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### 3.1 ANALOG MULTIPLIERS

A multiple produces an output V0 which is proportional to the product of two inputs $V_{x}$ and $V_{y}$.

$$
\mathrm{V}_{0}=K \mathrm{~V}_{\mathrm{x}} \mathrm{~V}_{\mathrm{y}}
$$

where K is the scaling factor $=(1 / 10) \mathrm{V}^{-1}$.
There are various methods available for performing analog multiplication. Four of such techniques, namely,

1. Logarithmic summing technique

Pulse height/width modulation Technique
2. Variable trans conductance Technique
3. Multiplication using Gilbert cell and
4. Multiplication using variable trans conductance technique.

An actual multiplier has its output voltage $\mathrm{V}_{0}$ defined by

$$
\text { Q } V_{0}=\frac{\left(v_{z}+\phi_{x}\right)\left(V_{y}+\phi_{y}\right)}{10(1+\varepsilon)} \phi_{0}
$$

where $\varphi \mathrm{x}$ and $\varphi \mathrm{y}$ are the offsets associated with signals Vx and $\mathrm{Vy}, \varepsilon$ is the error signal associated with K and $\varphi 0$ is the offset voltage of the multiplier output.

## TERMINOLOGIES ASSOCIATED VOLTAGE OF THE MULTIPLIER CHARACTERISTICS

## - Accuracy

This specifies the derivation of the actual output from the ideal output, for any combination of X and Y inputs falling within the permissible operating range of the multiplier.

## - Linearity

This defines the accuracy of the multiplier. The Linearity Error can be defined as the maximum absolute derivation of the error surface. This linearity error imposes a lower limit on the multiplier accuracy.

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Figure 3.1.1 Linearity of the multiplier
[source: "Linear Integrated Circuits"by S.Salivahanan\& V.S. Kanchana Bhaskaran, Page-396]

The figure 3.1.1 shows the response of the output as a function of one input voltage $\mathrm{V}_{\mathrm{x}}$ when the other $\mathrm{V}_{\mathrm{y}}$ is assumed constant. It represents the maximum percentage derivation from the ideal straight line output. An error surface is formed by plotting the output for different combinations of X and Y inputs.

## - Square law accuracy

The Square - law curve is obtained with the X and Y inputs connected together and applied with the same input signal. The maximum derivation of the output voltage from an ideal square -law curve expresses the squaring mode accuracy.This is shown in figure 3.1.2


Figure 3.1.2 Squaring mode Accuracy
[source: "Linear Integrated Circuits"by S.Salivahanan\& V.S. Kanchana Bhaskaran, Page-396]

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## - Bandwidth

The Bandwidth indicates the operating capability of an analog multiplier at higher frequency values. Small signal 3 dB bandwidth defines the frequency $\mathrm{f}_{0}$ at which the output reduces by 3 dB from its low frequency value for a constant input voltage. This is identified individually for the X and Y input channels normally.

The transconductance bandwidth represents the frequency at which the transconductance of the multiplier drops by 3 dB of its low frequency value. This characteristic defines the application frequency ranges when used for phase detection or AM detection.

## - Quadrant

The quadrant defines the applicability of the circuit for bipolar signals at its inputs. First quadrant device accepts only positive input signals, the two quadrant device accepts one bipolar signal and one unipolar signal and the four quadrant device accepts two bipolar signals.

## LOGARITHMIC SUMMING TECHNIQUE

This technique uses the relationship $\ln V_{x}+\ln V_{y}=\ln \left(V_{x} V_{y}\right)$


Figure 3.1.3. Multiplication using Logarithmic Summing Technique
[source: "Linear Integrated Circuits"by S.Salivahanan\& V.S. Kanchana Bhaskaran, Page-396]
As shown in figure 3.1.3 the input voltages $\mathrm{V}_{\mathrm{x}}$ and $\mathrm{V}_{\mathrm{y}}$ are converted to their logarithmic equivalent, which are then added together by a summer. An antilogarithmic converter produces the output voltage of the summer. The output is given by,

$$
\mathrm{V}_{\mathrm{z}}=\ln ^{-1}\left(\ln \left(\mathrm{~V}_{\mathrm{x}} \mathrm{~V}_{\mathrm{y}}\right)\right)=\mathrm{V}_{\mathrm{x}} \mathrm{~V}_{\mathrm{y}} .
$$

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The relationship between $\mathrm{I}_{0}$ and $\mathrm{V}_{\text {BE }}$ of the transistor is given by $\left.\mathrm{I}_{\mathrm{C}}=\mathrm{I}_{0} \mathrm{e}^{\left(\mathrm{V}_{\mathrm{BE}}\right.}{ }^{\prime} \mathrm{V}_{\mathrm{T}}\right) \cdot \mathrm{It}$ is found that the transistor follows the relationship very accurately in the range of 10 nA to 100mA. Logarithmic multiplier has low accuracy and high temperature instability. This method is applicable only to positive values of $\mathrm{V}_{\mathrm{x}}$ and $\mathrm{V}_{\mathrm{y}}$.

Limitation: this type of multiplier is restricted to one quadrant operation only.

## PULSE HEIGHT/ WIDTH MODULATION TECHNIQUE



Pulse Height/ Width Modulation Technique
Figure 3.1.4.Pulse Height/Width Modulation Technique
[source: "Linear Integrated Circuits"by S.Salivahanan\& V.S. Kanchana Bhaskaran, Page-397]
In this method, the pulse width of a pulse train is made proportional to one input voltage and the pulse amplitude is made proportional to the second input voltage. Therefore, $\mathrm{V}_{\mathrm{x}}=\mathrm{K}_{\mathrm{x}} \mathrm{A}$, $\mathrm{V}_{\mathrm{y}}=\mathrm{K}_{\mathrm{y}} \mathrm{t}$, and $\mathrm{V}_{\mathrm{z}}=\mathrm{K}_{\mathrm{z}} \mathrm{T}$ where $\mathrm{K}_{\mathrm{x}}, \mathrm{K}_{\mathrm{y}}, \mathrm{K}_{\mathrm{z}}$ are scaling factors. In figure 3.1.4 A is the amplitude of the pulse, t is the pulse width and T is the area of=the pulse=. Therefore,

$$
V_{z}=K_{z} T=\frac{V_{x} V_{y}}{k_{x} k_{y}}
$$

The modulated pulse train is passed through an integrated circuit. Therefore, the input of the integrator is proportional to the area of pulse, which in turn is proportional to the product of two input voltages.

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## ANALOG MULTIPLIER USING AN EMITTER COUPLED TRANSISTOR PAIR

An circuit using an emitter coupled pair is shown in figure 3.1.5.The output currents $\mathrm{I}_{\mathrm{C} 1}$ an $\mathrm{d}_{\mathrm{C} 2}$ are related to the differential input voltage $\mathrm{V}_{1}$ by


Figure 3.1.5 Multiplier circuit using an emitter coupled pair
[source: "Linear Integrated Circuits"by S.Salivahanan\& V.S. Kanchana Bhaskaran, Page-397]
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where $\mathrm{V}_{\mathrm{T}}$ is thermal voltage and the base currents have been neglected. Combining above eqn., difference between=the two output currents as

$$
\begin{gathered}
\Delta I_{C}=I_{C 1}-I_{C 2} \\
=I_{E E}\left(\frac{1}{1+t^{-V_{1} / V}}-\frac{1}{1+e^{-V_{1} / V_{T}}}\right) \\
=I_{E E} \tanh \left(\frac{V_{1}}{2 V_{T}}\right)
\end{gathered}
$$

The dc transfer characteristics of the emitter - coupled pair is shown in figure 3.1.6. It shows that the emitter coupled pair can be used as a simple multiplier using this configuration. When the differential input voltage $\mathrm{V}_{1} \ll \mathrm{~V}_{\mathrm{T}}$, we can appropriate as given by

$$
I_{E E} \tanh \left(\frac{V_{1}}{2 V_{T}}\right)=I_{E E}\left(\frac{V_{1}}{2 V_{T}}\right)
$$

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Then the equation becomes

$$
\Delta I_{C}=I_{E}\left(\frac{V_{1}}{2 V_{T}}\right)
$$

The current $\mathrm{I}_{\mathrm{EE}}$ is the bias current for the emitter - coupled pair. If the current $\mathrm{I}_{\mathrm{EE}}$ is made proportional to a second input signal $\mathrm{V}_{2}$, then

$$
\mathrm{I}_{\mathrm{EE}}=\mathrm{K}_{0}\left(\mathrm{~V}_{2}-\mathrm{V}_{\mathrm{BE}(\mathrm{on})}\right)
$$

Substituting above eqn., we get

$$
\Delta \mathrm{I}_{\mathrm{C}}=\mathrm{K}_{0} \mathrm{~V}_{1}\left(\mathrm{~V}_{2}-\mathrm{V}_{\mathrm{BE}(\text { on })}\right) / 2 \mathrm{~V}_{\mathrm{T}}
$$



Figure 3.1.6 DC Transfer Characteristics of emitter coupled pair
[source: "Linear Integrated Circuits"by S.Salivahanan\& V.S. Kanchana Bhaskaran, Page-398]
This arrangement is shown in figure 3.1.7. It is a simple modulator circuit constructed using a differential amplifier. It can be used as a multiplier, provided $V_{1}$ is small and much less than 50 mV and $\mathrm{V}_{2}$ is greater than $\mathrm{V}_{\text {BE }}(\mathrm{on})$. But, the multiplier circuit shown in figure has several limitations. The first limitation is that $\mathrm{V}_{2}$ is offset by $\mathrm{V}_{\mathrm{BE}}$ (on).

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Figure 3.1.7 A Simple modulator using a differential Amplifier
[source: "Linear Integrated Circuits"by S.Salivahanan\& V.S. Kanchana Bhaskaran, Page-398]
The second is that $\mathrm{V}_{2}$ must always be positive which results in only a two-quadrant multiplier operation. The third limitation is that, the $\tanh (\mathrm{X})$ is approximately as X , where X $=\mathrm{V}_{1} / 2 \mathrm{~V}_{\mathrm{T}}$. The first two limitations are overcome in the Gilbert cell.
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### 3.3 ANALOG MULTIPLIER ICs

Analog multiplier is a circuit whose output voltage at any instant is proportional to the product of instantaneous value of two individual input voltages. Important applications of these multipliers are multiplication, division, squaring and square - rooting of signals, modulation and demodulation. These analog multipliers are available as integrated circuits consisting of op-amps and other circuit elements. The Schematic of a typical analog multiplier, namely, AD633 is shown in figure 3.3.1.


Figure 3.3.1 Schematic of Analog Multiplier IC and its symbol
[source: https://www.brainkart.com/subject/Linear-Integrated-Circuits_220/]

The AD633 multiplier is a four - quadrant analog multiplier.

- It possesses high input impedance; this characteristic makes the loading effect on the signal source negligible.
- It can operate with supply voltages ranging from $\pm 18 \mathrm{~V}$.
- IC does not require external components.
- The typical range of the two input signals is $\pm 10 \mathrm{~V}$.


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## SCHEMATIC REPRESENTATION OF A MULTIPLIER

The schematic representation of an analog multiplier is shown in figure 3.3.1. The output $V_{0}$ is the product of the two inputs $V_{x}$ and $V_{y}$ is divided by a reference voltage $V_{\text {ref }}$. Normally, the reference voltage $\mathrm{V}_{\text {ref }}$ is internally set to 10 V . Therefore, $\mathrm{V} 0=\mathrm{V}_{\mathrm{x}} \mathrm{V}_{\mathrm{y}} / 10$. In other words, the basic input - output relationship can be defined by $\mathrm{KV}_{\mathrm{x}} \mathrm{V}_{\mathrm{y}}$ when $\mathrm{K}=1 / 10$, a constant. Thus for peak input voltages of 10 V , the peak magnitude of output voltage is $1 / 10$ $* 10 * 10=10 \mathrm{~V}$. Thus, it can be noted that, as long as $\mathrm{V}_{\mathrm{x}}<10 \mathrm{~V}$ and $\mathrm{V}_{\mathrm{y}}<10 \mathrm{~V}$, the multiplier output will not saturate.

## MULTIPLIER QUADRANTS

The transfer characteristics of a typical four-quadrant multiplier are shown in figure 3.3.2. Both the inputs can be positive or negative to obtain the corresponding output as shown in the transfer characteristics.


Figure 3.3.2. Transfer Characteristics of a typical four-quadrant multiplier
[source: https://www.brainkart.com/subject/Linear-Integrated-Circuits_220/]

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## APPLICATIONS OF MULTIPLIER ICs

The multiplier ICs are used for the following purposes:

1. Voltage Squarer
2. Frequency doublers
3. Voltage divider
4. Square rooter
5. Phase angle detector
6. Rectifier

## VOLTAGE SQUARER



Figure 3.3.3. Voltage squarer Circuit
[source: "Linear Integrated Circuits" by S.Salivahanan\& V.S. Kanchana Bhaskaran, Page-408]
Figure 3.3.3 shows the multiplier IC connected as a squaring circuit. The inputs can be positive or negative, represented by any corresponding voltage level between 0 and 10 V . The input voltage $\mathrm{V}_{\mathrm{i}}$ to be squared is simply connected to both the input terminals, and hence we have, $V_{x}=V_{y}=V_{i}$ and the output is $V_{0}=K V_{i}{ }^{2}$. The circuit thus performs the squaring operation. This application can be extended for frequency doubling applications.

## FREQUENCY DOUBLERS

Figure 3.3.4 a). shows the squaring circuit connected for frequency doubling operation. A sine-wave signal $\mathrm{V}_{\mathrm{i}}$ has a peak amplitude of Av and frequency of f Hz . Then, the output voltage of the doublers circuit is given by

$$
v_{0}=\frac{A_{v} \sin 2 \pi f t * A_{v} \sin 2 \pi f t-\frac{A_{v}^{2}}{10} \sin ^{2} 2 \pi f t=\frac{A_{v}^{2}}{20}(1-\cos 4 \pi f t), ~(10)}{10}
$$

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Assuming a peak amplitude Av of 5 V and frequency f of $10 \mathrm{KHz}, \mathrm{V}_{0}=1.25-1.25 \cos 2 \pi(20000)$ t . The first term represents the dc term of 1.25 V peak amplitude. The input and output waveforms are shown in figure. The output waveforms ripple with twice the input frequency in the rectified output of the input signal. This forms the principle of application of analog multiplier as rectifier of ac signals. Figure 3.3.4 b)Input-output waveform of frequency doubler.

(a)

Figure 3.3.4 a) Frequency Doubler circuit diagram
[source: "Linear Integrated Circuits"by S.Salivahanan\& V.S. Kanchana Bhaskaran, Page-408]

(b)

Figure 3.3.4 b)Input-output waveform
[source: "Linear Integrated Circuits"by S.Salivahanan\& V.S. Kanchana Bhaskaran, Page-408]
The dc component of output $\mathrm{V}_{0}$ can be removed by connecting a $1 \mu \mathrm{~F}$ coupling capacitor between the output terminal and a load resistor, across which the output can be observed.

## VOLTAGE DIVIDER

In voltage divider circuit the division is achieved by connecting the multiplier in the feedback loop of an op-amp. The voltages $\mathrm{V}_{\text {den }}$ and $\mathrm{V}_{\text {num }}$ represent the two input voltages, $\mathrm{V}_{\mathrm{dm}}$ forms one input of the multiplier, and output of op-amp $V_{o A}$ forms the second input.The output

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$\mathrm{V}_{\mathrm{OA}}$ forms the second input. The output VOM of the multiplier is connected back of op-amp in the feedback loop. Then the characteristic operation of the multiplier gives

$$
\begin{equation*}
\mathrm{V}_{\mathrm{om}}=\mathrm{K} \mathrm{~V}_{\mathrm{OA}} \mathrm{~V}_{\mathrm{dm}} \tag{1}
\end{equation*}
$$



Figure 3.3.5.Voltage Divider Circuit
[source: "Linear Integrated Circuits"by S.Salivahanan\& V.S. Kanchana Bhaskaran, Page-409]
As shown in figure 3.3 .5 no input signal current can flow into the inverting input terminal of op-amp, which_is at virtual ground. Therefore, at the junction a, $i_{1}+i_{2}=0$, the current $\mathrm{i} 1=\mathrm{V}_{\text {num }} / R$, where R is the input resistance and the current $i_{2}=V_{\mathrm{om}} / R$. With virtual ground existing at a,

$$
\begin{gathered}
i_{1}+i_{2}=\mathrm{V}_{\text {num }} / \mathrm{R}+\mathrm{V}_{\mathrm{om}} / \mathrm{R}=0 \\
K V_{O A} V_{\text {den }}=-V_{\text {num }} \\
o r \\
v o A=-v_{\text {num }} / K v_{\text {den }}
\end{gathered}
$$

where $V_{\text {num }}$ and $V_{\text {den }}$ are the numerator and denominator voltages respectively. Therefore, the voltage division operation is achieved. $V_{\text {num }}$ can be a positive or negative voltage and $V_{d e n}$ can have only positive values to ensure negative feedback. When $V_{d m}$ is changed, the gain $10 / V_{d m}$ changes, and this feature is used in automatic gain control (AGC) circuits.

## SQUARE ROOTER

The divider voltage can be used to find the square root of a signal by connecting both inputs of the multiplier to the output of the op-amp. Substituting equal in magnitude but

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opposite in polarity (with respect to ground) to Vi. But we know that $\mathrm{V}_{\text {om }}$ is one- term (Scale factor) of $\mathrm{V}_{0} * \mathrm{~V}_{0}$ or

$$
-\mathrm{V}_{\mathrm{i}}=\mathrm{V}_{\mathrm{om}}=\mathrm{V}^{2} / 10
$$

Solving for $\mathrm{V}_{0}$ and eliminating $\sqrt{ }-1$ yields. $\mathrm{V} 0=\sqrt{ } 10|\mathrm{Vi}|$
Eqn. states that $\mathrm{V}_{0}$ equals the square root of 10 times the absolute magnitude of $\mathrm{V}_{\mathrm{i}}$.The input voltage Vi must be negative, or else, the op-amp saturates.The range of Vi is between -1 and -10 V . Voltages less than -1 V will cause inaccuracies in the result.The diode prevents negative saturation for positive polarity Vi signals. For positive values of Vi the diode connections are reversed.

## PHASE ANGLE DETECTOR

The multiplier configured for phase angle detection measurement is shown in figure 3.3.6. When two sine-waves of the same frequency are applied to the inputs of the multiplier, the output V0 has a dc component and an AC component. The trigonometric identity shows that

$$
\sin A \sin B=1 / 2(\cos (A-B)=\cos (A+B)) .
$$

When the two frequencies are equal, but with different phase angles, e.g. $\mathrm{A}=2 \pi \mathrm{ft}+\theta$ for signal $V x$ and $B=2 \pi f t$ for signal $V_{y}$, then using the identity

$$
\begin{aligned}
& [\sin (2 \mathrm{ft}+)][\sin 2 \mathrm{ft})]=1 / 2[\cos -\cos (4 \mathrm{ft}+)] \\
& \quad=1 / 2(\mathrm{dc} \text { - the double frequency term })
\end{aligned}
$$

Therefore, when the two input signals $\mathrm{V}_{\mathrm{x}}$ and $\mathrm{V}_{\mathrm{y}}$ are applied to the multiplier, $\mathrm{V}_{0}(\mathrm{dc})$ is given by

$$
v_{v}(d c)=\frac{v_{x p} v_{y p}}{20} \cos \theta
$$

where $\mathrm{V}_{\mathrm{xp}}$ and $\mathrm{V}_{\mathrm{yp}}$ are the peak voltage amplitudes of the signals $\mathrm{V}_{\mathrm{x}}$ and $\mathrm{V}_{\mathrm{y}}$. Thus, the output $\mathrm{V}_{0}(\mathrm{dc})$ depends on the factor $\cos \theta$. A dc voltmeter can be calibrated as a phase angle meter when the product of $\mathrm{V}_{\mathrm{xp}}$ and $\mathrm{V}_{\mathrm{yp}}$ is made equal to 20. Then, a (0-1) V range dc voltmeter can

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directly read $\cos \theta$, with the meter calibrated directly in degrees from a cosine table. The input and output waveforms are shown in figure 3.3.7.

Then the above eqn becomes $\mathrm{V}_{0}(\mathrm{dc})=\cos \theta$, if we make the product $\mathrm{V}_{\mathrm{xp}} \mathrm{V}_{\mathrm{yp}}=20$ or in other words, $\mathrm{V}_{\mathrm{xp}}-\mathrm{V}_{\mathrm{yp}}=4.47 \mathrm{~V}$.


Figure 3.3.6 Phase angle measurement circuit diagram
[source: https://www.brainkart.com/subject/Linear-Integrated-Circuits_220/]


Figure 3.3.7.Input-output waveform
[source: https://www.brainkart.com/subject/Linear-Integrated-Circuits_220/]

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### 3.4 OPERATION OF BASIC PHASE LOCKED LOOP



Figure 3.4.1.Block Diagram of PLL
[source: https://www.brainkart.com/subject/Linear-Integrated-Circuits_220/]
Before the input is applied, the PLL is in free running state. Once the input frequency is applied the VCO frequency starts to change and PLL is said to be in the capture mode. The VCO frequency continuous to change until it equals the input frequency and the PLL is in phase lock mode. When Phase locked, the loop tracks any change in the input frequency through its repetitive action.

If an input signal ys of frequency fs is applied to the PLL, the phase detector compares the phase and frequency of the incoming signal to that of the output vo of the VCO. If the two signals differ in frequency of the incoming signal to that of the output vo of the VCO.The phase detector is basically a multiplier and produces the sum ( $f_{s}+f_{o}$ ) and difference ( $f_{s}-f_{o}$ ) components at its output.The high frequency component $\left(f_{s}+f_{0}\right)$ is removed by the low pass

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filter and the difference frequency component is amplified then applied as control voltage $\mathrm{v}_{\mathrm{c}}$ to VCO.

The signal $\mathrm{v}_{\mathrm{c}}$ shifts the VCO frequency in a direction to reduce the frequency difference between fs and $\mathrm{f}_{\mathrm{o}}$. Once this action starts, we say that the signal is in the capture range. The VCO continues to change frequency till its output frequency is exactly the same as the input signal frequency. The circuit is then said to be locked. Once locked, the output frequency $f_{o}$ of VCO is identical to fs except for a finite phase difference $\varphi$. This phase difference $\varphi$ generates a corrective control voltage vc to shift the VCO frequency from f0 to fs and thereby maintain the lock. Once locked, PLL tracks the frequency changes of the input signal. Thus, a PLL goes through three stages (i) free running, (ii) capture and (iii) locked or tracking.

Capture range: The range of frequencies over which the PLL can acquire lock with an input signal is called the capture range. This parameter is also expressed as percentage of fo.

Pull-in time: The total time taken by the PLL to establish lock is called pull-in time. This depends on the initial phase and frequency difference between the two signals as well as on the overall loop gain and loop filter characteristics.

Lock-in Range: Once the PLL is locked, It can track frequency changes in the incoming signals.The range of frequencies over which the PLL can maintain lock with the incoming signal is called the lock in range or tracking range.

## CLOSED LOOP ANALYSIS OF PLL



Figure 3.4.2.Detailed block diagram of PLL
[source: "Linear Integrated Circuits"by S.Salivahanan\& V.S. Kanchana Bhaskaran, Page-417]

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Figure 3.4.2 shown above is the detailed block diagram of PLL. The $\mathrm{i} / \mathrm{p}$ sinusoidal gain $\mathrm{V}_{\mathrm{i}}$ is given as

$$
V_{i}=V_{p} \sin (\omega t+\theta)
$$

If the phase shift of the signal at the $\mathrm{VCO} o / \mathrm{p}$ is $\theta_{\text {osc }}$, then the average value of the $\mathrm{o} / \mathrm{p}$ of phase detector is

$$
\begin{gathered}
V_{e}=K_{d}\left(\theta_{i}-\theta_{o s c}\right) \\
\text { where } \theta_{i} \& \theta_{\text {osc }} \rightarrow \text { phase shift }
\end{gathered}
$$

The phase of the signal at the $\mathrm{o} / \mathrm{p}$ of VCO as a function of time is equal to integral of the VCO o/p frequency \& can be expressed as

$$
\begin{gathered}
\omega_{o s c(t)}=\frac{d \theta_{\operatorname{osc}(t)}}{d t} \\
d \theta_{o s c(t)}=\omega_{o s c}(t) \\
\int d t \\
d \theta_{o s c(t)}=\int \omega_{o s c(t)} d t+\theta_{o s c ~ a t} t=0 \\
\theta_{o s c(t)}=\int \omega_{\operatorname{osc}(t)} d t+\theta_{o s c} \text { at } t=0
\end{gathered}
$$

The integral component is represented as $1 / \mathrm{S}$ inside the VCO block. Oscillator frequency $\omega_{\text {osc }} \&$ the dc control voltage $\mathrm{V}_{\mathrm{c}}$ are related by

$$
\omega_{o s c}=\omega_{c}+K_{o} V_{c}
$$

where, $\omega_{c} \rightarrow$ centre or free running angular frequency,
that results when $V_{c}=0 \& K_{o}$ is the VCO gain in rad/sec per volt.
Then the closed loop transfer function of PLL

$$
\frac{\underline{V}_{c}(s)}{\theta_{i}(s)}=\frac{K_{d} \underline{G}(S) A}{1+K_{d} A F(S) \frac{K_{o}}{S}}=\frac{S K_{d} \underline{G}(S) A}{S+K_{d} A F(S) K_{o}}
$$

Response of the loop to frequency variation at $\mathrm{i} / \mathrm{p}$ than phase.

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$$
\begin{aligned}
\frac{V_{c}(s)}{\omega_{i}(s)} & =\frac{V_{c}(S)}{S \theta_{i}(S)}=\frac{K_{d} F(S) A}{S+K_{d} A F(S) K_{o}} \\
\omega_{i} & =\frac{d \theta_{i}}{d t} \& \omega_{i}(s)=s \theta_{i}(s)
\end{aligned}
$$

$F(S)=0$, with the loop having a first order low pass frequency response.

$$
\begin{gathered}
\frac{V_{c}(s)}{\omega_{i}(s)}=\frac{K_{v}}{S+K_{v}} X \frac{1}{K_{o}} \\
K_{v} \rightarrow \text { loop bandwidth. } \\
K_{v}=K_{o} K_{d} A
\end{gathered}
$$

## SECOND ORDER PLL

The first order loop without loop-filter has several limitations.

1. Both the sum \& difference frequency components are fed to the o/p from the phase detector
2. All out-of band interfering signals from the i/p will appear shifted in frequency at the $\mathrm{o} / \mathrm{p}$.

The most common configuration of monolithic PLL in the second order loop with a loop -filter F(s) of a simple single-pole LPF realized with a resistor R \& a Capacitor C in Figure 3.4.3.


Figure 3.4.3 Single pole loop filter
[source: https://www.electronics-tutorials.ws/filter/filter_2.html]

$$
\left.F_{S}^{( }\right)=\frac{1 / S C}{R+1 / S C}=\frac{1}{1+S R C}
$$

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$$
\begin{gathered}
=\frac{1}{1+S / \omega}=\frac{1}{1+S \tau} \\
\omega=\frac{1}{R C}=\frac{1}{\tau}
\end{gathered}
$$

The resulting block diagram of the second order PLL using single-pole loop filter is shown in Figure 3.4.4.


Figure 3.4.4. block diagram of the second order PLL using single-pole loop filter.
[source: "Linear Integrated Circuits"by S.Salivahanan\& V.S. Kanchana Bhaskaran, Page-419]

## LOOP LOCK-RANGE \& CAPTURE RANGE

The loop lock-range is represented as the range of frequency about $\omega_{o}$ for which PLL maintains the relationship

$$
\omega_{i}=\omega_{o s c}
$$

If the phase detector can determine the phase difference between $\theta_{i} \& \theta_{\text {osc }}$
Over a $\pm \frac{\pi}{2}$ range, then the lock-range is defined as

$$
\begin{gathered}
\omega_{L}= \pm \Delta \omega_{o s c} \\
=\boldsymbol{K}_{d} \boldsymbol{A} \boldsymbol{K}_{o}\left( \pm \frac{\boldsymbol{\pi}}{\mathbf{2}}\right)= \pm \boldsymbol{K}{ }_{v}\left(\frac{\pi}{\mathbf{2}}\right)
\end{gathered}
$$

The capture range is the range of $\mathrm{i} / \mathrm{p}$ frequencies with in which a initially unlocked loop will get locked with an $\mathrm{i} / \mathrm{p}$ signal. When $\mathrm{F}(\mathrm{s})=1$,the capture range equals the lock-range. If $\boldsymbol{F}(\boldsymbol{S})=$ $\frac{1}{1+{ }^{s} / \omega_{i}}$,then the capture range is smaller than the lock-range.

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### 3.5 VOLTAGE CONTROLLED OSCILLATOR

The timing capacitor $\mathrm{c}_{\mathrm{T}}$ is linearly charged or discharged by a constant current source/sink. The amount of current can be controlled by changing the voltage $\mathrm{v}_{\mathrm{c}}$ applied at the modulating input (pin 5) or by changing the timing resistor $\mathrm{R}_{\mathrm{T}}$ external to the IC chip.The voltage at pin 6 is held at the same voltage as pin 5.Thus, if the modulating voltage at pin 5 is increased, the voltage at pin 6 also increases, resulting in less voltage across $R_{T}$ and thereby decreasing the charging current. Pin configuration of VCo is shown in figure 3.5.1.

Figure 3.5.2 shown below is the block diagram of VCO.A small capacitor of $0.001 \mu \mathrm{f}$ should be connected between pin 5 \& 6 to eliminate possible oscillations.A VCO is commonly used in converting low frequency signals such as EEG,ECG in to an audio frequency range.These audio signals can be transmitted over telephone lines or a two way radio communication system for diagnostic purposes or can be recorded on a magnetic tape for further references.


Figure 3.5.1.Pin configuration
[source: https://www.elprocus.com/voltage-controlled-oscillator-working-application/]
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Figure 3.5.2 Voltage controlled oscillator Block diagram
[source: https://sthcphy.files.wordpress.com/2015/09/vco-566.pdf]
The voltage across the capacitor $\mathrm{C}_{\mathrm{T}}$ is applied to the inverting $\mathrm{i} / \mathrm{p}$ terminal of Schmitt trigger $A_{2}$ via buffer amplifier $A_{1}$.The o/p voltage swing of the Schmitt trigger is designed to $V_{c c} \&$ $0.5 \mathrm{~V}_{\mathrm{cc}}$. If $\mathrm{R}_{\mathrm{a}}=\mathrm{R}_{\mathrm{b}}$ in the +ive feedback loop, the voltage at the non-inverting $\mathrm{i} / \mathrm{p}$ terminal of $\mathrm{A}_{2}$ swings from $0.5 \mathrm{~V}_{\mathrm{cc}}$ to $0.25 \mathrm{~V}_{\mathrm{cc}}$.Figure 3.5 .3 a ). when the voltage on the capacitor $\mathrm{C}_{\mathrm{T}}$ exceeds $0.5 \mathrm{~V}_{\mathrm{cc}}$ during charging, the o/p of the Schmitt trigger goes low $\left(0.5 \mathrm{~V}_{\mathrm{cc}}\right)$ The capacitor now discharges \& when it is at $0.25 \mathrm{~V}_{\mathrm{cc} \text {. The }} \mathrm{o} / \mathrm{p}$ of Schmitt trigger goes high $\left(\mathrm{V}_{\mathrm{cc}}\right)$.Since the source \& sink currents are equal, capacitor for the same amount of time.This gives a triangular voltage waveform across CT which is also available at pin 4.The square wave o/p of the Schmitt trigger is inverted by inverter A3 \& is available at pin 3.The inverter A3 is basically a current amplifier used to drive the load.

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Figure 3.5.3 a)output waveform b)Typical connection diagram
[source: "Linear Integrated Circuits"by D.Roy Choudhry, Shail Bala Jain, Page-386]
The total voltage on the capacitor changes from $0.25 \mathrm{~V}_{\mathrm{cc}}$ to $0.5 \mathrm{~V}_{\mathrm{cc}}$.Thus $\Delta \mathrm{v}=0.25 \mathrm{~V}_{\mathrm{cc}}$ The capacitor charges with a constant current source.

$$
\begin{gathered}
\frac{\Delta v}{\Delta t}=\frac{i}{C_{T}} \\
\frac{0.25 \mathrm{~V}_{c c}}{\Delta t}=\frac{i}{C_{T}} \\
\Delta t=\frac{0.25 V_{c c} C_{T}}{i}
\end{gathered}
$$

The time period T of the triangular waveform $=2 \Delta t$. The freq of oscillator $f_{o}$ is

$$
\begin{gathered}
f_{o}=\frac{1}{T}=\frac{1}{2 \Delta t}=\frac{i}{.5 V_{c c} C_{T}} \\
\text { But } i=\frac{V_{c c}-V_{c}}{R_{T}} \\
\text { Where } V_{c} \rightarrow \text { Voltage at pin } 5 \\
f_{o}=\frac{2\left(V_{c c}-V_{c}\right)}{C_{T} R_{T} V_{c c}}----(1)
\end{gathered}
$$

The $\mathrm{o} / \mathrm{p}$ freq of VCO can be changed either by (i) $\mathrm{R}_{\mathrm{T}}\left(\right.$ (ii) $\mathrm{C}_{\mathrm{T}}$ or (iii) the voltage $\mathrm{V}_{\mathrm{c}}$ at the modulating $\mathrm{i} / \mathrm{p}$ terminal pin 5 . The voltage vc can be varied by connecting a $\mathrm{R}_{1} \mathrm{R}_{2}$ circuit as

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shown in the figure 3.5 .3 b ). The components R1 and c 1 are first selected so that VCO output frequency lies in the centre of the operating frequency range. Now the modulating input voltage is usually varied from $0.75 \mathrm{~V}_{\mathrm{cc}}$ to Vcc which can produce a frequency variation of about 10 to 1 .

The signetics NE/SE 560 series is monolithic phase locked loops. The SE/NE 560, 561, $562,564,565$ \& 567 differ mainly in operating frequency range, poser supply requirements \& frequency \& bandwidth adjustment ranges.

With no modulating $\mathrm{i} / \mathrm{p}$ signal if the voltage at pin 5 is biased at ${ }_{8}^{7} V_{c c}$ (1) gives the $\mathrm{VCO} \mathrm{o} / \mathrm{p}$ frequency

$$
f_{o}=\frac{2\left(V_{c c}-7 / 8 V_{c c}\right)}{C_{T} R_{T} V_{C C}}=\frac{1}{4 R_{T} C_{T}}=\frac{0.25}{R_{T} C_{T}}---(2)
$$

## VOLTAGE TO FREQUENCY CONVERSION FACTOR

Voltage to frequency conversion factor $\mathrm{K}_{\mathrm{v}}$ \& is defined as

$$
\bigcirc \quad K_{v}=\frac{\Delta f_{o}}{\Delta C} \bigcirc \bigcirc
$$

$\Delta V_{c} \rightarrow$ modulation voltage required to produce the frequency shift $\Delta f_{o}$ for a VCO Original frequency is $f_{o}$ \& the new frequency is $f_{1}$ then

$$
\begin{gather*}
\Delta f_{o}=f_{1}-f_{o} \\
=\frac{2\left(V_{c c}-V_{c}+\Delta V_{c}\right)}{C_{T} R_{T} V_{c c}}-\frac{2\left(V_{c c}-V_{c}\right)}{C_{T} R_{T} V_{c c}} \\
=\frac{2 \Delta V_{c}}{C_{T} R_{T} V_{c c}} \\
\Delta V_{c}=\Delta f_{o} \frac{c T R T V c c \Delta f o-\cdots--(3)}{2} \tag{3}
\end{gather*}
$$

From (2)

$$
f_{o}=\frac{0.25}{R_{T} C_{T}}
$$

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$$
\begin{aligned}
& R_{T} C_{T}=\frac{0.25}{f_{o}} \\
& \Delta V_{c}=4^{f} \frac{V_{c c}}{{ }_{o}} \frac{8 f_{o}}{K_{v}}=\frac{\Delta f_{o}}{\Delta V_{c}}=\frac{8 f_{o}}{V_{c c}}
\end{aligned}
$$

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### 3.6 MONOLITHIC PHASE LOCKED LOOPS

The signetics NE/SE 560 series is monolithic phase locked loops. The SE/NE 560, 561, $562,564,565 \& 567$ differ mainly in operating frequency range, poser supply requirements \& frequency \& bandwidth adjustment ranges. IC 565 is available in a 14 pin DIP package and 10 pin metal can package. Figure 3.6.1 shows the 14 pin DIP package of NE/SE 565.

NE/SE 565 PLL Block Diagram is shown in figure 3.6.2.The o/p frequency of the VCO is given by equation

$$
f_{o}=\frac{0.25}{R_{T} C_{T}} H Z
$$

where $\mathrm{R}_{1} \& \mathrm{C}_{1}$ are an external resistor \& a capacitor connected to pins $8 \& 9$


Figure 3.6.1.Pin Configuration of NE/SE565
[source: https://sites.google.com/site/learneasyyourself/home/lic/phase-locked-loop-ic-s]

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Figure 3.6.2 NE/SE 565 PLL Block Diagram
[source: https://sites.google.com/site/learneasyyourself/home/lic/phase-locked-loop-ic-s]
A value between $2 \mathrm{~K} \Omega \& 20 \mathrm{~K} \Omega$ is recommended for $\mathrm{R}_{1}$. The VCO free running freq is adjusted with $R_{1} \& C_{1}$ be at the centre of the $i / p$ frequency range. A short circuit between pins $4 \& 5$ connects the $\mathrm{VCO} \mathrm{o} / \mathrm{p}$ to the phase comparator so as to compare $\mathrm{f}_{\mathrm{o}}$ with $\mathrm{i} / \mathrm{p}$ signal $\mathrm{f}_{\mathrm{s}} \mathrm{A}$ capacitor C is connected between pin 7 \& pin 10 to make a low pass filter with the internal resistance of $3.6 \mathrm{~K} \Omega$.The important electrical characteristics of the 565 PLL are, •

- Operating frequency range: 0.001 Hz to 500 Khz .
- Operating voltage range: $\pm 6$ to $\pm 12 \mathrm{v}$.
- Input level required for tracking: 10 mv rms min to $3 \mathrm{Vpp} \max$ -
- Input impedance: 10 K ohms typically. .
- Output sink current: 1 mA .
- Output source current: 10 mA


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## DERIVATION OF LOCK-IN RANGE

If $\Phi$ radians- phase difference between the signal \& the VCO voltage. The $\mathrm{o} / \mathrm{p}$ voltage of the analog phase detector is given by

$$
V_{e}=K_{\Phi}\left(\Phi-\frac{\pi}{2}\right)-\cdots--(1)
$$

where $K_{\Phi}$
$\rightarrow$ phase angle to voltage transfer coefficient of the phase detector.
The o/p voltage of VCO is

$$
V_{c}=A K_{\Phi}\left(\Phi-\frac{\pi}{2}\right)-\cdots---(2)
$$

where $A \rightarrow$ voltage gain of the amplifier
This $V_{c}$ shifts VCO frequency from its free running frequency $f_{o}$ to a frequency $f$ given by

$$
f=f_{o}+K_{v} V_{c^{---}}(3)
$$

Where $\mathrm{K}_{\mathrm{v}}$-voltage to freq transfer coefficient of the VCO
When PLL is locked in to signal frequency $f_{s}$

$$
\begin{gathered}
f=f_{s}=f_{o}+K_{v} V_{c} \\
f_{s}-f_{o}=K_{v} V_{c}
\end{gathered}
$$

comparing (4)\&(2)

$$
\begin{gathered}
\frac{\left(f_{s}-f_{o}\right)}{K V}=A K_{\Phi}\left(\Phi-\frac{\pi}{2}\right) \\
\left(\Phi-\frac{\pi}{2}\right)=\frac{\left(f_{s}-f_{o}\right)}{K V A K_{\Phi}} \\
(\Phi)=\frac{\pi}{2}+\frac{\left(f_{s}-f_{o}\right)}{K V A K_{\Phi}}-\cdots-(5)
\end{gathered}
$$

Max o/p voltage magnitude available from the phase detector occurs for $\Phi=\boldsymbol{\pi} \& 0$ radians.

$$
(1) \rightarrow V_{e(\max )}= \pm K_{\boldsymbol{\Phi}} \frac{\pi}{2}
$$

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The corresponding value of the max control voltage available to drive VCO will be

$$
(2) \rightarrow V_{c(\max )}= \pm A K_{\Phi} \frac{\pi}{2}
$$

The max VCO frequency swing that can be obtained is given by

$$
\begin{gathered}
(3) \rightarrow\left(f-f_{o}\right)_{\max }=K_{V} V_{C(\max )} \\
\left(f-f_{o}\right)_{\max }=K_{V} A K_{\Phi} \frac{\pi}{2}
\end{gathered}
$$

The max range of signal frequencies over which PLL can remain locked will be

$$
\begin{gathered}
f_{s}=f_{o} \pm\left(f-f_{o}\right)_{\max } \\
f_{s}=f_{o} \pm K_{V} A K_{\Phi}^{\pi} \overline{2} \\
f_{s}=f_{o} \pm \Delta f_{L}
\end{gathered}
$$

Lock -in Range

$$
\begin{gathered}
\Delta f_{L}= \pm K_{V} A K_{\Phi} \pi \\
2 \Delta f_{L}= \pm K_{V} A K_{\Phi} \pi \\
\text { w.k.t, } K_{v}=\frac{8 f_{o}}{V} \\
V=+V_{c c}-\left(-V_{c c}\right) \\
K_{\Phi}=\frac{1.4}{\pi}
\end{gathered}
$$

$\mathrm{A}=1.4$

$$
\begin{gathered}
\text { lock - in range } \Delta f_{L}= \pm \frac{8 f_{o}}{V} \times \frac{1.4}{\pi} X 1.4 X \pi \\
\Delta f_{L}= \pm \frac{7.8 f_{o}}{V}
\end{gathered}
$$

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## DERIVATION OF CAPTURE RANGE

When PLL is not locked to the signal, the frequency of the VCO will be free running frequency $f_{0}$. The phase angle difference between the signal \& the $\mathrm{VCO} \mathrm{o} / \mathrm{p}$ voltage

$$
\begin{gathered}
\Phi=\left(w_{s} t+\theta_{s}\right)-\left(w_{o} t+\theta_{o}\right) \\
\Phi=\left(w_{s} t\right)-\left(w_{o} t\right)+\left(\theta_{s}-\theta_{o}\right) \\
\Phi=\left(w_{s}-w_{o}\right) t+\Delta \theta
\end{gathered}
$$

The phase angle difference does not remain constant but will change with time at a rate given by

$$
\frac{d \Phi}{d t}=w_{s}-w_{0}
$$

The phase detector $\mathrm{o} / \mathrm{p}$ voltage will not have a dc component but will produces on ac voltage with a triangular waveform of peak amplitude $\left(K_{\Phi} \frac{\pi}{2}\right) \&$ a fundamental frequency

LPF is a simple RC network having transfer function

$$
\begin{aligned}
T(j f) & =\frac{1}{1+j \frac{f}{f_{1}}} \\
f_{1} & =\frac{1}{2 \pi R C}
\end{aligned}
$$

$$
\left(\frac{f^{\prime}}{f_{1}}\right)^{2} \gg 1 \text { then } T \stackrel{()}{f}=\frac{1}{j \frac{f}{f_{1}}}
$$

$$
T(f)=\frac{f_{1}}{\overline{j f}}
$$

Fundamental frequency term supplied to LPF by the phase Detector will be the difference frequency

$$
\Delta f=f_{s}-f_{o}
$$

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LPF Transfer function will be

$$
\begin{gathered}
T(\Delta f)=\frac{f_{1}}{\Delta f} \\
T(\Delta f)=\frac{f_{1}}{f_{s}-f_{o}}
\end{gathered}
$$

Voltage $\mathrm{V}_{\mathrm{c}}$ to drive the VCO is

$$
\begin{aligned}
V_{c} & =V_{e} X T(f) X A \\
V_{c(\max )} & =V_{e(\max )} X T(f) X A \\
& = \pm K_{\Phi}{ }^{\pi} A\left(\frac{f_{1}}{\overline{\Delta f}}\right)
\end{aligned}
$$

Then the corresponding value of the max VCO frequency shift is

$$
\begin{gathered}
\left(f-f_{o}\right)_{\max }=K_{v} V_{c(\max )} \\
= \pm K K \pi A\left(f_{1}\right)
\end{gathered}
$$

Sub $f=f_{s} \rightarrow$ max signal freq range that can be acquired by PLL is

$$
\left(f_{s}-f_{o \max }\right)_{v}= \pm K_{\overline{2}}^{\pi} A\left(\overline{f_{1}}\right)
$$

Now, $\Delta f_{c}=\left(f_{s}-f_{o}\right)_{\max }$

$$
\begin{gathered}
\left.\Delta f_{c}= \pm K_{v} K_{\Phi \overline{2}}^{\pi} \frac{f_{1}}{\Delta f_{c}}\right) \\
\Delta f_{c}^{2}=K_{v} K_{\Phi}^{\pi} \overline{2}_{1}^{\pi} A f \\
\text { where, } \Delta f_{L}= \pm K_{v}^{K}{ }_{\Phi} \frac{\pi}{2} A \\
\Delta f_{c}^{2}=f_{1} \Delta f_{L} \\
\Delta f_{c}= \pm \sqrt{f_{1} \Delta f_{L}}
\end{gathered}
$$

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The total capture range is

$$
\begin{aligned}
2 \Delta f_{c} & =2 \sqrt{f_{1} \Delta f_{L}} \\
f_{1} & =\frac{1}{2 \pi R C}
\end{aligned}
$$

IC PLL 565, $\mathrm{R}=3.6 \mathrm{~K} \Omega$
Capture range is $\pm 2\left[\frac{\Delta f L}{2 \pi\left(3.6 \times 10^{3}\right) C}\right] \frac{1}{2} \quad$, Where $\mathrm{C} \rightarrow$ farads
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