}^{x2}\ d_y d_z$$

$$\iiint_{x1}^{x_2} \frac{\partial A_x}{\partial_x} d_x d_y d_z = \iint [A_{x2} - A_{x1}] d_y d_z$$

$$\iiint_v \frac{\partial A_x}{\partial_x} d_x d_y d_z = \iint [A_{x2} - A_{x1}] d_y d_z$$

Considered the second portion of equation

$$\iiint_v [\frac{\partial A_y}{\partial_y} d_x d_y d_z]$$

$$\iiint_v [\frac{\partial A_y}{\partial_y} d_x d_y d_z] = \iiint_{y1}^{y_2} \frac{\partial A_y}{\partial_y} d_x d_y d_z$$

$$= \iint [A_y]_{y1}^{y2}\ d_x d_z$$

$$\iiint_{y1}^{y_2} \frac{\partial A_y}{\partial_y} d_x d_y d_z = \iint [A_{y2} - A_{y1}] d_x d_z$$

$$\iiint_v [\frac{\partial A_y}{\partial_y} d_x d_y d_z] = \iint [A_{y2} - A_{y1}] d_x d_z$$

Considered the third portion of equation

$$\iiint_v \left[\frac{\partial A_z}{\partial_z} d_x d_y d_z\right]$$

$$\iiint_v \left[\frac{\partial A_z}{\partial_z} d_x d_y d_z\right] = \iiint_{z1}^{z_2} \frac{\partial A_z}{\partial_z} d_x d_y d_z$$

$$\iiint_{z1}^{z_2} \frac{\partial A_z}{\partial_z} d_x d_y d_z = \iint [A_z]_{z1}^{z2} d_x d_y$$

$$\iiint_v \left[\frac{\partial A_z}{\partial_z} d_x d_y d_z\right] = \iint [A_{z2} - A_{z1}] d_x d_y$$

Substitute the equation in below equation

$$\iiint_v \nabla . A\, dv = \iiint_v \left[\frac{\partial A_x}{\partial_x} d_x d_y d_z\right] + \iiint_v \left[\frac{\partial A_y}{\partial_y} d_x d_y d_z\right] + \iiint_v \left[\frac{\partial A_z}{\partial_z} d_x d_y d_z\right]$$

$$\iiint_v \nabla . A\, dv = = \iint [A_{x2} - A_{x1}]\, d_y d_z + \iint [A_{y2} - A_{y1}]\, d_x d_z + \iint [A_{z2} - A_{z1}]\, d_x d_y$$

Assume $A_{x2} - A_{x1} = A_x$

$$A_{y2} - A_{y1} = A_y$$

$$A_{z2} - A_{z1} = A_z$$

Then $d_x d_y = ds_z$

$$d_y d_z = ds_x$$

$$d_z d_x = ds_y$$

Sub all these in above equation

$$\iiint_v \nabla . A\, dv = = \iint [A_x]\, ds_x + \iint [A_y]\, ds_y + \iint [A_z]\, ds_z$$

$$\iiint_v \nabla . A\, dv = \iint [A_x + A_y + A_z]\, d_s$$

Assume $\quad A_x + A_y + A_z = A$

Substitute $A_x + A_y + A_z = A$ in above equation

$$= \oiint_s A . ds$$

$$\iiint_v \nabla . A\, dv = \oiint_s A . ds$$

STROKES THEOREM:

Curl of a Vector

The curl of **A** is an axial (or) rotational vector whose magnitude is the maximum circulation of **A** per unit area as the area tends to zero and whose direction is the normal direction of the area when the area is oriented to make the circulation maximum.

The curl of vector **A** at any point is defined as the limit of its surface integral of its cross product with normal over a closed surface per unit volume as the volume shrinks to zero.

$$|\text{Curl } A| = \lim_{v \to 0} \frac{1}{v} \oiint_s \vec{n} \times A\, ds$$

It can expressed as

$$\nabla = \frac{\partial}{\partial_x}\vec{a}_x + \frac{\partial}{\partial_y}\vec{a}_y + \frac{\partial}{\partial_z}\vec{a}_z$$

$$A = A_x\vec{a}_x + A_y\vec{a}_y + A_z\vec{a}_z$$

$$\nabla \times \mathrm{A} = \begin{vmatrix} \vec{a}_x & \vec{a}_y & \vec{a}_z \\ \frac{\partial}{\partial_x} & \frac{\partial}{\partial_y} & \frac{\partial}{\partial_z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla \times A = \left(\frac{\partial A_z}{\partial_y} - \frac{\partial A_y}{\partial_z}\right)\vec{a}_x - \left(\frac{\partial A_z}{\partial_x} - \frac{\partial A_x}{\partial_z}\right)\vec{a}_y + \left(\frac{\partial A_y}{\partial_x} - \frac{\partial A_x}{\partial_z}\right)\vec{a}_z$$

$$\nabla \times A = \left(\frac{\partial A_z}{\partial_y} - \frac{\partial A_y}{\partial_z}\right)\vec{a}_x + \left(\frac{\partial A_x}{\partial_z} - \frac{\partial A_z}{\partial_x}\right)\vec{a}_y + \left(\frac{\partial A_y}{\partial_x} - \frac{\partial A_x}{\partial_z}\right)\vec{a}_z$$

This operation is called curl.

$$\nabla \times A = Curl\ A$$

STROKES THEOREM:

Strokes's theorem states that the circulation of a vector field A around a (closed) path L is equal to the surface integral of the curl of A over the open surface S bounded by L, provided A and $\nabla \times A$ are continuous on S.

The line integral of a vector around a closed path is equal to the surface integral of the normal component of its curl over any closed surface.

$$\oint H.dl = \iint_s \nabla \times H\ ds$$

Proof:

Consider an arbitrary surface this is broken up into incremental surface of area Δs shown in figure. 1.4.2

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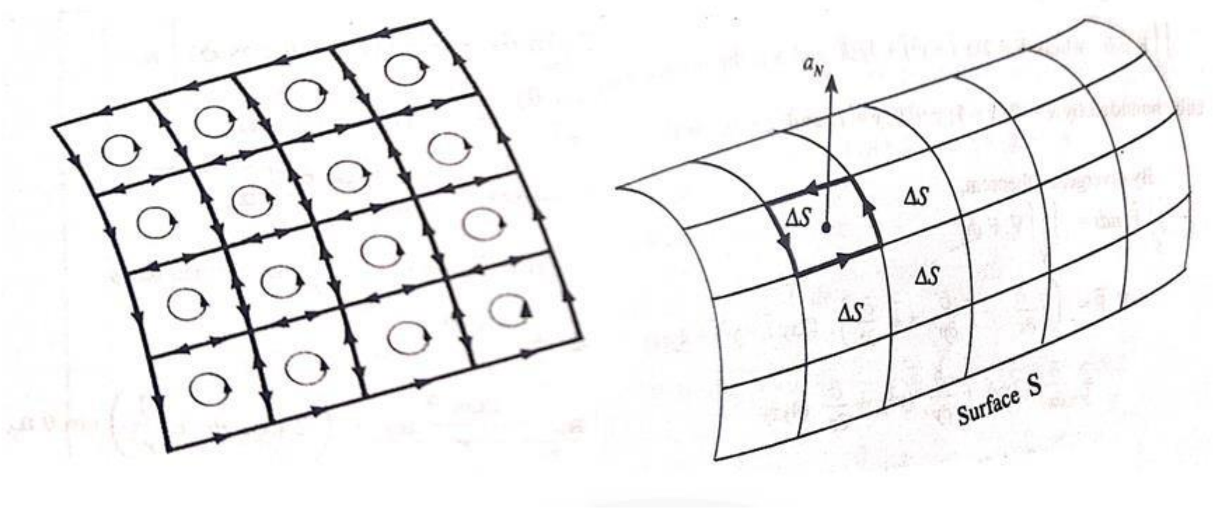

Figure 1.4.2 Illustration of Stroke's theorem

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-81]

If $\boldsymbol{H}$ is any field vector, then by definition of the curl to one of these incremental surfaces.

$$\frac{\oint \boldsymbol{H}.\boldsymbol{dl}\Delta\boldsymbol{s}}{\Delta\boldsymbol{s}} = (\nabla \times \boldsymbol{H})_N$$

Where $\boldsymbol{N}$ indicates normal to the surface and $\boldsymbol{dl}\Delta\boldsymbol{s}$ indicate that the closed path of an incremental area $\Delta\boldsymbol{s}$

The curl of $\boldsymbol{H}$ normal to the surface can be written as

$$\frac{\oint \boldsymbol{H}.\boldsymbol{dl}\Delta\boldsymbol{s}}{\Delta\boldsymbol{s}} = (\nabla \times \boldsymbol{H}) \,.\, \boldsymbol{a_N}$$

$$\oint \boldsymbol{H}.\boldsymbol{dl}\,\Delta\boldsymbol{s} = (\nabla \times \boldsymbol{H}) \,.\, \boldsymbol{a_N}\,\Delta\boldsymbol{s}$$

$$\oint \boldsymbol{H}.\boldsymbol{dl}\,\Delta\boldsymbol{s} = (\nabla \times \boldsymbol{H}) \,.\, \Delta\boldsymbol{s}$$

Where $\boldsymbol{a_N}$ is the unit vector normal to $\Delta\boldsymbol{s}$

The closed integral for whole surface $\boldsymbol{S}$ is given by the surface integral of the normal component of curl $\boldsymbol{H}$

$$\oint \boldsymbol{H}.\boldsymbol{dl} = \iint_{s} \nabla \times \boldsymbol{H}\, \boldsymbol{ds}$$

1.5 COULOMB'S LAW

Coulomb's Law:

Coulomb's law is an experimental law formulated in 1785 by Charles Augustin de coulomb. It deals with the force a point charge exerts on another point charge.A point charge means a charge that is located on a body whose dimensions are much smaller than other relevant dimensions.

Charge is generally measured in coulomb $(\boldsymbol{C})$.One Coulomb is approximately equivalent to **6×10^{18} electrons.** It is very large unit charge because one electron charge $\boldsymbol{e} = -\mathbf{1.6019} \times \mathbf{10^{-19}}\ \boldsymbol{C}$.

Coulomb's law states that the force between two very small objects separated by a distance which is large compared to their size is proportional to the charge on each and inversely proportional to the square of the distance between them.

The coulomb's law can be stated that "The force of attraction or repulsion between any two point charges is directly proportional to the product of two charges and inversely proportional to the square of the distance between them."

Consider the two charges $Q_1$and Q_2 separated by a distance $\boldsymbol{r}$.This force of interaction between two point charges is given as follows:

$$\boldsymbol{F} \propto \frac{\boldsymbol{Q_1 Q_2}}{\boldsymbol{r^2}}$$

$$\boldsymbol{F} = \frac{\boldsymbol{Q_1 Q_2}}{\boldsymbol{4\pi\varepsilon r^2}}\ Newtons$$

Where $\boldsymbol{\varepsilon}$ is permittivity of the medium or dielectric constant which is written as

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon_0 \varepsilon_r}\ Farads/meter$$

Relative permittivity of the medium

$$\boldsymbol{\varepsilon_r} = \frac{\boldsymbol{\varepsilon}}{\boldsymbol{\varepsilon_0}}$$

$$\boldsymbol{\varepsilon_r} = \frac{\boldsymbol{Permittivity\ of\ the\ medium}}{\boldsymbol{Permittivity\ of\ the\ free\ space\ (or) vaccum}} = \frac{\boldsymbol{\varepsilon}}{\boldsymbol{\varepsilon_0}}$$

$$Permittivity\ of\ the\ free\ space\ (or) vaccum\ (\varepsilon_0) = \frac{1}{36\pi \times 10^9}$$

$$= 8.854 \times 10^{-12}\ F/m$$

Coulom's law in vector form:

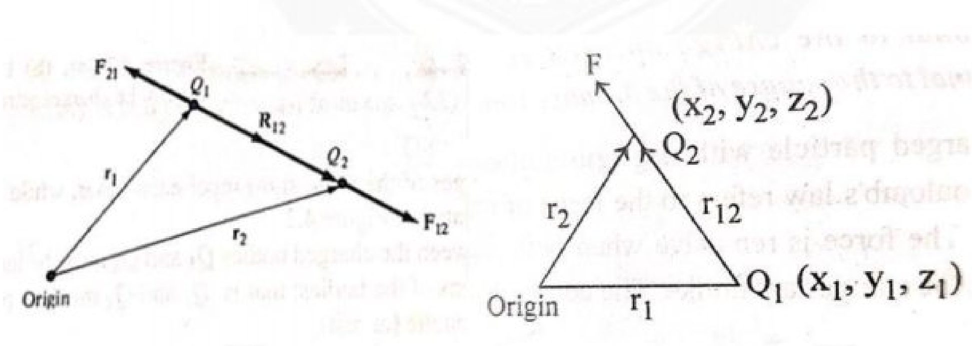

Figure 1.5.1 Coulomb vector force on point charges Q_1 and Q_2

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-107]

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon r^2} \times Unit\ vector$$

Consider two charges Q_1 and Q_2 at a distance vectors r_1 and r_2 from the origin respectively. r_{12} represents the distant vector form Q_1 to Q_2

$$r_{12} = r_2 - r_1$$

The vector F is the force between Q_1 and Q_2

The vector form of coulomb's law is

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon r^2} \times \vec{a}_{12}$$

The unit vector $$\vec{a}_{12} = \frac{r_2 - r_1}{|r_2 - r_1|}$$

Substitute $\vec{a}_{12}$ vector and ε value in above equation

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0\varepsilon_r r^2} \times \vec{a}_{12}$$

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0\varepsilon_r r^2} \times \frac{r_2 - r_1}{|r_2 - r_1|}$$

r is the distance between the charge.

The distance between the charges is r_{12}

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 r_{12}^2} \times \frac{r_2 - r_1}{|r_2 - r_1|}$$

That is $r_{12} = r_2 - r_1$

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 (r_2 - r_1)^2} \times \frac{r_2 - r_1}{|r_2 - r_1|}$$

$$r_2 - r_1 = (x_2 - x_1)\vec{a}_x + (y_2 - y)\vec{a}_y + (z_2 - z_1)\vec{a}_z$$

$$(r_2 - r_1)^2 = [(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]^2$$

$$|r_2 - r_1| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 [(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]^2} \times \frac{[(x_2 - x_1)\vec{a}_x + (y_2 - y)\vec{a}_y + (z_2 - z_1)\vec{a}_z]}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0 [(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]^2} \times \frac{[(x_2 - x_1)\vec{a}_x + (y_2 - y)\vec{a}_y + (z_2 - z_1)\vec{a}_z]}{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]^{1/2}}$$

$$F = \frac{Q_1Q_2}{4\pi\varepsilon_0} \times \frac{[(x_2 - x_1)\vec{a}_x + (y_2 - y)\vec{a}_y + (z_2 - z_1)\vec{a}_z]}{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]^{5/2}}$$

The force is attractive if the charges are of opposite sign and is repulsive if the charges are alike in sign

1.6 ELECTRIC FIELD DUE TO DISCRETE AND CONTINUOUS CHARGES

If the charge are distributed instead of concentrated at one point, it is better to define charge distribution in terms of charge density. It is also possible to have continuous charge distribution along line, on a surface or in a volume.

It is customary to denote the line charge density by ρ_l in(C/m), surface charge density by ρ_s in (C/m^2) and volume charge density by ρ_v in (C/m^3) respectively.

LINE OR LINEAR CHARGE DENSITY:

It is defined as the total charge distributed over a line or curve.

$$\rho_l = \lim_{\Delta l \to 0} \left(\frac{\Delta Q}{\Delta l}\right)$$

This gives the total charge per length. It is given by

$$\rho_l = \frac{Q}{l} \quad Coulomb/meter(c/m)$$

SURFACE CHARGE DENSITY:

It is defined as the total charge distributed over a surface.

$$\rho_s = \lim_{\Delta s \to 0} \left(\frac{\Delta Q}{\Delta s}\right)$$

This gives the total charge per area. It is given by

$$\rho_s = \frac{Q}{S} = \frac{Q}{A} \quad Coulomb/squaremeter(c/m^2)$$

VOLUME CHARGE DENSITY:

It is defined as the total charge distributed over a volume.

$$\rho_{sv} = \lim_{\Delta v \to 0} \left(\frac{\Delta Q}{\Delta v}\right)$$

This gives the total charge per volume. It is given by

$$\rho_v = \frac{Q}{V} \quad Coulomb/cubicmeter(c/m^3)$$

ELECTRIC FILED OR ELECTRIC FIELD INTENSITY:

The electric field or electric field intensity is defined as the electric force per unit charge .It is given by

$$E = \frac{F}{q}$$

According to coulomb's law

$$F = \frac{Qq}{4\pi\varepsilon r^2}$$

Electric Filed

$$E = \frac{F}{q}$$

Substitute F value in above equation

$$E = \frac{\frac{Qq}{4\pi\varepsilon r^2}}{q}$$

$$E = \frac{Q\cancel{q}}{4\pi\varepsilon r^2 \cancel{q}}$$

$$E = \frac{Q}{4\pi\varepsilon r^2} V/m$$

The another unit of electric field is $Volts/meter$

ELECTRIC POTENTIAL DIFFERENCE AND POTENTIAL:

Consider a uniform electric field E and a unit positive charge q .There is a force act on the charge due to the electric field. The force is given by

$$F = qE$$

There is a movement of charge in the electric field from one point r_1 to another r_2, there will be work done against the force

$$W = -\int_{r_1}^{r_2} qE.dr$$

$$W = -q\int_{r_1}^{r_2} E.dr$$

Potential difference (V) is defined as the work done in moving a unit positive charge from one point to another in an electric field.

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Work done on unit positive charge per charges

$$V = \frac{W}{q}$$

$$V = -\int_{r_1}^{r_2} E.\,dr \qquad Joules\,/Coulomb$$

But

$$E = \frac{Q}{4\pi\varepsilon r^2}$$

Substitute $\boldsymbol{E}$ in $\boldsymbol{V}$

$$V = -\frac{Q}{4\pi\varepsilon}\int_{r1}^{r2} \frac{1}{r^2}\,dr$$

$$V = -\frac{Q}{4\pi\varepsilon}\left[\frac{1}{r}\right]_{r_1}^{r_2}$$

$$V = -\frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$V = \frac{Q}{4\pi\varepsilon}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] \quad Volts$$

This is the potential difference between two points $\boldsymbol{r_1}$ and $\boldsymbol{r_2}$

$$V = \left[\frac{Q}{4\pi\varepsilon r_1} - \frac{Q}{4\pi\varepsilon r_2}\right] \quad Volts$$

$$V_1 = \frac{Q}{4\pi\varepsilon r_1}$$

$$V_2 = \frac{Q}{4\pi\varepsilon r_2}$$

$$V = V_1 + V_2$$

If the charge is moving form infinity to a given point in the electric filed

$$V_2 = 0$$

Then

$$V = V_1 + 0$$

$$V = V_1$$

Absolute potential or potential at a point is defined s the work done in moving a unit positive charge from infinity to a given point in an electric field.

$$V = \frac{Q}{4\pi\varepsilon r} \quad Volts$$

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Any field where the closed line integral of the field is zero, is said to be a conservative field

$$\boldsymbol{E.dr = 0}$$

Thus the electric field strength at any points just the negative of the potential gradient at that point. The negative sign shows that the direction of E is opposite to the direction in which V increases.

$$\boldsymbol{E = -\nabla V}$$

1.7 ELECTRIC FIELD INTENSITY

ELECTRIC FILED OR ELECTRIC FIELD INTENSITY:

The electric field or electric field intensity is defined as the electric force per unit charge .It is given by

$$E = \frac{F}{q}$$

According to coulomb's law

$$F = \frac{Qq}{4\pi\varepsilon r^2}$$

Electric Filed

$$E = \frac{F}{q}$$

Substitute F value in above equation

$$E = \frac{\frac{Qq}{4\pi\varepsilon r^2}}{q}$$

$$E = \frac{Q\cancel{q}}{4\pi\varepsilon r^2\cancel{q}}$$

$$E = \frac{Q}{4\pi\varepsilon r^2}\, V/m$$

The another unit of electric field is $Volts/meter$

ELECTRIC FIELD INTENSITY DUE TO LINE CHARGE:

Considered uniformly charged line of length L whose linear charge density is ρ_l Coulomb/meter. Consider a small element dl at a distance l from one end of the charged line as shown in figure 1.7.1 .Let P be any point at a distance r from the element dl.

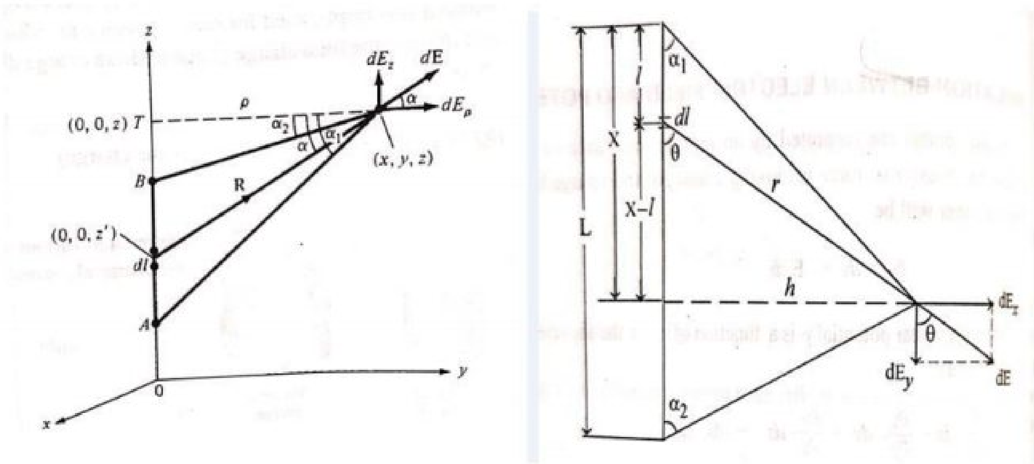


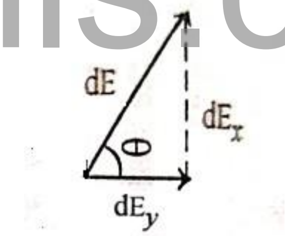
Figure 1.7.1 Evaluation of the *E* field due to a line charge

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-114]

The electric field at a point $\boldsymbol{P}$ due to the charge element $\boldsymbol{\rho dl}$ is given

$$dE = \frac{\rho_l dl}{4\pi\varepsilon r^2}$$

The $\boldsymbol{x}$ and $\boldsymbol{y}$ components of electric field $\boldsymbol{dE}$ are given by



From the above diagram find $\boldsymbol{\sin\theta}$ *and* $\boldsymbol{\cos\theta}$

$$\sin\theta = \frac{dE_x}{dE}$$

$$dE_x = dE\sin\theta$$

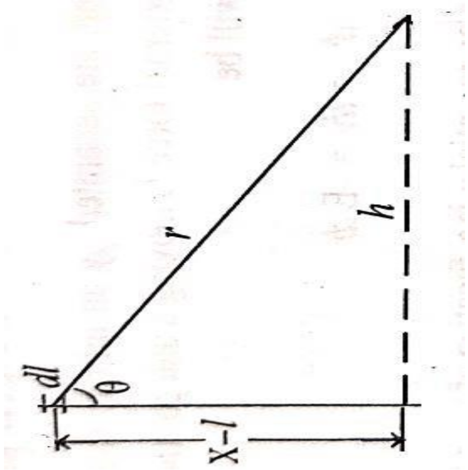
$$\cos\theta = \frac{dE_y}{dE}$$

$$dE_y = dE\cos\theta$$

Substitute $\boldsymbol{dE}$ expression in $\boldsymbol{dE_x}$

$$dE_x = \frac{\rho_l dl\sin\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_l dl\cos\theta}{4\pi\varepsilon r^2}$$



From the above diagram find $\tan\theta$

$$\tan\theta = \frac{h}{x-l}$$

$$x-l = \frac{h}{\tan\theta}$$

$$x-l = h\cot\theta$$

Differentiate above equation on both sides

$$0 - dl = h(-\operatorname{cosec}^2\theta)$$

$$-dl = -h(\operatorname{cosec}^2\theta)$$

$$dl = h(\operatorname{cosec}^2\theta).d\theta$$

From the above diagram find $\sin\theta$

$$\sin\theta = \frac{h}{r}$$

$$r = \frac{h}{\sin\theta}$$

$$r = h\operatorname{cosec}\theta$$

Substitute dl and r value in dE_x

$$dE_x = \frac{\rho_l dl \sin\theta}{4\pi\varepsilon r^2}$$

$$dE_x = \frac{\rho_l h(\operatorname{cosec}^2\theta)\, d\theta \sin\theta}{4\pi\varepsilon\,(h\operatorname{cosec}\theta)^2}$$

$$dE_x = \frac{\rho_l h(\operatorname{cosec}^2 \theta)\, d\theta \sin\theta}{4\pi\varepsilon h^2 \operatorname{cosec}^2 \theta}$$

$$dE_x = \frac{\rho_l \sin\theta d\theta}{4\pi\varepsilon h}$$

Integrate the above equation dE_x considered the limit as α_1 to $\pi - \alpha_2$

The electric field E_x due to the entire length of line charge is given by

$$\int dE_x = \int_{\alpha_1}^{\pi-\alpha_2} \frac{\rho_l \sin\theta d\theta}{4\pi\varepsilon h}$$

$$E_x = \int_{\alpha_1}^{\pi-\alpha_2} \frac{\rho_l \sin\theta d\theta}{4\pi\varepsilon h}$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} \int_{\alpha_1}^{\pi-\alpha_2} \sin\theta d\theta$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} [-\cos\theta]_{\alpha_1}^{\pi-\alpha 2}$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} [-\cos(\pi - \alpha_2) - (\cos\ \alpha_1)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} [(\cos \alpha_2) + (\cos\ \alpha_1)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} [(\cos \alpha_1) + (\cos\ \alpha_2)]$$

Substitute dl and r value in dE_x

$$dE_y = \frac{\rho_l h(\operatorname{cosec}^2 \theta) d\theta \cos\theta}{4\pi\varepsilon\ (h \operatorname{cosec}\theta)^2}$$

$$dE_y = \frac{\rho_l h(\operatorname{cosec}^2 \theta)\, d\theta \cos\theta}{4\pi\varepsilon h^2 \operatorname{cosec}^2 \theta}$$

$$dE_y = \frac{\rho_l h(\operatorname{cosec}^2 \theta)\, d\theta \cos\theta}{4\pi\varepsilon h^2 \operatorname{cosec}^2 \theta}$$

$$dE_y = \frac{\rho_l \, d\theta \cos\theta}{4\pi\varepsilon h}$$

$$dE_y = \frac{\rho_l \cos\theta \, d\theta}{4\pi\varepsilon h}$$

Similarly for $\boldsymbol{y}$ component of $\boldsymbol{E}$

Integrate the above equation $\boldsymbol{dE_y}$ considered the limit as $\boldsymbol{\alpha_1}$ *to* $\boldsymbol{\pi - \alpha_2}$

The electric field $\boldsymbol{E_y}$ due to the entire length of line charge is given by

$$\int dE_y = \int_{\alpha_1}^{\pi-\alpha_2} \frac{\rho_l \cos\theta \, d\theta}{4\pi\varepsilon h}$$

$$E_y = \int_{\alpha_1}^{\pi-\alpha_2} \frac{\rho_l \cos\theta \, d\theta}{4\pi\varepsilon h}$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h} \int_{\alpha_1}^{\pi-\alpha_2} \cos\theta \, d\theta$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h} [\sin\theta]_{\alpha_1}^{\pi-\alpha 2}$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h} [\sin(\pi - \alpha_2) - (\sin \alpha_1)]$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h} [(\sin \alpha_2) - (\sin \alpha_1)]$$

Case (i): If the point $\boldsymbol{P}$ is at bisector of a line, then $\boldsymbol{\alpha_1 = \alpha_2 = \alpha}$

$\boldsymbol{E_y = 0}$ $\quad$ $\boldsymbol{E}$ becomes $\boldsymbol{E_x}$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} [(\cos \alpha_1) + (\cos \alpha_2)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} [(\cos\alpha) + (\cos\alpha)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h} (2\cos\alpha)$$

$$E_x = \frac{\rho_l}{2\pi\varepsilon h}(\cos\alpha)$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[(\sin\alpha_2) - (\sin\alpha_1)]$$

Substitute $\alpha_1 = \alpha_2 = \alpha$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[(\sin\alpha) - (\sin\alpha)]$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[0]$$

$$E_y = 0$$

E becomes E_x $\quad E = E_x$

$$E = E_x = \frac{\rho_l}{2\pi\varepsilon h}(\cos\alpha)$$

$$E = \frac{\rho_l}{2\pi\varepsilon h}(\cos\alpha)$$

Case (ii): If the line is infinitely long then $\alpha_1 = \alpha_2 = \alpha = 0$

$E_y = 0$ $\quad E$ becomes E_x

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}[(\cos\alpha_1) + (\cos\alpha_2)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}[(\cos 0) + (\cos 0)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}[(1) + (1)]$$

$$E_x = \frac{\rho_l}{4\pi\varepsilon h}[2]$$

$$E_x = \frac{\rho_l}{2\pi\varepsilon h}$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[(\sin\alpha_2) - (\sin\alpha_1)]$$

Substitute $\alpha_1 = \alpha_2 = \alpha = 0$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[(\sin 0) - (\sin 0)]$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[(0) - (0)]$$

$$E_y = \frac{\rho_l}{4\pi\varepsilon h}[0]$$

$$E_y = 0$$

E becomes E_x $\quad E = E_x$

$$E = E_x = \frac{\rho_l}{2\pi\varepsilon h}$$

$$E = \frac{\rho_l}{2\pi\varepsilon h}$$

ELECTRIC FIELD INTENSITY DUE TO CIRCULAR DISC:

Consider a circular disc of radius R is charged uniformly with a charge density of ρ_s $coulomb/m^2$ .Let P be any point on the axis of the disc at a distance from the centre. Consider an annular ring of radius r and of radial thickness dr as shown in figure 1.7.2.The area of the annular ring is $ds = 2\pi r dr$.

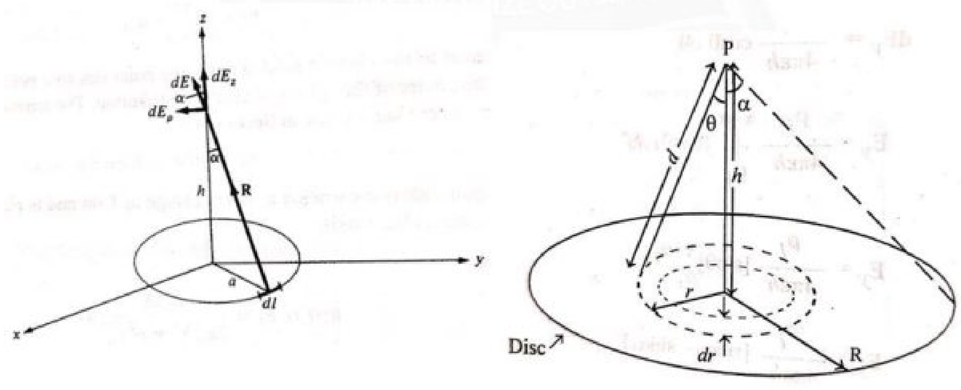


Figure 1.7.2 Evaluation of the *E* field due to a charged ring

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-120]

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The field intensity at point P due to the charged annular ring is given by

$$dE = \frac{\rho_S ds}{4\pi\varepsilon d^2}$$

Since the horizontal component of electric field intensity is zero, The horizontal components and vertical components are dE_x and dE_y

The horizontal components of angular ring is zero

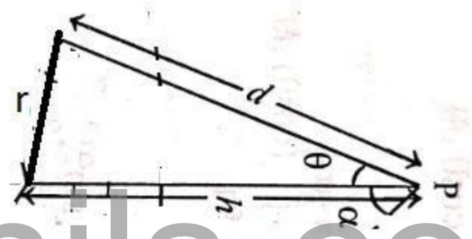
$$dE_x = 0$$

$$E_x = 0$$

The horizontal components of angular ring E_y have to find for circular ring.

the vertical component is given by

$$dE_y = \frac{\rho_S ds \cos\theta}{4\pi\varepsilon d^2}$$



From the above diagram find $\tan\theta$ *and* $\sin\theta$

$$\tan\theta = \frac{r}{h}$$

$$r = h\tan\theta$$

$$\sin\theta = \frac{r}{d}$$

$$d = \frac{r}{\sin\theta}$$

Assume $$ds = 2\pi r dr$$

$$dE_y = \frac{\rho_S ds \cos\theta}{4\pi\varepsilon d^2}$$

Substitute ds in dE_y

$$dE_y = \frac{\rho_S 2\pi r dr \cos\theta}{4\pi\varepsilon d^2}$$

$$r = h \tan\theta$$

Differentiate above equation

$$dr = h \sec^2\theta d\theta$$

Substitute dr and d in dE_y

$$dE_y = \frac{\rho_S(2\pi r)h\sec^2\theta d\theta\cos\theta}{4\pi\varepsilon d^2}$$

$$dE_y = \frac{\rho_S(2\pi r)h\sec^2\theta d\theta\cos\theta}{4\pi\varepsilon\left(\frac{r}{\sin\theta}\right)^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h\sec^2\theta)d\theta\cos\theta\ \sin^2\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h\sec^2\theta)\sin^2\theta\cos\theta\, d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\sin^2\theta\cos\theta\, d\theta}{4\pi\varepsilon r^2\ \cos^2\theta}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\sin^2\theta d\theta}{4\pi\varepsilon r^2\cos\theta}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\tan\theta\sin\theta\, d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\tan\theta\sin\theta\, d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(h)\tan\theta\sin\theta\, d\theta}{2\varepsilon r}$$

Substitute r in dE_y

$$dE_y = \frac{\rho_S(h)\tan\theta\sin\theta\, d\theta}{2\varepsilon r}$$

$$dE_y = \frac{\rho_S(h)\tan\theta\sin\theta\, d\theta}{2\varepsilon\, h\tan\theta}$$

$$dE_y = \frac{\rho_S\sin\theta\, d\theta}{2\varepsilon}$$

Integrate the above equation dE_y considered the limit as 0 *to* α

$$\int dE_y = \int_0^\alpha \frac{\rho_S \sin\theta \, d\theta}{2\varepsilon}$$

$$\int dE_y = \frac{\rho_S}{2\varepsilon} \int_0^\alpha \sin\theta \, d\theta$$

$$E_y = \frac{\rho_S}{2\varepsilon} [-\cos\theta]_0^\alpha$$

$$E_y = \frac{\rho_S}{2\varepsilon} [(-\cos\alpha) - (-\cos 0)]$$

$$E_y = \frac{\rho_S}{2\varepsilon} [(-\cos\alpha) + (1)]$$

$$E_y = \frac{\rho_S}{2\varepsilon} [(1) + (-\cos\alpha)]$$

$$E_y = \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

The total electric field $E = E_x + E_y$

$$E = E_x + E_y$$

$$E_x = 0$$

$$E_y = \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

$$E = 0 + \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

$$E = \frac{\rho_S}{2\varepsilon} [1 - \cos\alpha]$$

ELECTRIC FIELD INTENSITY DUE TO INFINITE SHEET OF CHARGE:

Consider an infinite plane sheet which is uniformly charged with a charge density of ρ_s ***Coulom/m²*** as shown in figure 1.7.3.

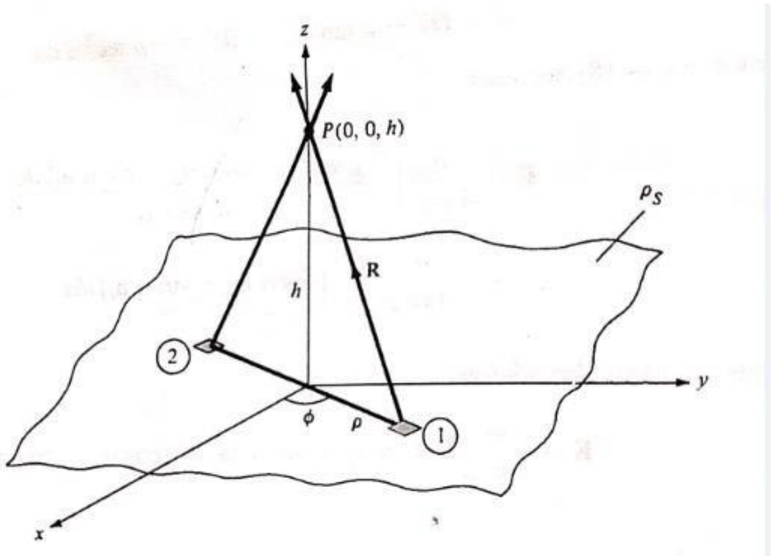


Figure 1.7.2 Evaluation of the *E* field due to an infinite sheet of charge

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-116]

The field intensity at any point ***P*** due to infinite plane sheet of charge can be evaluated by applying expression of charged circular disc.

The field intensity at point ***P*** due to the charged annular ring is given by

$$dE = \frac{\rho_S ds}{4\pi\varepsilon d^2}$$

Since the horizontal component of electric field intensity is zero, The horizontal components and vertical components are dE_x and dE_y

The horizontal components of angular ring is zero

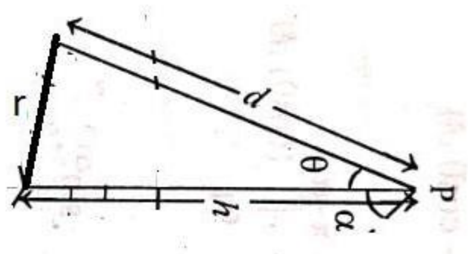
$$dE_x = 0$$

$$E_x = 0$$

The horizontal components of angular ring E_y have to find for circular ring.

the vertical component is given by

$$dE_y = \frac{\rho_S ds \cos\theta}{4\pi\varepsilon d^2}$$



From the above diagram find $\tan\theta$ *and* $\sin\theta$

$$\tan\theta = \frac{r}{h}$$

$$r = h\tan\theta$$

$$\sin\theta = \frac{r}{d}$$

$$d = \frac{r}{\sin\theta}$$

Assume $$ds = 2\pi r dr$$

$$dE_y = \frac{\rho_S ds\cos\theta}{4\pi\varepsilon d^2}$$

Substitute ds in dE_y

$$dE_y = \frac{\rho_S 2\pi r dr\cos\theta}{4\pi\varepsilon d^2}$$

$$r = h\tan\theta$$

Differentiate above equation

$$dr = h\sec^2\theta d\theta$$

Substitute dr and d in dE_y

$$dE_y = \frac{\rho_S(2\pi r)h\sec^2\theta d\theta\cos\theta}{4\pi\varepsilon d^2}$$

$$dE_y = \frac{\rho_S(2\pi r)h\sec^2\theta d\theta\cos\theta}{4\pi\varepsilon\left(\frac{r}{\sin\theta}\right)^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h\sec^2\theta)d\theta\cos\theta\ \sin^2\theta}{4\pi\varepsilon r^2}$$

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$$dE_y = \frac{\rho_S(2\pi r)(h\sec^2\theta)\sin^2\theta\cos\theta\, d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\sin^2\theta\cos\theta\, d\theta}{4\pi\varepsilon r^2\ \cos^2\theta}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\sin^2\theta d\theta}{4\pi\varepsilon r^2\cos\theta}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\tan\theta\sin\theta\, d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(2\pi r)(h)\tan\theta\sin\theta\, d\theta}{4\pi\varepsilon r^2}$$

$$dE_y = \frac{\rho_S(h)\tan\theta\sin\theta\, d\theta}{2\varepsilon r}$$

Substitute r in dE_y

$$dE_y = \frac{\rho_S(h)\tan\theta\sin\theta\, d\theta}{2\varepsilon r}$$

$$dE_y = \frac{\rho_S(h)\tan\theta\sin\theta\, d\theta}{2\varepsilon\, h\tan\theta}$$

$$dE_y = \frac{\rho_S\sin\theta\, d\theta}{2\varepsilon}$$

Integrate the above equation dE_y consider the limit as 0 *to* α

$$\int dE_y = \int_0^\alpha \frac{\rho_S\sin\theta\, d\theta}{2\varepsilon}$$

$$\int dE_y = \frac{\rho_S}{2\varepsilon}\int_0^\alpha \sin\theta\, d\theta$$

$$E_y = \frac{\rho_S}{2\varepsilon}[-\cos\theta]_0^\alpha$$

$$E_y = \frac{\rho_S}{2\varepsilon}[(-\cos\alpha) - (-\cos 0)]$$

$$E_y = \frac{\rho_S}{2\varepsilon}\,[(-\cos\alpha) + (1)]$$

$$E_y = \frac{\rho_S}{2\varepsilon}\,[(1) + (-\cos\alpha)]$$

$$E_y = \frac{\rho_S}{2\varepsilon}\,[1 - \cos\alpha]$$

The total electric field $E = E_x + E_y$

$$E = E_x + E_y$$

$$E_x = 0$$

$$E_y = \frac{\rho_S}{2\varepsilon}\,[1 - \cos\alpha]$$

$$E = 0 + \frac{\rho_S}{2\varepsilon}\,[1 - \cos\alpha]$$

$$E = \frac{\rho_S}{2\varepsilon}\,[1 - \cos\alpha]$$

The electric field due to infinite uniformly charge sheet $\alpha = 90°$

$$E = \frac{\rho_S}{2\varepsilon}\,[1 - \cos\alpha]$$

$$E = \frac{\rho_S}{2\varepsilon}\,[1 - \cos 90°]$$

$$E = \frac{\rho_S}{2\varepsilon}\,[1 - 0]$$

$$E = \frac{\rho_S}{2\varepsilon}\,[1]$$

$$E = \frac{\rho_S}{2\varepsilon}$$

1.8 GAUSS'S LAW AND ITS APPLICATIONS

ELECTRIC FLUX ($\boldsymbol{x}$)

If the test charge is moved towards the charge $\boldsymbol{Q}$,the test charge will experience force due to the main charge $\boldsymbol{Q}$.The lines of force can be designated as electric flux which is equal to the charge itself. The electric flux $(\boldsymbol{x})$ eminates from electric charge$\boldsymbol{Q}$.

$$\boldsymbol{x} = \boldsymbol{Q}$$

ELECTRIC FLUX DENSITY($\boldsymbol{D}$):

Electric flux density or Displacement density is defined as the electric flux per unit area

$$\boldsymbol{D} = \frac{\boldsymbol{Q}}{\boldsymbol{A}} \quad \boldsymbol{Coulomb/metre^2}$$

For sphere surface area

$$\boldsymbol{A} = 4\pi r^2$$

Substitute $\boldsymbol{A}$ in $\boldsymbol{D}$

$$\boldsymbol{D} = \frac{\boldsymbol{Q}}{4\pi r^2}$$

But

$$\boldsymbol{E} = \frac{\boldsymbol{Q}}{4\pi\varepsilon r^2}$$

Substitute $\boldsymbol{D}$ in $\boldsymbol{E}$

$$\boldsymbol{E} = \frac{\boldsymbol{D}}{\varepsilon}$$

$$\boldsymbol{D} = \varepsilon\boldsymbol{E}$$

GAUSS'S LAW:

The gauss law states that the electric flux passing through any closed surface is equal to the total charge enclosed by the surface.

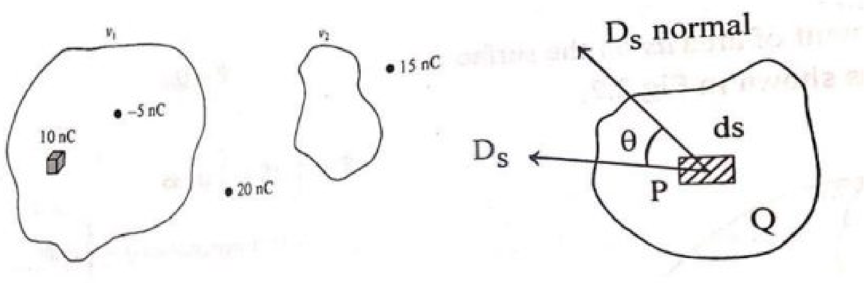

Figure 1.8.1 Illustration of Gauss's law

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-128]

Consider a small element of area ds in a plane surface having a charge Q and P be a point in the element. At every point on the surface the electric flux density D will have value D_s. Let D_s make an angle θ with ds as shown in figure 1.8.1.The flux crossing ds is the product of the normal component D_s and ds.

$$dx = D_{s\,normal} \,.\, ds$$

$$D_{s\,normal} = D_s \cos\theta$$

$$dx = D_s \cos\theta \, ds$$

For dot product

$$dx = D_s \,.\, ds$$

a small element of area ds can also be written as dA

$$dx = D_s \,.\, dA$$

The total flux passing through the closed surface is given by

$$x = \int dx = \oint_s D_s \,.\, dA$$

$$x = D_s A$$

Substitute D_s value in above equation

$$D_s = \frac{Q}{A}$$

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$$x = \frac{Q}{A} A$$

$$x = Q$$

Proof:

Consider a charge Q at the origin of a spherical co-ordinate system, whose co-ordinates as r, θ, φ as shown in figure1.8.2.The radius of the sphere is r.

The electric flux density due to the charge Q is

$$D = \frac{Q}{A} \quad Coulomb/metre^2$$

For sphere surface area

$$A = 4\pi r^2$$

Substitute A in D

$$D = \frac{Q}{4\pi r^2}$$

But

$$E = \frac{Q}{4\pi\varepsilon r^2}$$

Substitute D in E

$$E = \frac{D}{\varepsilon}$$

$$D = \varepsilon E$$

$$E = \frac{Q}{4\pi\varepsilon r^2}$$

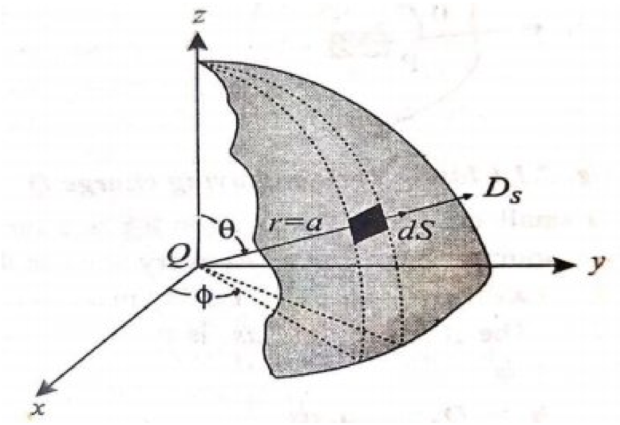

Figure 1.8.2 Surface element ds in the spherical co-ordinate

[Source: "Electromagnetic Theory" by Dr.P.Dananjayan, page-2.3]

Consider a small element of area $\boldsymbol{ds}$ on the surface of the sphere at a distance $\boldsymbol{r}$ from the origin as shown in figure. The sides of spherical co-ordinate system are $\boldsymbol{dr, rd\theta\ and\ r\sin\theta d\varphi}$.

The differential area $\quad \boldsymbol{d_s = dr.rd\theta = r\,dr\,d\theta}$

$$\boldsymbol{d_s = rd\theta . r\sin\theta d\varphi = r^2\sin\theta\, d\theta\, d\varphi}$$

$$\boldsymbol{d_s = r\sin\theta d\varphi\,.\,dr = r\sin\theta d\varphi\ dr}$$

For dot product

$$\boldsymbol{dx = D_s\,.ds}$$

Integrate the above equation on both sides

$$\int \boldsymbol{dx} = \int_s \boldsymbol{D_s\,.ds}$$

$$\int \boldsymbol{dx} = \iint \boldsymbol{D_s\,.ds}$$

Substitute $\boldsymbol{D_s}$ in above equation

$$\boldsymbol{D_s = \frac{Q}{4\pi r^2}}$$

$$\int \boldsymbol{dx} = \iint_s \boldsymbol{\frac{Q}{4\pi r^2}.ds}$$

Substitute $\boldsymbol{ds}$ in above equation

$$\boldsymbol{d_s = r^2\sin\theta\, d\theta\, d\varphi}$$

$$\int \boldsymbol{dx} = \iint_s \boldsymbol{\frac{Q}{4\pi r^2}\,.r^2\sin\theta\, d\theta\, d\varphi}$$

$$\boldsymbol{x} = \iint_s \boldsymbol{\frac{Q}{4\pi r^2}\,.r^2\sin\theta\, d\theta\, d\varphi}$$

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$$x = \iint_s \frac{Q}{4\pi} . \sin\theta \, d\theta \, d\varphi$$

The limit for θ is $0 \; to \; \pi$

The limit for φ is $0 \; to \; 2\pi$

First integrate with respect to θ

$$x = \iint_s \frac{Q}{4\pi} . \sin\theta \, d\theta \, d\varphi$$

$$x = \frac{Q}{4\pi} \int_{\varphi=0}^{\varphi=2\pi} \int_{\theta=0}^{\theta=\pi} \sin\theta \, d\theta \, d\varphi$$

$$x = \frac{Q}{4\pi} \int_{\varphi=0}^{\varphi=2\pi} [-\cos\theta]_0^{\pi} \; d\varphi$$

$$x = \frac{Q}{4\pi} \int_{\varphi=0}^{\varphi=2\pi} [(-\cos\pi) - (-\cos 0)] \; d\varphi$$

$$x = \frac{Q}{4\pi} \int_{\varphi=0}^{\varphi=2\pi} [-(-1) - (-1)] \; d\varphi$$

$$x = \frac{Q}{4\pi} \int_{\varphi=0}^{\varphi=2\pi} [(1) + (1)] \; d\varphi$$

$$x = \frac{Q}{4\pi} \int_{\varphi=0}^{\varphi=2\pi} [(2)] \; d\varphi$$

$$x = \frac{Q}{2\pi} \int_{\varphi=0}^{\varphi=2\pi} d\varphi$$

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$$x = \frac{Q}{2\pi} \int_{\varphi=0}^{\varphi=2\pi} d\varphi$$

Next integrate with respect to φ

$$x = \frac{Q}{2\pi} \int_{\varphi=0}^{\varphi=2\pi} d\varphi$$

$$x = \frac{Q}{2\pi} [\varphi]_0^{2\pi}$$

$$x = \frac{Q}{2\pi} [(2\pi) - (0)]$$

$$x = \frac{Q}{2\pi} [(2\pi)]$$

$$x = Q$$

The electric flux crossing the surface is equal to the charge enclosed by the surface.

The gauss's law can be stated in the point form as follows.

The divergence of electric flux density is equal to the volume charge density.

$$\nabla . D = \rho_v$$

APPLICATION OF GAUSS'S LAW:

The surface over which the gauss law is applied is called as Gaussian surface**.**

The gauss law is applied to the surface in following condition.

- The surface is closed
- The electric flux density D is either normal to the surface at each other.
- The electric flux density D is constant over the part of the surface.

INFINITE LINE CHARGE:

The infinite line of uniform charge $\rho_l\, c/m$ lies along the Z-axis. The line charge act on x, y – axis is zero. To determine D at a point P.Choose a cylindrical surface containing P to satisfy the symmetry condition as shown in figure 1.8.3.The electric flux density D is constant on and normal to the cylindrical Gaussian surface.

Consider a cylindrical surface .The axis of cylindrical are (x, y, z).The line charge act on the cylinder isρ_l.

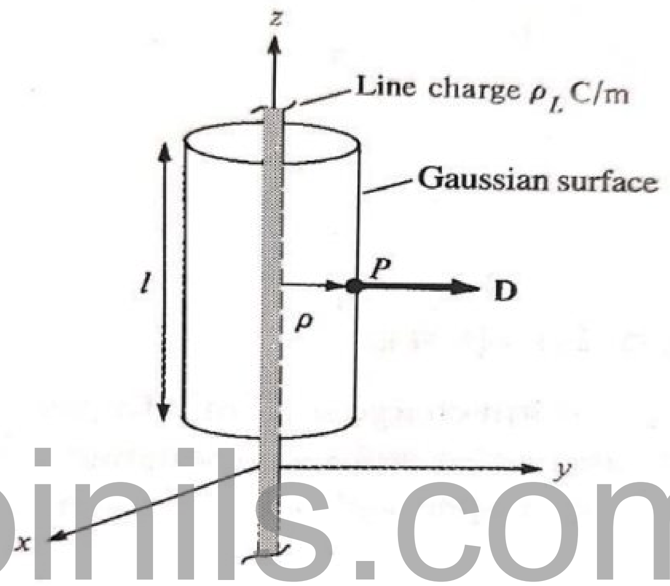

Figure 1.8.3 Gaussian surface about infinite line charge

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-129]

The electric flux density

$$D = \frac{Q}{A}$$

$$Q = DA$$

Surface area can be written as

$$A = \iint ds$$

Substitute A in Q

$$Q = \iint D\, ds$$

Substitute

$$D = D_s$$

$$Q = \iint D_s\, ds$$

$$Q = \iint D_s\, ds + \iint D_s\, ds + \iint D_s\, ds$$

(Side) (Top) (Front)

The electric flux density in x *and* y *direction* is zero.

The electric flux density only occur in the side of the cylinder

$$Q = \iint D_s\, ds + 0 + 0$$

$$Q = \iint D_s\, ds$$

The sides of cylindrical co-ordinate systems are $dr, rd\varphi$ and dz

The differential area $d_s = dr\, .rd\varphi = rdrd\varphi$

$$d_s = rd\varphi dz$$

$$d_s = dz\, dr = drdz$$

Consider

$$d_s = rd\varphi dz$$

Substitute d_s in Q

$$Q = \iint D_s rd\varphi dz$$

$$Q = D_s \iint rd\varphi dz$$

The limit for the Z *is* 0 *to* l

The limit for the φ *is* 0 *to* 2π

$$Q = D_s \int_{z=0}^{z=l} \int_{\varphi=0}^{\varphi=2\pi} r d\varphi dz$$

$$Q = D_s r \int_{z=0}^{z=l} \int_{\varphi=0}^{\varphi=2\pi} d\varphi dz$$

$$Q = D_s r \int_{z=0}^{z=l} [\varphi]_0^{2\pi} dz$$

$$Q = D_s r \int_{z=0}^{z=l} [(2\pi) - (0)] dz$$

$$Q = D_s r \int_{z=0}^{z=l} [(2\pi)] dz$$

$$Q = D_s r \int_{z=0}^{z=l} 2\pi dz$$

$$Q = 2\pi D_s r \int_{z=0}^{z=l} dz$$

$$Q = 2\pi D_s r [z]_0^l$$

$$Q = 2\pi D_s r [(l) - (0)]$$

$$Q = 2\pi D_s r [(l)]$$

$$Q = 2\pi r l D_s$$

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$$D_s = \frac{Q}{2\pi\, rl}$$

Area of Cylinder $A = 2\pi rl$

The line charge density

$$\rho_l = \frac{Q}{l} \quad Coulomb/meter(c/m)$$

$$\rho_l = \frac{Q}{l}$$

$$Q = \rho_l l$$

Substitute Q value in D_s equation

$$D_s = \frac{Q}{2\pi\, rl}$$

$$D_s = \frac{\rho_l l}{2\pi\, rl}$$

$$D_s = \frac{\rho_l}{2\pi\, r}$$

The electric field for infinite line charge

$$D = \varepsilon E$$

Substitute $D = D_s$

$$D_s = \varepsilon E$$

$$E = \frac{D_s}{\varepsilon}$$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

$$E = \frac{D_s}{\varepsilon_0 \varepsilon_r}$$

$$Relative\ Permitivity\ (\varepsilon_r) = 1$$

$$E = \frac{D_s}{\varepsilon_0 \times 1}$$

$$E = \frac{D_s}{\varepsilon_0}$$

Substitute D_s value in E equation

$$E = \frac{\frac{\rho_l}{2\pi r}}{\varepsilon_0}$$

$$E = \frac{\rho_l}{\varepsilon_0 \times 2\pi r}$$

$$E = \frac{\rho_l}{2\pi r \varepsilon_0}$$

COAXIAL CYLINDER

Consider the two coaxial cylindrical conductors forming a coaxial cable. The radius of the inner cylinder is a while the radius of the outer cylinder is b.The coaxial cable is shown in figure 1.8.4.The length of cable is L.

The line charge density of inner cylinder is ρ_l. The line charge density of inner cylinder is$-\rho_l$.

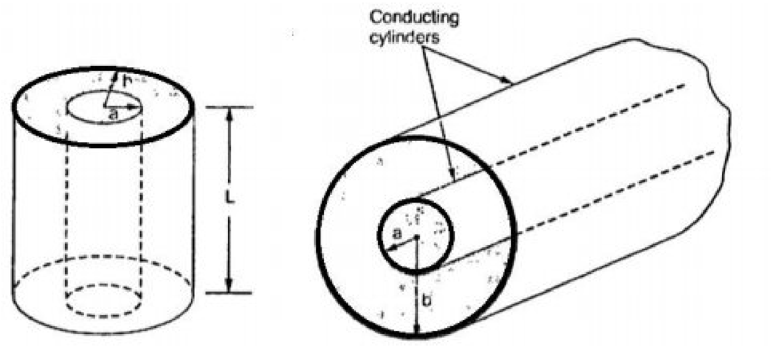

Figure 1.8.4 Coaxial Cable

[Source: "Electromagnetic Theory" by U.A.Bakshi, page-3.19]

In outer side the integral of electric flux density over a space is equal to charge.

$$\int D d_s = Q$$

The line charge density

$$\rho_l = \frac{Q}{l} \quad Coulomb/meter(c/m)$$

$$\rho_l = \frac{Q}{l}$$

$$Q = \rho_l l$$

Substitute Q`in $\int Dd_s$

$$\int Dd_s = Q$$

$$\int Dd_s = \rho_l l$$

Substitute D`in $\int Dd_s$ equation

$$D = \varepsilon E$$

$$\int \varepsilon E\, d_s = \rho_l l$$

$$\varepsilon E \int d_s = \rho_l l$$

$$\int d_s = S = A$$

Substitute $\int d_s$`value in above equation

$$\varepsilon EA = \rho_l l$$

$$E = \frac{\rho_l l}{\varepsilon A}$$

Area of Cylinder $A = 2\pi rl$

$$E = \frac{\rho_l l}{\varepsilon 2\pi rl}$$

$$E = \frac{\rho_l}{\varepsilon 2\pi r}$$

$$E = \frac{\rho_l}{2\pi\varepsilon r}$$

INFINITE SHEET OF CHARGE

Consider an infinite sheet of uniform charge $\rho_S\ C/m^2$ lying on the $Z = 0$ plane.To determine D at point P.Choose rectangular box that is cut symmetrically by the sheet of charge and has two of its faces parallel to the sheet as shown in figure 1.8.5.As D is normal to the sheet.

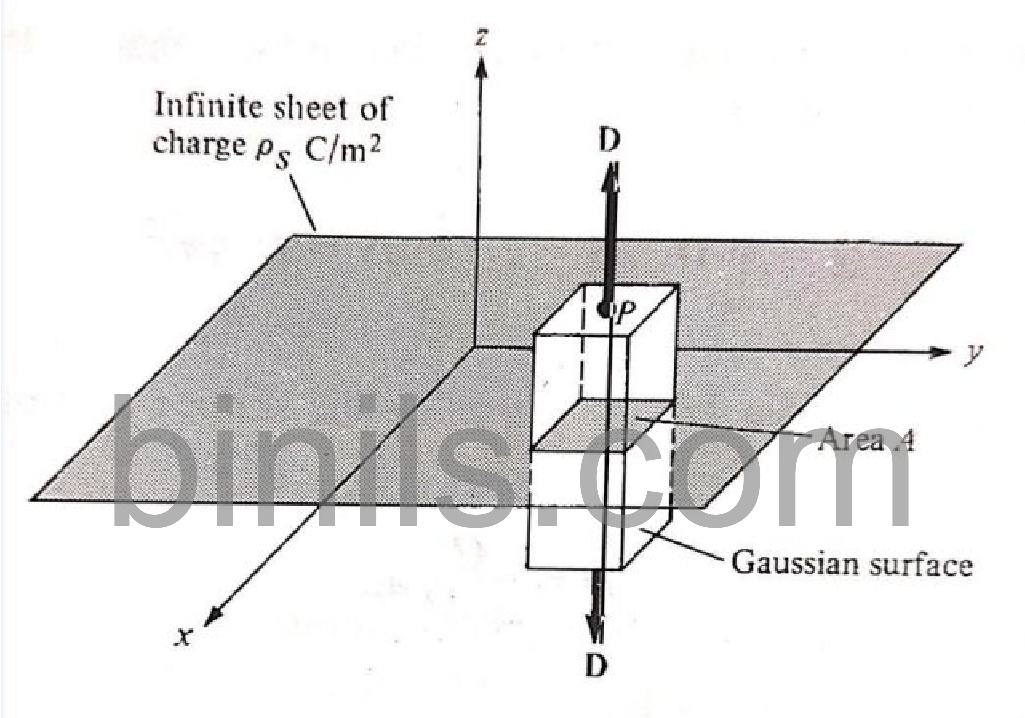

Figure 1.8.5 Gaussian surface about an infinite line sheet of charge

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-130]

$$Q = \iint D\, ds$$

Substitute

$$D = D_s$$

$$Q = \iint D_s\, ds$$

$$Q = \iint D_s\, ds + \iint D_s\, ds + \iint D_s\, ds$$

(Side) (Top) (Front)

The electric flux density in x *and* y direction is zero.

The electric flux density only occur in the side of the infinite sheet

$$Q = \iint D_s\, ds + 0 + 0$$

$$Q = \iint D_s\, ds$$

The sides of rectangular co-ordinate systems are dx, dy, dz

The differential area $d_s = d_x d_y$

$$d_s = d_y d_z$$

$$d_s = d_z d_x$$

Consider

$$d_s = d_x d_y$$

Substitute d_s in Q

$$Q = \iint D_s d_x d_y$$

$$Q = D_s \iint d_x d_y$$

$$\iint d_x d_y = A$$

Substitute $\iint d_x d_y$ in Q

$$Q = D_s A$$

$$Q = \iint D_s d_x d_y + \iint D_s d_x d_y$$

(Top) (Bottom)

$$Q = \iint D_s d_x d_y + \iint D_s d_x d_y$$

Substitute $\iint d_x d_y$ in Q

$$\iint d_x d_y = A$$

$$Q = D_s A + D_s A$$

$$Q = 2\, D_s A$$

Surface charge density:

$$\rho_s = \frac{Q}{S} = \frac{Q}{A} \quad Coulomb/squaremeter(c/m^2)$$

$$\rho_s = \frac{Q}{S} = \frac{Q}{A}$$

$$Q = \rho_s S = \rho_s A$$

Substitute Q value in above Q equation

$$Q = 2\, D_s A$$

$$\rho_s A = 2\, D_s A$$

$$\rho_s = 2\, D_s$$

$$D_s = \frac{\rho_s}{2}$$

Substitute D_s value in E

$$D = \varepsilon E$$

$$E = \frac{D}{\varepsilon}$$

Consider

$$D = D_s$$

$$E = \frac{D_s}{\varepsilon}$$

$$E = \frac{D_s}{\varepsilon_0 \varepsilon_r}$$

$\varepsilon = \varepsilon_0 \varepsilon_r \quad \varepsilon_r = 1$

$$E = \frac{D_s}{\varepsilon_0 \times 1}$$

$$E = \frac{D_s}{\varepsilon_0}$$

Substitute D_s equation in E

$$D_s = \frac{\rho_s}{2}$$

$$E = \frac{\rho_s}{2\varepsilon_0}$$

UNIFORMLY CHARGED SPHERE

Consider a sphere of radius a with a uniform charge $\rho_v\, C/m^3$ .To determine D everywhere. Construct a Gaussian surfaces for case $r \le a$ and $r \ge a$ separately. Since charge has spherical symmetry. It is obvious that a spherical surface is an appropriate Gaussian surface.

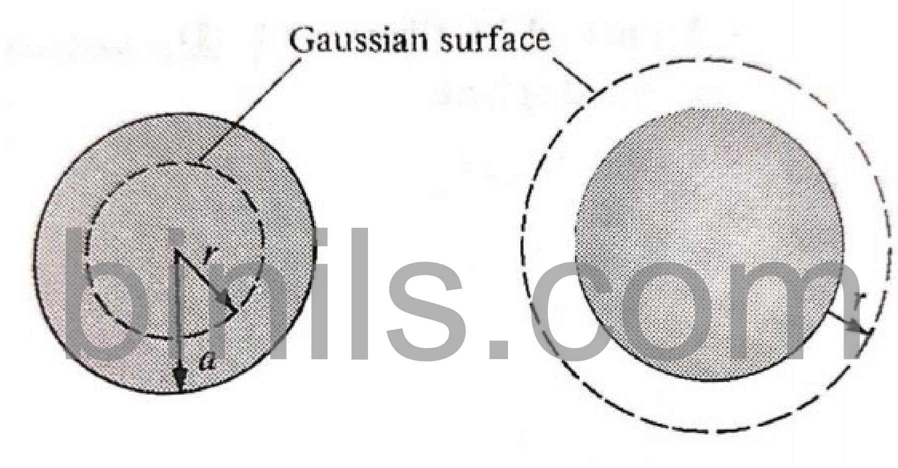

Figure 1.8.6 Gaussian surface for a uniformly charged sphere

[Source: "Elements of Electromagnetics" by Matthew N.O.Sadiku, page-131]

Case (i): The point P is outside the sphere $(r > a)$

The Gaussian surface passing through through point P is a spherical surface of radius r The sides of spherical co-ordinate system are $dr, rd\theta$ and $r\sin\theta d\varphi$.

The differential area $d_s = dr.rd\theta = r\,dr\,d\theta$

$$d_s = rd\theta . r\sin\theta d\varphi = r^2 \sin\theta\, d\theta\, d\varphi$$

$$d_s = r\sin\theta d\varphi\,.\,dr = r\sin\theta d\varphi\ dr$$

Consider differential area

$$d_s = r^2 \sin\theta\, d\theta\, d\varphi$$

$$x = Q$$

$$Q = \iint D\, ds$$

$$x = Q = \iint D\, ds$$

The limit for θ is 0 *to* π

The limit for φ is 0 *to* 2π

First integrate with respect to θ

$$x = Q = \iint D\, ds$$

$$Q = \iint D\, r^2 \sin\theta\, d\theta\, d\varphi$$

$$Q = D\, r^2 \int_{\varphi=0}^{\varphi=2\pi} \int_{\theta=0}^{\theta=\pi} \sin\theta\, d\theta\, d\varphi$$

$$Q = D\, r^2 \int_{\varphi=0}^{\varphi=2\pi} [-\cos\theta]_0^{\pi}\, d\varphi$$

$$Q = D\, r^2 \int_{\varphi=0}^{\varphi=2\pi} [(-\cos\pi) - (-\cos 0)]\, d\varphi$$

$$Q = D\, r^2 \int_{\varphi=0}^{\varphi=2\pi} [-(-1) - (-1)]\, d\varphi$$

$$Q = D\, r^2 \int_{\varphi=0}^{\varphi=2\pi} [(1) + (1)]\, d\varphi$$

$$Q = D\, r^2 \int_{\varphi=0}^{\varphi=2\pi} [(2)]\, d\varphi$$

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$$Q = 2D\,r^2 \int_{\varphi=0}^{\varphi=2\pi} d\varphi$$

$$Q = 2D\,r^2 \int_{\varphi=0}^{\varphi=2\pi} d\varphi$$

Next integrate with respect to $\boldsymbol{\varphi}$

$$Q = 2D\,r^2 \int_{\varphi=0}^{\varphi=2\pi} d\varphi$$

$$x = \frac{Q}{2\pi}\,[\varphi]_0^{2\pi}$$

$$Q = 2D\,r^2\ [(2\pi) - (0)]$$

$$Q = 2D\,r^2\ [(2\pi)]$$

$$Q = 4\pi D\,r^2$$

$$D = \frac{Q}{4\pi\,r^2}$$

Consider

$$D = \varepsilon E$$

$$E = \frac{D}{\varepsilon}$$

$\varepsilon = \varepsilon_0 \varepsilon_r \quad \varepsilon_r = 1$

$$E = \frac{D}{\varepsilon_0 \varepsilon_r}$$

$$E = \frac{D}{\varepsilon_0 \times 1}$$

$$E = \frac{D}{\varepsilon_0}$$

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Substitute $\boldsymbol{D}$ in $\boldsymbol{E}$

$$D = \frac{Q}{4\pi r^2}$$

$$E = \frac{\frac{Q}{4\pi r^2}}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi r^2 \varepsilon_0}$$

Total charge enclosed by volume

$$Q = \int_v^0 \rho_v \, dv$$

The sides of spherical co-ordinate system are $dr, rd\theta \text{ and } r\sin\theta d\varphi.$

The differential Volume $\quad d_v = dr.rd\theta.r\sin\theta d\varphi$

$$d_v = r^2 \sin\theta \, d\theta \, d\varphi \; dr$$

Substitute d_v in Q

$$Q = \int_v^0 \rho_v \, dv$$

$$Q = \int_v^0 \rho_v r^2 \sin\theta \, d\theta \, d\varphi \; dr$$

$$Q = \int_{\varphi=0}^{\varphi=2\pi} \int_{\theta=0}^{\theta=\pi} \int_{r=0}^{r=a} \rho_v r^2 \sin\theta \, d\theta \, d\varphi \; dr$$

The limit for θ is $0 \text{ to } \pi$

The limit for φ is $0 \text{ to } 2\pi$

The limit for r is $0 \text{ to } a$

First integrate with respect to θ

$$Q = \rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{\theta=0}^{\theta=\pi} \int_{r=0}^{r=a} r^2 \sin\theta \, d\theta \, d\varphi \, dr$$

$$Q = \rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{\theta=0}^{\theta=\pi} \int_{r=0}^{r=a} r^2 \sin\theta \, d\theta \, d\varphi \, dr$$

$$Q = \rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{r=0}^{r=a} [-\cos\theta]_0^{\pi} \, d\varphi \, r^2 \, dr$$

$$Q = \rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{r=0}^{r=a} [(-\cos\pi) - (-\cos 0)] \, d\varphi \, r^2 \, dr$$

$$Q = \rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{r=0}^{r=a} [[-(-1) - (-1)]] \, d\varphi \, r^2 \, dr$$

$$Q = \rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{r=0}^{r=a} [(1) + (1)] \, d\varphi \, r^2 \, dr$$

$$Q = \rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{r=0}^{r=a} [(2)] \, d\varphi \, r^2 \, dr$$

$$Q = 2\rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{r=0}^{r=a} d\varphi \, r^2 \, dr$$

Next integrate with respect to φ

$$Q = 2\rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{r=0}^{r=a} d\varphi \, r^2 \, dr$$

$$Q = 2\rho_v \int_{r=0}^{r=a} [\varphi]_0^{2\pi} \, r^2 \, dr$$

$$Q = 2\rho_v \int_{r=0}^{r=a} [(2\pi) - (0)]\, r^2\, dr$$

$$Q = 2\rho_v \int_{r=0}^{r=a} [(2\pi) - (0)]\, r^2\, dr$$

$$Q = 2\rho_v \int_{r=0}^{r=a} [(2\pi) - (0)]\, r^2\, dr$$

$$Q = 2\rho_v \int_{r=0}^{r=a} [(2\pi)]\, r^2\, dr$$

$$Q = 4\pi\rho_v \int_{r=0}^{r=a} r^2\, dr$$

Next integrate with respect to r

$$Q = 4\pi\rho_v \int_{r=0}^{r=a} r^2\, dr$$

$$Q = 4\pi\rho_v \left[\frac{r^3}{3}\right]_0^a$$

$$Q = 4\pi\rho_v \left[\left(\frac{a^3}{3}\right) - \left(\frac{0^3}{3}\right)\right]$$

$$Q = 4\pi\rho_v \left[\left(\frac{a^3}{3}\right) - 0\right]$$

$$Q = 4\pi\rho_v \left[\left(\frac{a^3}{3}\right)\right]$$

$$Q = \frac{4\pi\rho_v a^3}{3}$$

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Consider electric field

$$E = \frac{Q}{4\pi\varepsilon r^2}$$

$\varepsilon = \varepsilon_0\varepsilon_r \quad \varepsilon_r = 1$

$$E = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}$$

$$E = \frac{Q}{4\pi\varepsilon_0 \times 1 \times r^2}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

Substitute Q in E

$$E = \frac{\frac{4\pi\rho_v a^3}{3}}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{4\pi\rho_v a^3}{3 \times 4\pi\varepsilon_0 r^2}$$

$$E = \frac{\rho_v a^3}{3\varepsilon_0 r^2}$$

Electric flux density

$$D = \varepsilon E$$

$\varepsilon = \varepsilon_0\varepsilon_r \quad \varepsilon_r = 1$

$$D = \varepsilon_0\varepsilon_r E$$

$$D = \varepsilon_0 \times 1 \times E$$

$$D = \varepsilon_0 E$$

Substitute E in D

$$E = \frac{\rho_v a^3}{3\varepsilon_0 r^2}$$

$$D = \varepsilon_0 E$$

$$D = \varepsilon_0 \frac{\rho_v a^3}{3\varepsilon_0 r^2}$$

$$D = \frac{\rho_v a^3}{3r^2}$$

Case (ii): The point P on the sphere $r = a$

Gaussian surface is same as the surface of the charged sphere

$$D = \frac{\rho_v a^3}{3r^2}$$

Substitute $r = a$ in above equation

$$D = \frac{\rho_v a^3}{3r^2}$$

$$D = \frac{\rho_v a^3}{3a^2}$$

$$D = \frac{\rho_v a}{3}$$

Case (ii): The point P is inside the sphere $r < a$

Gaussian surface is spherical surface of the r where $r < a$

The differential area $\quad d_s = dr.rd\theta = r\,dr\,d\theta$

$$d_s = r d\theta . r\sin\theta d\varphi = r^2 \sin\theta\, d\theta\, d\varphi$$

$$d_s = r\sin\theta d\varphi\, .\, dr = r\sin\theta d\varphi\ dr$$

Consider differential area

$$d_s = r^2 \sin\theta\, d\theta\, d\varphi$$

$$x = Q$$

$$Q = \iint D\, ds$$

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$$x = Q = \iint D\, ds$$

The limit for θ is $0 \; to \; \pi$

The limit for φ is $0 \; to \; 2\pi$

First integrate with respect to θ

$$x = Q = \iint D\, ds$$

$$Q = \iint D\, r^2 \sin\theta \, d\theta \, d\varphi$$

$$Q = D\, r^2 \int_{\varphi=0}^{\varphi=2\pi} \int_{\theta=0}^{\theta=\pi} \sin\theta \, d\theta \, d\varphi$$

$$Q = D\, r^2 \int_{\varphi=0}^{\varphi=2\pi} [-\cos\theta]_0^{\pi} \, d\varphi$$

$$Q = D\, r^2 \int_{\varphi=0}^{\varphi=2\pi} [(-\cos\pi) - (-\cos 0)] \, d\varphi$$

$$Q = D\, r^2 \int_{\varphi=0}^{\varphi=2\pi} [-(-1) - (-1)] \, d\varphi$$

$$Q = D\, r^2 \int_{\varphi=0}^{\varphi=2\pi} [(1) + (1)] \, d\varphi$$

$$Q = D\, r^2 \int_{\varphi=0}^{\varphi=2\pi} [(2)] \, d\varphi$$

$$Q = 2D\, r^2 \int_{\varphi=0}^{\varphi=2\pi} d\varphi$$

$$Q = 2D\,r^2 \int_{\varphi=0}^{\varphi=2\pi} d\varphi$$

Next integrate with respect to φ

$$Q = 2D\,r^2 \int_{\varphi=0}^{\varphi=2\pi} d\varphi$$

$$x = \frac{Q}{2\pi}\,[\varphi]_0^{2\pi}$$

$$Q = 2D\,r^2\ [(2\pi) - (0)]$$

$$Q = 2D\,r^2\ [(2\pi)]$$

$$Q = 4\pi D\,r^2$$

$$D = \frac{Q}{4\pi\,r^2}$$

Consider D equation as

$$D = \varepsilon E$$

$$E = \frac{D}{\varepsilon}$$

$\varepsilon = \varepsilon_0 \varepsilon_r \quad \varepsilon_r = 1$

$$E = \frac{D}{\varepsilon_0 \varepsilon_r}$$

$$E = \frac{D}{\varepsilon_0 \times 1}$$

$$E = \frac{D}{\varepsilon_0}$$

Substitute D in E

$$D = \frac{Q}{4\pi\,r^2}$$

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$$E = \frac{\frac{Q}{4\pi r^2}}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\, r^2 \varepsilon_0}$$

Electric flux density

$$D = \varepsilon E$$

$\varepsilon = \varepsilon_0 \varepsilon_r \quad \varepsilon_r = 1$

$$D = \varepsilon_0 \varepsilon_r\, E$$

$$D = \varepsilon_0 \times 1 \times E$$

$$D = \varepsilon_0 E$$

Substitute E value in above equation

$$D = \varepsilon_0 E$$

$$E = \frac{Q}{4\pi\, r^2 \varepsilon_0}$$

$$D = \varepsilon_0 \frac{Q}{4\pi\, r^2 \varepsilon_0}$$

$$D = \frac{Q}{4\pi\, r^2}$$

The enclosed by the sphere of radius r only and not by the entire sphere. The charge outside the Gaussian surface will not affect$D$.

Total charge enclosed by volume

$$Q = \int_v^0 \rho_v\, dv$$

The sides of spherical co-ordinate system are$dr, rd\theta \; and \; r \sin\theta d\varphi$.

The differential Volume $\quad d_v = \; dr.rd\theta.r\sin\theta d\varphi$

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$$d_v = r^2 \sin\theta \, d\theta \, d\varphi \, dr$$

Substitute d_v in Q

$$Q = \int_{v}^{0} \rho_v \, dv$$

$$Q = \int_{v}^{0} \rho_v \, r^2 \sin\theta \, d\theta \, d\varphi \, dr$$

$$Q = \int_{\varphi=0}^{\varphi=2\pi} \int_{\theta=0}^{\theta=\pi} \int_{r=0}^{r=a} \rho_v \, r^2 \sin\theta \, d\theta \, d\varphi \, dr$$

The limit for θ is $0 \; to \; \pi$

The limit for φ is $0 \; to \; 2\pi$

The limit for r is $0 \; to \; r$

First integrate with respect to θ

$$Q = \rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{\theta=0}^{\theta=\pi} \int_{r=0}^{r=r} r^2 \sin\theta \, d\theta \, d\varphi \, dr$$

$$Q = \rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{\theta=0}^{\theta=\pi} \int_{r=0}^{r=r} r^2 \sin\theta \, d\theta \, d\varphi \, dr$$

$$Q = \rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{r=0}^{r=r} [-\cos\theta]_0^{\pi} \; d\varphi \, r^2 \, dr$$

$$Q = \rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{r=0}^{r=r} [(-\cos\pi) - (-\cos 0)] \; d\varphi \, r^2 \, dr$$

$$Q = \rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{r=0}^{r=r} [\,[-(-1)-(-1)]\,] \; d\varphi\, r^2\, dr$$

$$Q = \rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{r=0}^{r=r} [\,(1)+(1)] \; d\varphi\, r^2\, dr$$

$$Q = \rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{r=0}^{r=r} [\,(2)] \; d\varphi\, r^2\, dr$$

$$Q = 2\rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{r=0}^{r=r} d\varphi\, r^2\, dr$$

Next integrate with respect to φ

$$Q = 2\rho_v \int_{\varphi=0}^{\varphi=2\pi} \int_{r=0}^{r=r} d\varphi\, r^2\, dr$$

$$Q = 2\rho_v \int_{r=0}^{r=r} [\varphi]_0^{2\pi}\, r^2\, dr$$

$$Q = 2\rho_v \int_{r=0}^{r=r} [(2\pi) - (0)]\, r^2\, dr$$

$$Q = 2\rho_v \int_{r=0}^{r=r} [(2\pi) - (0)]\, r^2\, dr$$

$$Q = 2\rho_v \int_{r=0}^{r=r} [(2\pi) - (0)]\, r^2\, dr$$

$$Q = 2\rho_v \int_{r=0}^{r=r} [(2\pi)]\, r^2\, dr$$

$$Q = 4\pi\rho_v \int_{r=0}^{r=r} r^2\, dr$$

Next integrate with respect to r

$$Q = 4\pi\rho_v \int_{r=0}^{r=r} r^2\, dr$$

$$Q = 4\pi\rho_v \left[\frac{r^3}{3}\right]_0^r$$

$$Q = 4\pi\rho_v \left[\left(\frac{r^3}{3}\right) - \left(\frac{0^3}{3}\right)\right]$$

$$Q = 4\pi\rho_v \left[\left(\frac{r^3}{3}\right) - 0\right]$$

$$Q = 4\pi\rho_v \left[\left(\frac{r^3}{3}\right)\right]$$

$$Q = \frac{4\pi\rho_v r^3}{3}$$

Substitute Q in D

$$D = \frac{Q}{4\pi\, r^2}$$

$$Q = \frac{4\pi\rho_v r^3}{3}$$

$$D = \frac{1}{4\pi\, r^2}\, \frac{4\pi\rho_v r^3}{3}$$

$$D = \frac{\rho_v r}{3}$$

Electric flux density

$$D = \varepsilon E$$

$\varepsilon = \varepsilon_0 \varepsilon_r \quad \varepsilon_r = 1$

$$D = \varepsilon_0 \varepsilon_r \, E$$

$$D = \varepsilon_0 \times 1 \times E$$

$$D = \varepsilon_0 E$$

Substitute D value in above equation

$$D = \varepsilon_0 E$$

$$\frac{\rho_v r}{3} = \varepsilon_0 E$$

$$E = \frac{\rho_v r}{3\varepsilon_0}$$