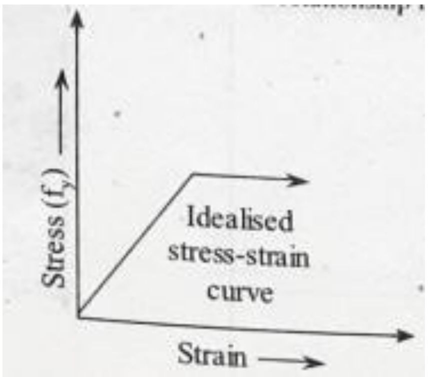
PLASTIC THEORY

Assumptions made in plastic analysis of structure



The strain hardening effect are ignored and the stress – strain relationship is expressed through two straight lines.

a. The effect of axial load on fully plastic moment capacity of the section are ignored.
b. The plane section before bending continuous to remain plane even after the bending. The shear deformation are ignored.
c. There is an identical relationship between compressive stress, compressive strain and tensile stress, tensile strain.
d. The effect of shear on a fully plastic moment capacity of the section is ignored.
e. A plastic hinge is formed at the cross section where the plastic moment is attained. This plastic hinge is allowed to undergo rotation of any magnitude, but at a fully plastic value, the bending moment remains uniform.
f. The materials is assumed to be homogeneous and isotropic in both the elastic and plastic states.
g. The resultant axial force on beam is zero
h. Total compression = total tension
i. The value of modulus of elasticity is same in both tension and compression.

j. The fibers in the lateral direction remains unaffected due to the expansion or contraction of longitudinal fibers

Limitation of plastic analysis

The limitation of the theory of plastic analysis are as follows

a. Only the material of ductile steel can be analyzed by plastic analysis. Therefore this method is not recommended for high strength steel.
b. It is difficult to recognize the unstable plastic structure than the identical elastic structure.
c. The connection provided in the plastic structure should be strong enough to transfer the plastic moments. Thus, plastic structure requires greater provision, care and good materials when compared to elastically designed structure.
d. Theory of simple plastic analysis is not applicable to the trusses in which the structural member carry axial forces instead of the bending. Hence, special methods are required to analysis such structure.

STATICALLY INDETERMINATE STRUCTURE

Statically indeterminate structure

When the number of independent equations of equilibrium are less than the number of independent reaction components. Then such structure is known as statically indeterminate structure.

Degree of redundancy,

$$Ds = (r_e + m) - 2j$$

PLASTIC MOMENT OF RESISTANCE

Plastic moment

The moment of resistance of a beam or any structure when thee plastic hinges are formed is known as the plastic moment or plastic moment of resistance

Plastic moment, Mp = Fy / Zp

Where,

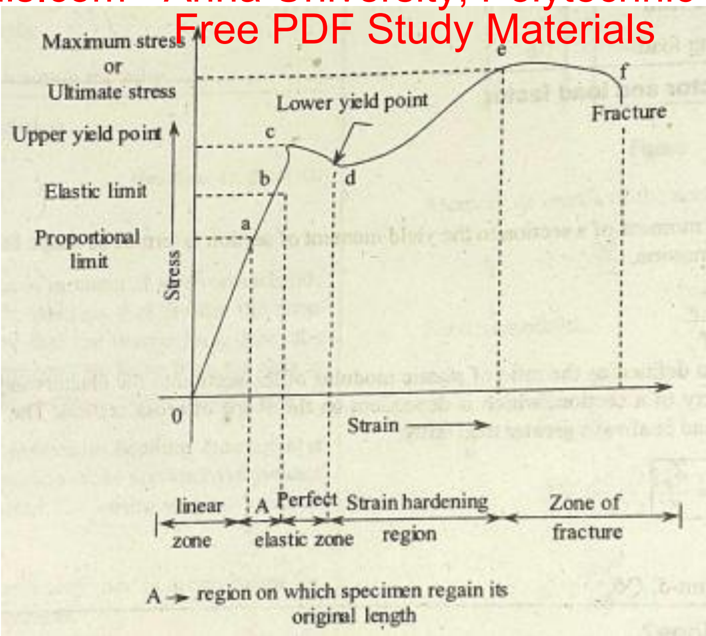
Fy – Yield stress

Zp – Plastic sectional modulus

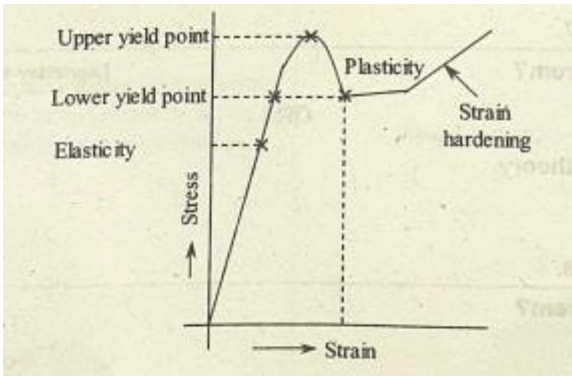
Stress-strain curve for mild steel

The stress - strain curve of the mild steel specimen of typical form and their corresponding elements are explained below, with the help of the graph.

As the strain in the specimen increases, the stress will also increase with linear variation till the proportional limit(within the linear region). Beyond this point, the stress reaches a point at which the specimen can regain its original length. When stress in increased above this perfectly elastic stage, it reaches a point called the upper yield point (c) from which it falls to lower yield point. The strain in the specimen keeps on increasing at a constant rate when the stress drops from c to d. Beyond those point the stress increase rapidly within strain hardening range and reaches the maximum or ultimate’s stress. Further increase in stress results in fracture. The behavior of stress-strain curve can be understood easily by the following stress- strain scale.



The upper yield stress varies with strain as it depends upon the rate of application of the load and its method. Due to this reason the fully plastic moment remains un-effected. While designing the steel member by plastic methods, the elastic limit and the lower yield point is assumed to be numerically identical. The achieved values indicate that elastic deformation is only about 1/12th of the strain starting point of the strain hardening zone reaching to the value of 1/200 of the strain atmaximum strain of 0.25.



According to the elastic theory, the stress - strain relationship is assumed to linear. depending upon this theory, the structure fails when the stress at any point reaches the yield stress. By neglecting the strain idealized by two straight lines as shown in fig. Shear deformation is neglected so that the plane section before and after the bending remains plane and compressive stress and the compressive strain are directly proportional to the tensile stress and tensile strain. Bending moment is assumed as constant even after the application of fully plastic moment.

PLASTIC MODULUS

Plastic section modulus

It is defined as the sum of moment of areas on each side of plastic neutral axis. An axis that divides the cross section in such a way that the tensile force from the area in the tension and compressive force from the area in compression are equal, is called plastic neutral axis.

It is used to find the maximum bending moment at a point when all the fibers in a cross section have yielded elastically. At this point, the entire section behaves plastically.

Plastic section modulus is only used in the problems for the design of steel structures.

It is denoted by the letter 'Zp'.

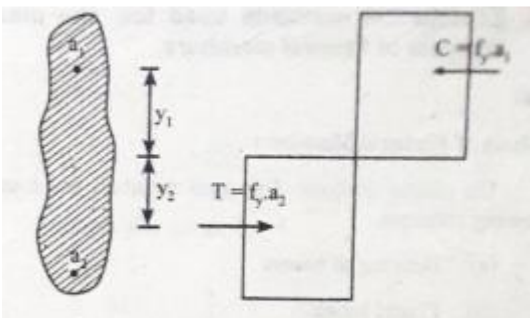
Fully plastic moment of a section

The maximum moment of resistance in a fully yielded cross section is termed as fully plastic moment. To determine the value of fully plastic moment, the following assumption are made,

The ultimate load is achieved without causing any instability to the structure.

The connections in the structure are assumed to give full continuity in order to transmit the fully plastic moment safety.

Each load increase in fixed proportion to the other i.e..loading is proportional.



1

The zone above the neutral axis is in compression and the zone below the neutral axis is in tension, such that the nature of the bending moment is sagging.

Force in compression

$$F_c=f_y.a_1$$

Force in tension

$$F_t=f_y.a_2$$

Consider the equilibrium condition where the force of compression is equal to force of tension

$$F_c=F_t.$$

$$f_y.a_1 = f_y.a_2$$

$$a1 = a2 = a/2$$

Where,

a1 -- Area of the section above N.A

a2 - Area of the section below N.A

a - Area of cross section of the beam

The "equal area axis" is the neutral axis of the plasticized section. The two forces tensile and compressive form a couple and resists the plastic moment.

$$M_p=f_y.a_1\,\bar{y}_1+f_y.a_2\,\bar{y}_2$$

$$M_p=f_y.a/2.\,\bar{y}_1+f_y.a/2.\,\bar{y}_2$$

$$M_p=f_y.a/2(\,\bar{y}_1+\bar{y}_2\,)$$

$$M_p=f_y.Z_p$$

Where

Z_p - Plastic section modulus

$=a/2(\bar{y}_1 + \bar{y}_2)$

$\bar{y}_1$= distance of centre of gravity above the neutral axis

$\bar{y}_2$ =distance of centre of gravity below the neutral axis

SHAPE FACTOR

Shape factor

Shape factor is also defined as the ratio of plastic modulus of the section to the elastic modulus of the section.

$$S = Zp / Z$$

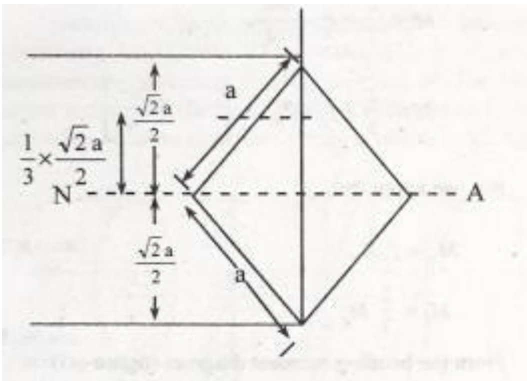
The ratio of plastic moment of the section to the yield moment of the section is termed as shape factor.

$$S = Mp / My$$

Example :

One diagonal horizontal

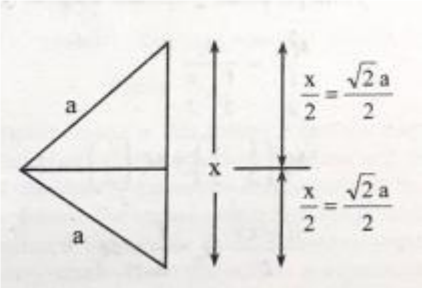
A square section with one diagonal lying horizontally is shown below. Length of diagonal = x



$X^2 \quad = a^2+b^2$

$X \quad =\sqrt{2a^2}$

$\quad =a\sqrt{2}$



Moment of inertia about the neutral axis is given by,

I =2 x (√2a x[√2a/2]^3/12)

= √2a/6 x 2√2a^3/8

=a^4/12

Section modulus

Z =I/Y

Z =a^4/12)/(√2a/2)

=√2a^3/12

Plastic section modulus

Zp = A/2 ($\bar{Y}_1+\bar{Y}_2$)

=a x a /2 [1/3 x √2a/3 + 1/3 x √2a/2]

=a^2/2[√2/3 a] =a^3/√2√2 x √2/3

Zp =a^3 / 3√2

Shape factor,

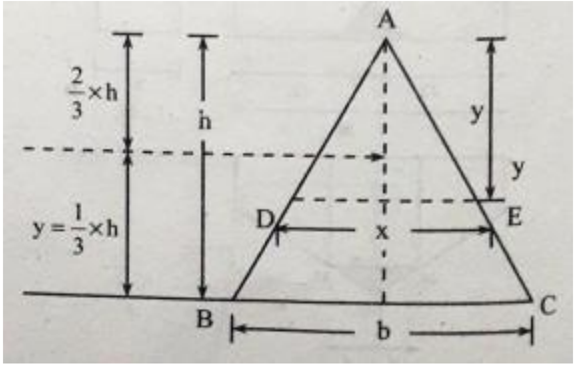
s =Zp / Z

S =(a^3/3√2) / (√2 /a^3)

= 2

Example :

Triangle section of base b and height h



Moment of inertia about neutral axis,

I $=bh^3/36$

Section modulus

Z $=I/Y$

$=(bh^3/36)/(2/3\ h)$

$=bh^2/24$

Zp $=A/2(\bar{Y}_1+\bar{Y}_2)$

The area of triangle ADE is the half of the area of triangle ABC. Therefore,

$\frac{1}{2} \times X \times y = \frac{1}{2} \times [1/2 \times bxh]$

Xy $=bh/2$

Y $=bh/2x$ ------------1

Due to properties of similar triangles,

h/b $= y/x$

y $=hx/b$-------------2

Equating the equation 1 and 2 , we get

$bh/2x = hx/b$

$2x^2 \quad = b^2$

$X^2 \quad = b^2/2$

$X \quad = \sqrt{b^2/2}$

$= b/\sqrt{2}$

Sub in eqn 1

$Y \quad = bh/2 \times \sqrt{2}/b$

$= h\sqrt{2} / \sqrt{2}.\sqrt{2}$

$= h / \sqrt{2}$

$\bar{y}1 \quad = 1/3 \times y$

$= 1/3 \times h/\sqrt{2}$

$= h / 3\sqrt{2}$

$= 0.236\ h$

$\bar{y}2 \quad = [h - y/3] \times [x+2b / x + b]$

$= [h - h/\sqrt{2}] \times [b/\sqrt{2} + 2b / b/\sqrt{2} + b]$

$= \sqrt{2}h - h / 3\sqrt{2} \times [(b + 2\sqrt{2}b) / \sqrt{2}] / [b+\sqrt{2}b) / \sqrt{2}]$

$= 0.414h / 3\sqrt{2} \times b(1+2\sqrt{2}/ b(1+\sqrt{2})$

$= 0.098h \times 1.586$

$\bar{y}2 \quad = 0.155\ h$

Plastic section modulus,

$Zp = A/2(\bar{Y}_1+\bar{Y}_2)$

$=1/2bh / 2 [h/3\sqrt{2} +0.155h]$

$=bh /4 [0.3907 h]$

$Zp =0.0976bh^2$

Shape factor,

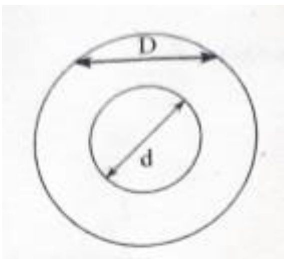
$S = Zp/Z$

$=0.0976bh^2/[bh^2/24]$

$S =2.342$

Example :

Find the shape factor of a tubular section with outer diameter equal to twice the inner diameter



Outer diameter of the tubular section =D

Inner diameter of the tubular section =d

Let,

$d/ D =K$

$d /2d =k$

½ $=k$

K $=0.5$

Z $=I / Y$

$=\pi/64[D^4 - d^4] / D/2$

$= \pi/64[16d^4 - d^4] / 2d/2$

$= \pi/64[15d^4] / d$

For a circular section

Zp $=d^3/6$

Therefore, for a hollow circular section,

Zp $=D^3/6 - d^3/6$

$=8d^3/6 - d^3/6$

$=7d^3/6$

Shape factor

S $=Zp / Z$

$=(7d^3/6) /[\pi/64[15d^4] /d]$

$=(7d^3/6) \times d / [\pi/64[15d^4]$

$=(7d^3/6) \times d / [\pi/64(15d)^4]$

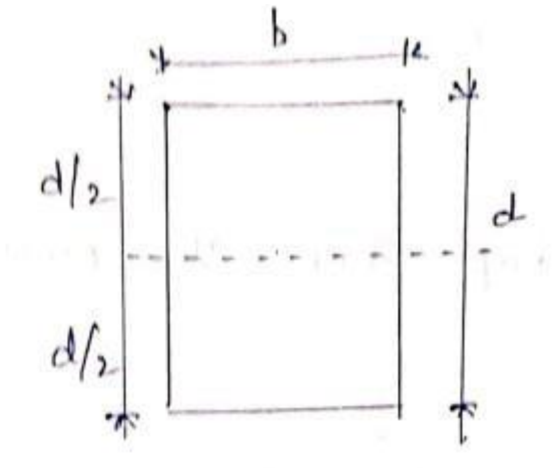
$=7 d^4/6 \times 64/15\pi d^4$

$=64 \times 7 / 6 \times 15 \times \pi$

S $=1.584$

Example :

Find shape factor for rectangular section



Solution

Shape factor

S =ZP/Z

ZP =Plastic modulus of section

=A/2(Ŷ1+Ŷ2)

=bd/2(d/4 +d/4)

=bd/2(1/2 d)

$=bd^2/4$

Z =I/Y

I $=bd^3/12$

Y =d/2

Z $=bd^3/12 \times 2/d$

$=bd^2/6$

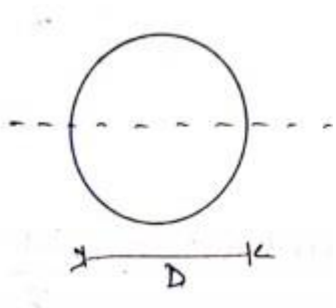
S $=(bd^2/4)/(bd^2/6)$

$=bd^2/4 \times 6/bd^2$

S $=1.5$

Example :

Find shape factor of circular section



Solution

Shape factor

S $=ZP/Z$

ZP =Plastic modulus of section

ZP $= A/2(\hat{Y}1+\hat{Y}2)$

$=\Pi d^2/4]/2\ [2D/3\pi + 2D/3\pi]$

$=\Pi d^2/8]\ [2D/3\pi + 2D/3\pi]$

$=\Pi d^2/8]\ [4D/3\pi\ ]$

$=D^3/6$

Z $=I/Y$

$=[\Pi d^4/64]/\ [D/2]$

$=\Pi d^4/64 \times 2/D$

$=\Pi d^3/32$

Shape factor

S $=z_p/z$

$=[D^3/6]/[\Pi d^3/32]$

S $=1.69$

LOAD FACTOR

Load factor

The ratio of the collapse load to the working load is termed as load factor. It is represented by 'F; and is similar to factor of safety in the elastic design. The load factor is expressed as

$$F = Pu / Pw$$

Where,

Pu – Collapse load

Pw – Working load

UPPER AND LOWER BOUND THEOREMS

Different types of plastic theorems

The various type of plastic theorems, depending on which, the plastic analysis has been developed are,

i. Lower bound theorem
ii. Upper bound theorem
iii. Uniqueness theorem

Lower bound theorem

This theorem is also known as "statically theorem" or "safe theorem". It is the maximum principle which demonstrates that of all the equilibrium states which are satisfying the conditions. The plastic moment does not exceed and gives a load, which is less than or equal to the collapse load. Hence the value of loads which are obtained are always less than or equal to collapse load which is expressed as

$W \le W_c$

Where

W - set of loads

Wc - collapse load

Upper bound theorem

This theorem is also known as "kinematic theorem" which states that a given structure, which is subjected to a set of loads (W) and the value of "W", corresponding to any mechanism must be greater than or equal to the collapse load. Which is expressed as

$W \ge W_c$

where ,

W - set of loads

Wc - collapse load

According to kinematic theorem, the correct collapse mechanism requires the minimum load among all the mechanism formed by assuming hinge positions . Thus, the collapse load can be estimated from any of the hinge positions, which is greater than or equal to correct one.

Uniqueness Theorem

The uniqueness theorem is formed by combining the static and kinematic theorems which gives a unique value for collapse load. Hence by static theorem which gives a unique value for collapse load. Hence, by static theorem. It is not possible to figure out a bending moment which is not possible to establish any mechanism for which the corresponding loads is less than Wc. When these two theorems are combined they form a unique value of load which is equal to Wc. According to the uniqueness theorem, the bending moment distribution which, satisfies the following conditions are unique.

a. A mechanism is produced, when the plastic hinge are formed at an adequate number of points
b. The bending moments are supposed to be in balance with the applied load
c. The allowable plastic moment must not exceed at any point in the structure.