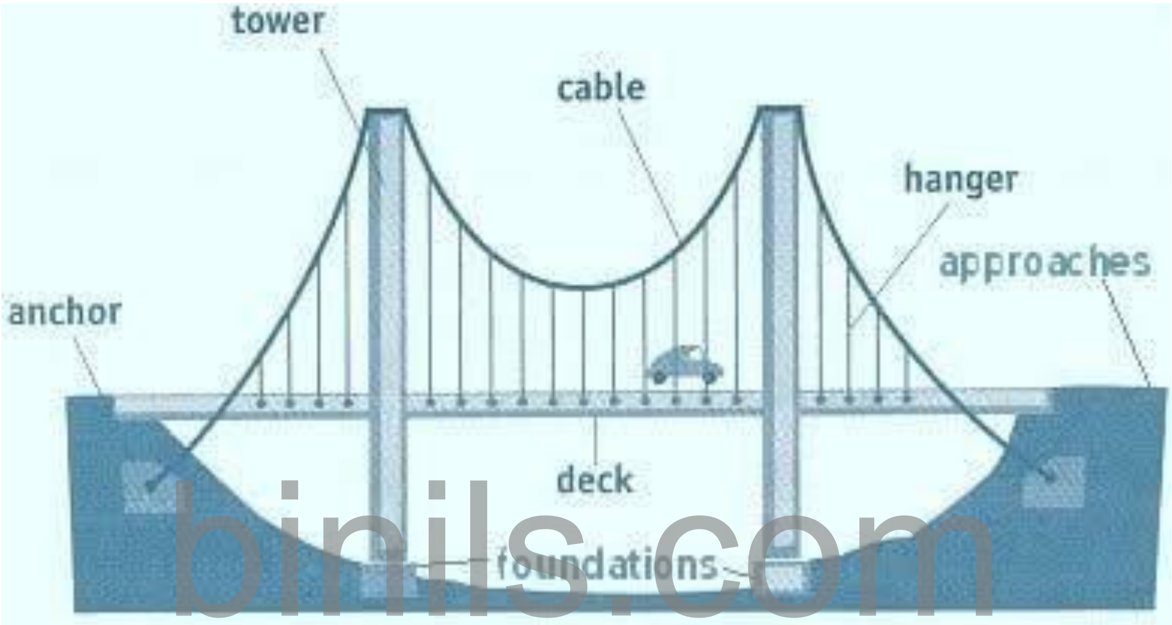
EQUILIBRIUM OF CABLE

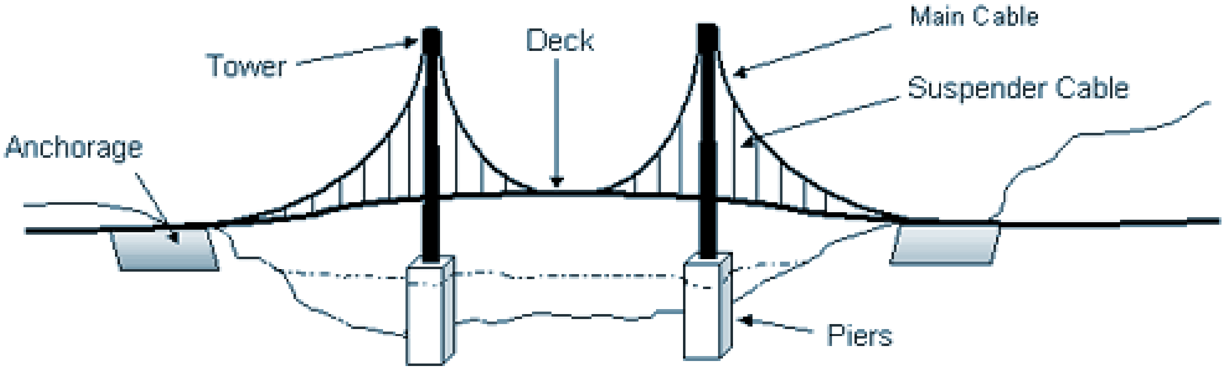
Cable structures

Long span structures subjected to tension and uses suspension cables for supports. Examples of cable structures are suspension bridges, cable stayed roof.



True shape of cable structures

Cable structures especially the cable of a suspension bridge is in the form of a catenary. Catenary is the shape assumed by a string / cable freely suspended between two points.

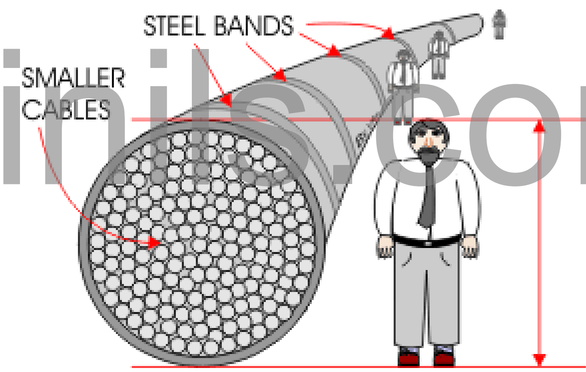


Catenary

Catenary is the shape taken up by a cable or rope freely suspended between two supports and under its own self weight.

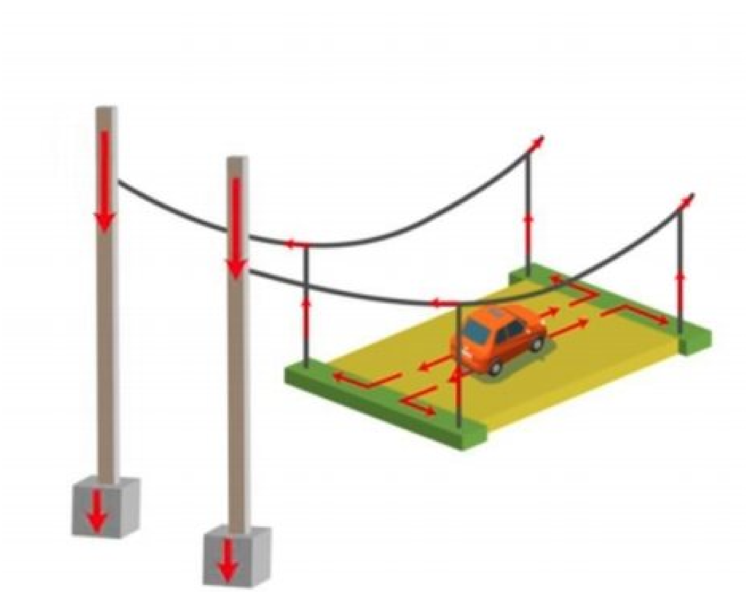
Cables made of

Cables can be of mild steel, high strength steel, stainless steel, or polyester fibres. Structural cables are made of a series of small strands twisted or bound together to form a much larger cable. Steel cables are either spiral strand, where circular rods are twisted together or locked coil strand, where individual interlocking steel strands form the cable (often with a spiral strand core)



Nature of force in the cables

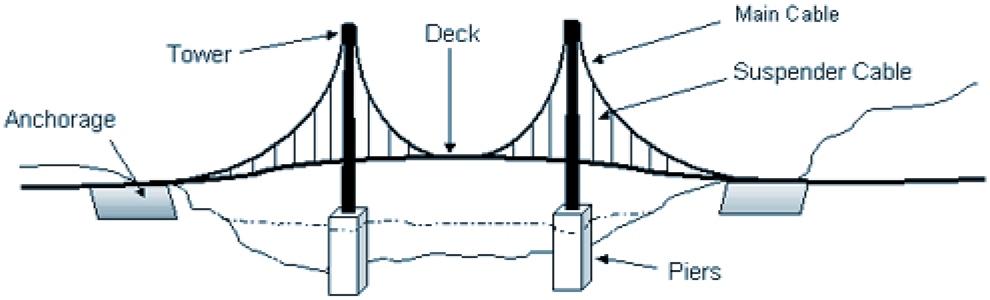
Cables of cable structures have only tension and no compression or bending.



LENGTH OF CABLE

Cable

The cables are flexible structures which carry loads in tension only. The cables vary vertical loads and are suspended between the supports.



Range of central dip of a cable

The central dip of a cable ranges from 1/10 to 1/15 of the span.

Assumptions made in the analysis of cables

a. Cable is considered to be stable and flexible.
b. When external loads act on the cable, self weight of cable is not considered.
c. The length of cable is always constant and therefore it is assumed as a rigid body.
d. The force in the cable is tangential to the cable profile as it carries only axial tensile forces.
e. The load acting on the cable is assumed to be uniformly distributed even though if it is moving load.

Simple suspension bridge

Suspension bridge has got two cables which are stretched over the span. Each cable run over two towers and is anchored by anchor to have a firm foundation. Cable is flexible throughout the span and its bending moment at every point is taken as zero. The load transferred by hangers or suspenders are assumed to be UDL. When the span is more than 200mts for a road way and 300mts for light way traffic suspension bridge is preferred.

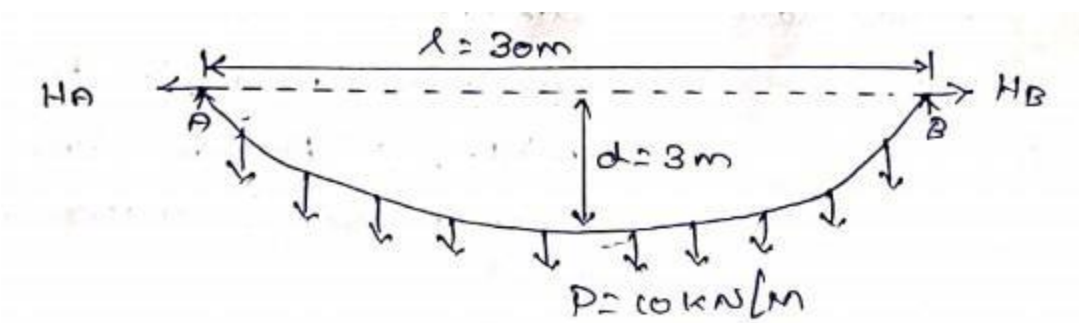
Stresses in suspended wires due to self weight

The dip is very small in suspended wire occurring at the centre. If ‘w’ is considered o be the weight of wire per unit length, then the horizontal tension in the wire given by,

$$H = wl^2 / 8d$$

Example :

A suspension cable having support at same level, has a span of 30m and a maximum dip of 3m. The cable is loaded with a UDL of 10KN/m throughout its length. Find the maximum tension in cable



Given data

Span ‘l’ = 30m

Dip ‘d’ = 30

UDL ‘P’ = 10 KN/m

To find

Max Tension in cable

Solution

Max Tension in cable

Tmax = $\sqrt{(VA^2 + H^2}$

VA = VB = $\frac{PL}{2}$

Find Vertical Reaction

VA =VB = P l / 2

= (10× 30) / 2

=150KN

Horizontal pull in the cable

H = $pl^2/8d$

= $10\times30^2/8\times3$

= 375 KN

Find Max tension in cable

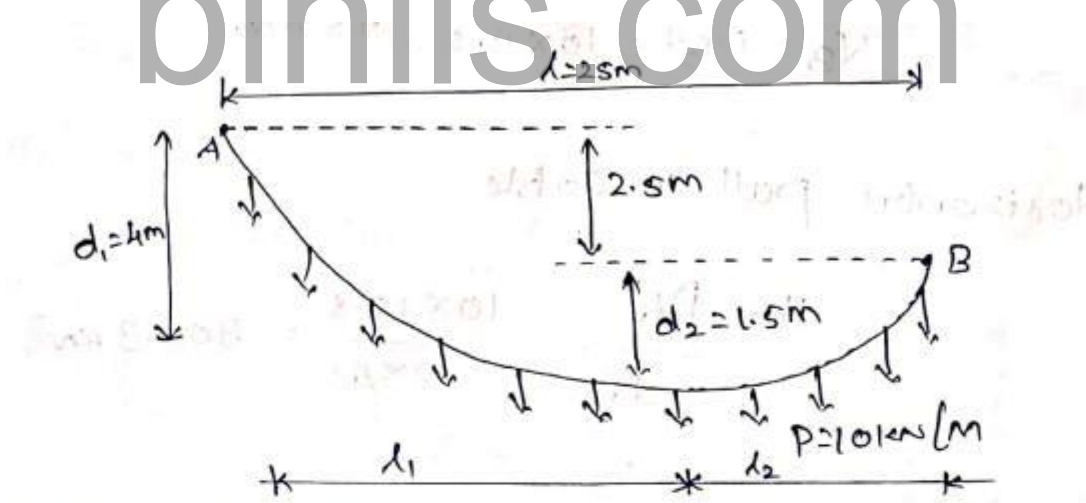
$T_{max} = \sqrt{VA^2 + H^2}$

$= \sqrt{150^2+375^2}$

$= 403.88KN$

Example :

A suspension cable is supported at two panel 25m apart the left support is 2.5m above the right support. The cable is loaded with a uniformly distributed load by 10KN/m throughout the span. The max dip in cable from the left support is 4m. Find maximum and minimum tension in cable.



Given data

UDL = 10KN/m

d1 = 4m

To find

Maximum and Minimum Tension in cable

SOLUTION:

Max Tension in cable

VA = p l1

VB = p l2

L = l1 +l2

Tmax = $\sqrt{(VA^2 + H^2}$

Find the length

L1 and L2

l1 / l2 = √ d1 / d2

l1 = √ (4 / 1.5) × l2

l1 =1.63 x l2

L = l1 + l2

25 = 1.63 l2 + l2

25 = 2.63 x l2

l2 = 9.5m

l1 $= L - l2$

$= 25 - 9.5$

$= 15.5$ m

Find Vertical Reaction

VA $= P\ l1$

$= 10\times 15.5$

$= 155$ KN

VB $= P\ l2$

$= 10\times 9.5$

$= 95$ KN

Horizontal pull in cable

H $= P\ l1^2 / 2\ d1$

$= 10\times 15.5^2 / 2\times 4$

$= 300.3$ KN

H $= P\ l2^2 / 2\ d2$

$= 10\times 9.495^2 / 2\times 1.5$

$= 300.5$KN

Find Tension in cable

TA $= \sqrt{VA^2+H^2}$

$= \sqrt{(155^2+300^2}$

$=$ 377.9 KN

TB $= \sqrt{VB^2 + H^2}$

$= \sqrt{(300.3^2+95^2}$

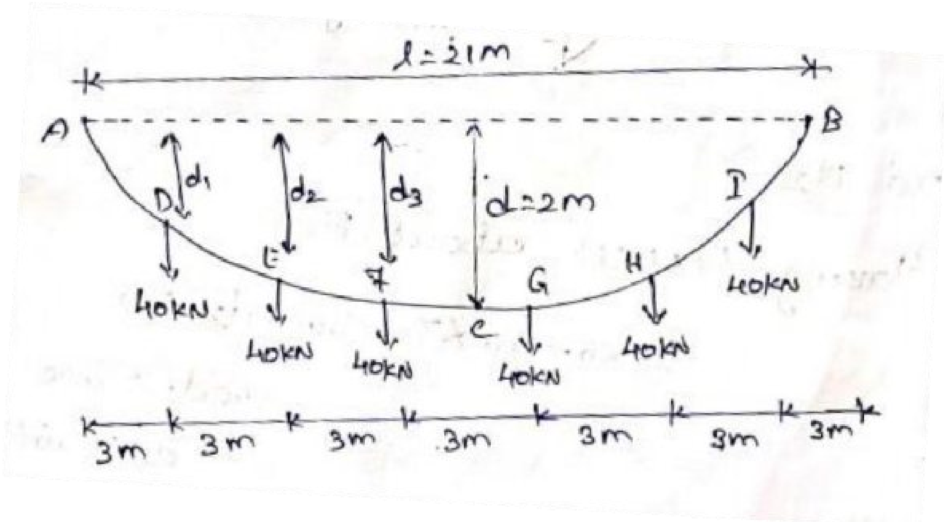
=314.96 KN

Max Tension = 337.9KN

Min Tension = 300.3 KN

Example :

A cable of horizontal span 21m is to be used to support six equal loads of 40KN each at 3m spacing the central dip of the cable is limited to 2m. Find the length of the cable required and also its sectional area if the safe tensile stress is 750N/mm^2



Given:

span = 21m

dip 'd' = 2m

stress =750 N/mm^2

To find :

a. length of cable

b. sectional area

Solution:

Vertical reaction

VA =VB

= total load / 2

= 6×40 / 2

=120KN

Horizontal Pull

Taking moment about C

$VA \times 10.5 - 40\times7.5 - 40\times4.5 - 40\times1.5 - H\times2 = 0$

$120\times10.5 - 540 - 2H = 0$

$720 = 2H$

$H = 360KN$

Find d1

Taking moment about D

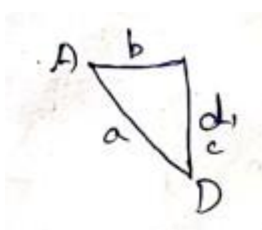
$120\times3 - 360\times d1 = 0$

$d1 = 1m$

$AD = \sqrt{b^2+C^2}$

$= \sqrt{3^2+1^2}$

$= 3.16$ m



Find d2

Taking moment about E

$120\times6 - 40\times3 - 360\times d2 = 0$

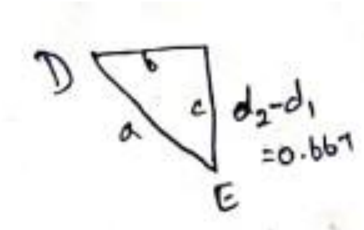
$360d2 = 600$

$d2 = 1.667$ m

DE $= \sqrt{b^2+c^2}$

$= \sqrt{3^2+0.667^2}$

$= 3.073$ m



Find d3

Taking moment about F

$120 \times 9 - 40\times6 - 40\times3 - 360\times d3 = 0$

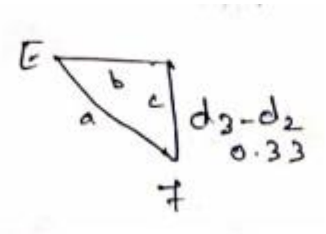
$720 - 360d3 = 0$

$d3 = 2$m

EF $= \sqrt{b^2+c^2}$

$= \sqrt{3^2+0.33^2}$

$= 3.018$ m



Find length of cable

length of cable =2(AD+DE+EF+FC)

=2(3.162+3.073+3.018+1.5)

=21.506m

Max Tension in cable

T_{max} = $\sqrt{V^2+H^2}$

$=\sqrt{120^2+360^2}$

=379.67 KN

Find Area

Stress $=T_{max} / A$

750 $=379.47\times10^3/A$

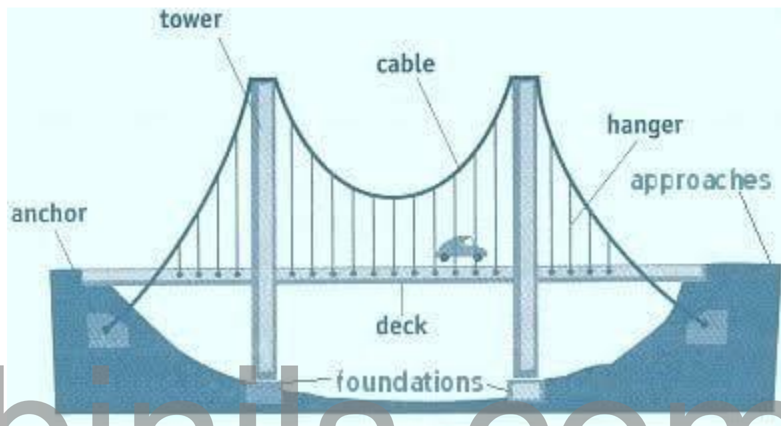
A $=0.505\ m^2$

A $=505\ mm^2$

ANCHORAGE OF SUSPENSION CABLES

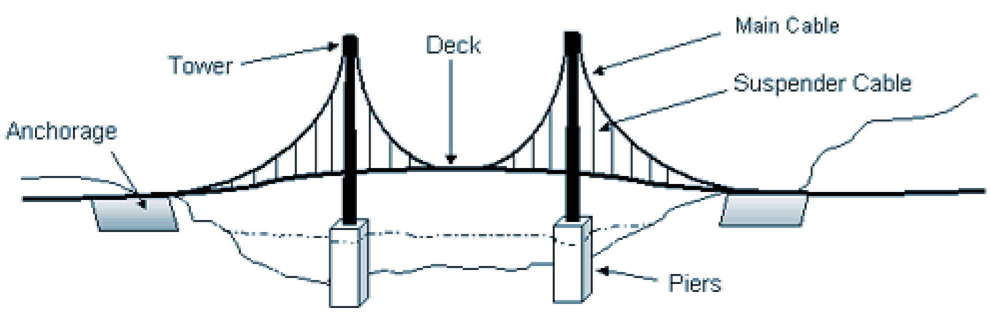
Anchor cable

The cable tension in suspension cable are of the order of several hundred tones and due to this reason, the anchoring of the suspension cable becomes a perplex task. The suspension cable are to be anchored to the bed rock, after they have been passed over the tall pylons.



Supporting towers

The supporting towers are basically designed for strength, stability and for architectural value of the structure. It provides foundation and glory to the bridge. The suspension cable is supported on the towers on its either sides and height of the tower is about 20 to 200m. For passing the suspension cable on the either side of the tower a saddle placed on rollers or a guide pulley is provided over the towers.

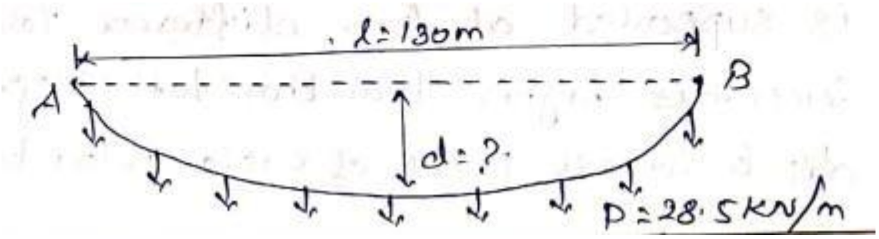


Suspenders

The suspenders are provided to transfer the traffic load on the deck slab to the suspension cable as a UDL. These suspenders are closely spaced.

Example :

A suspension cable of 130m horizontal span is supported at the same level it is subjected at to a uniformly distributed load of 28.5 KN/horizontal meter it the max tension in the cable is limited to 5000KN.calculated central dip needed.



Given:

span = 130m

UDL = 28.5KN/m

T_{max} = 5000KN

To find :

The central dip of cable

solution :

Vertical Reaction

VA =VB

$= P\,l / 2$

$= 28.5 \times 130 / 2$

$= 1852.5 KN$

Horizontal pull (tension)

$H = p\,l^2 / 8\,d$

$= 28.5 \times 130^2 / 8d$

$= 60206.25 / d$ KN

Max Tension

$T_{max} = \sqrt{V^2 + H^2}$

$5000 = \sqrt{185.5^2 + (60206.25/d)^2}$

$5000^2 = 1852.5^2 + 60206.25^2 / d^2$

$(5000^2 - 1852.5^2) / 60206.25^2 = 1/d^2$

$5.95 \times 10^{-3} = 1/d^2$

$d^2 = 1 / 5.95 \times 10^{-3}$

$d^2 = 168$

$d = \sqrt{168}$

$= 12.96m$

Example :

The suspension cable of horizontal span 95m is supported at two different level the right support is higher than left support by 4m. The dip to lowest point of cable below the left support 5m the cross sectional area of the cable is 3500mm^2. Find the uniformly distributed load that can be carried by the cable if the max stress is limited to 600N/mm^2

Given:

span 'l' =95m

d1 =5m

stress =600N/mm^2

A = 3500mm^2

To find :

Uniformly Distributed Load

SOLUTION:

Find Uniformly distributed load

VA = p l1

VB = p l2

L = l1 +l2

stress = T_{max} / A

Tmax = $\sqrt{(VA^2 + H^2}$

H = $\frac{Pl1^2}{2d1}$

H = $\frac{Pl2^2}{2d2}$

Find the length l_1 and l_2

l_1 / l_2 = √d1/d2

l_1 / l_2 = √5/9

l1 = 0.745 l2

L = l1+l2

95 =0.745 l2+ l2

95 = 1.745 l2

l_2 = 54.4 m

$l_1 = L - l2$

$= 95 - 54.4$

$= 40.56$ m

Vertical Reaction

$VA = p\, l_1$

$= 40.56$ p N

$VB = p\, l_2$

$= 54.4$ P N

Horizontal Pull

$H = pl_2^2 / 2d_2$

$= p(54.4)^2 / 2\times9$

$H = 164.4$ P N

Max tension will occur act right support

$VB>VA \qquad TB>TA$

$T_{max} = TB$

$=\sqrt{VB^2 + H^2}$

$=\sqrt{(54.4P)^2+(164.65P)^2}$

$= \sqrt{2963.71}\sqrt{p^2}+\sqrt{27109.62}\sqrt{p^2}$

T_{max} $=173.4$ p N

stress $=T_{max} / A$

600 N/mm^2 $=173.4$ p / 3500 mm^2

P = 12110 N/m

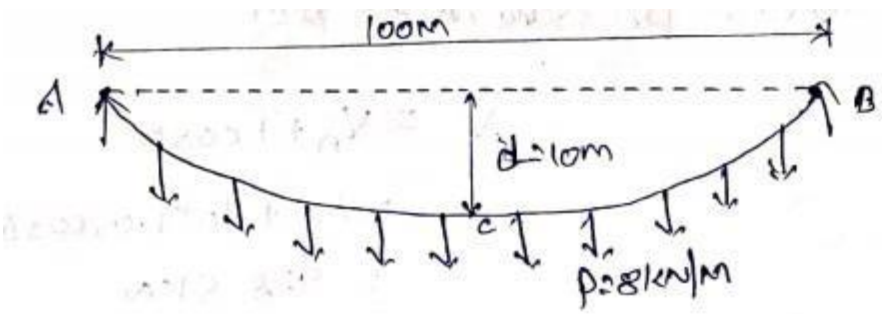
= 12.11KN/m

Example :

A suspension cable of span 100m and dip 10m carries a uniformly distributed load of 8KN/m of horizontal span over the full span. Find the vertical and horizontal forces transmitted to the supporting pylons.

a) If the cable is passed over a smooth pulley

b)If the cable is clamped to a saddle with roller the top of piers the anchor cable is make 30° the horizontal at a pylon



Given:

span l = 100m

dip d = 10 m

P = 8KN/m

Ø =30°

To find :

(i) vertical and horizontal forces

a) If the cable is passed over a smooth pulley

b)If the cable is clamped to a saddle with roller

Solution:

Vertical reaction

VA =VB

= Pl/2

= 8×100/2

=400 KN

Horizontal Pull

H $=Pl^2/8d$

$=8\times100^2/8\times10$

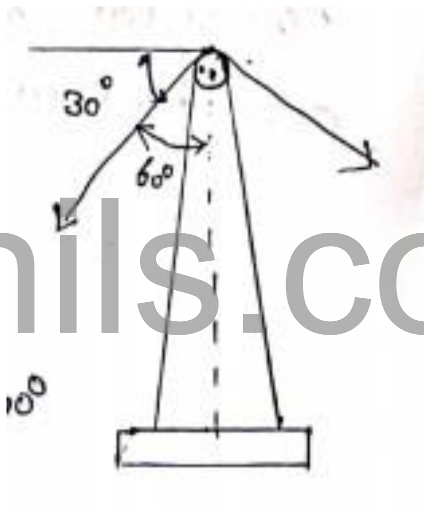
$= 1000$ KN

Tension in cable

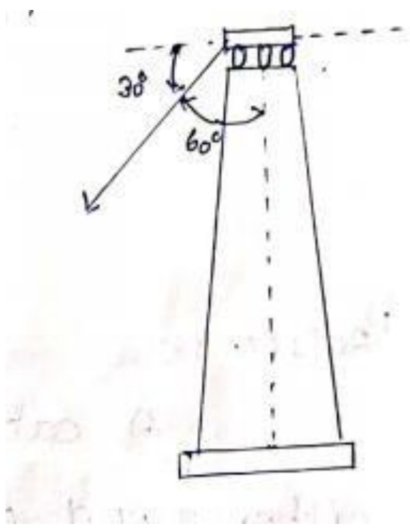
$T = \sqrt{V^2+H^2}$

$= \sqrt{400^2+1000^2}$

$T = 1077.03$ KN



a) Anchor cable passing over pulley



b) cable passing over saddle support

a) Anchor cable passing over pulley

vertical pressure on pier

V =VA + Tcos Ø

=400+1077.03 cos60°

=938.5 KN

Horizontal force at top of pylon

= H-T sin

= 1000-1077 sin60°

= 67.29 KN

b) cable passing over saddle support

$T_1 = H/\sin 60°$

$=1000/\sin 60°$

$=1154.7$ KN

Vertical pressure

$=V+T_1 \cos 60°$

$=977.35$ KN

Example :

A suspension cable of horizontal span 210mm is supported at the same level and has a central dip of 20mm. Find the increase in dip of the cable if the cable is subjected to a rise in temperature of 28°c. Take $\alpha=12\times10^{-6}$ per ° c.

Given data

Span ‘l’ = 210m

Dip ‘d’ = 20

‘t’ = 28°c

‘α’ $=12\times10^{-6}$ per°c

To find :

Increase in dip of the cable

Solution

Change in dip

$$\Delta d = \frac{3l^2}{16d} \propto t$$

$$\frac{3\times 210^2}{16\times 20}\times 12\times 10^{-6}\times 28$$

= 0.138m

= 138mm

Example :

A cable supported at the same level on either end is of 140m horizontal span with a central dip of 14 mm. It carries a load of 15KN/m on the horizontal span. Calculate the change in the horizontal tension when the temperature rises through 28° c. Co-efficient of linear expansion of the cable materials. $\alpha=4\times10^{-6}/^{o}c$.

Given data

SPAN 'L' = 140m

'd' = 14m

'P' =15KN/M

't' =28°c

'α' =$4\times10^{-6}/°c$

To find :

Change in the horizontal tension

SOLUTION:

$$H = \frac{pl^2}{8d}$$

$$H = \frac{15\times140^2}{8\times14}$$

$$= 2625 \text{ KN}$$

Change in horizontal tension

$$\Delta h = \frac{3l^2}{16d^2} \propto tH$$

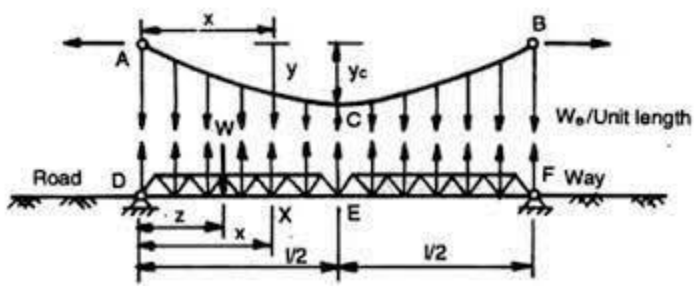
$$= \frac{3\times140^2}{16\times14^2}\times4\times10^{-6}\times28\times2625$$

$$= -5.513$$

STIFFENING GIRDERS

Girder

Stiffening girder are the major load bearing members in suspension bridges. As they are flexible, they change their shape with the nature and position of the moving live load on deck slab.



Functions of stiffening girder

a. They help in keeping the cables in shape
b. They resist part of shear force and bending moment due to live loads.
c. The cables take directly the dead load of the girder.
d. The dead load of the girder does not cause any shear force o bending moment in the girder.
e. The stiffening girder are subjected to shear force and bending moment due to live load and they should resist them safely.
f. Stiffening girder allow the suspension bridge deck to remain in its actual position even after the application of load.

Types of stiffening girders.

a. Two hinged stiffening girder
b. Three hinged stiffening girder

Two hinged stiffening girder

a. These are used to decrease the sag under the rolling load.
b. Suspension cable bridges are stiffened with two hinged stiffening girder to make them stiff.
c. These structures are statically indeterminate and by using energy methods, the forces in the cable may be obtained.
d. When the girder is assumed to be rigid, the load at any position is transferred in the form of UDL.

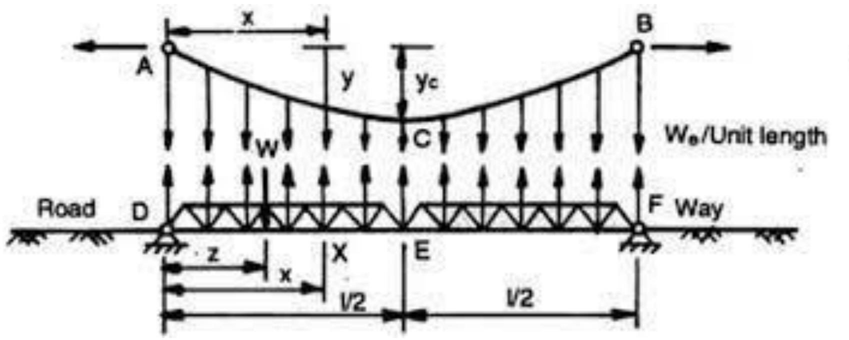
Three hinged stiffening girder

a. If the bridge is stiffened with three hinged stiffening girder, it maintains its parabolic shape during the movement off loads over the bridge.
b. If moving the loads are involved, then the cable are assumed to carry uniform load and hence the stiffening girder will be subjected to bending moment and shear force.

CABLES WITH THREE HINGED STIFFENING GIRDERS

Girder

Stiffening girder are the major load bearing members in suspension bridges. As they are flexible, they change their shape with the nature and position of the moving live load on deck slab.



Tension coefficient of a truss member

The tension coefficient for a member of a truss is defined as the pull or tension in the member divided by its length, i. e. the force in the member per unit length

Forces developed in beams curved in plan

Beams curved in plan will have the following forces developed in them:

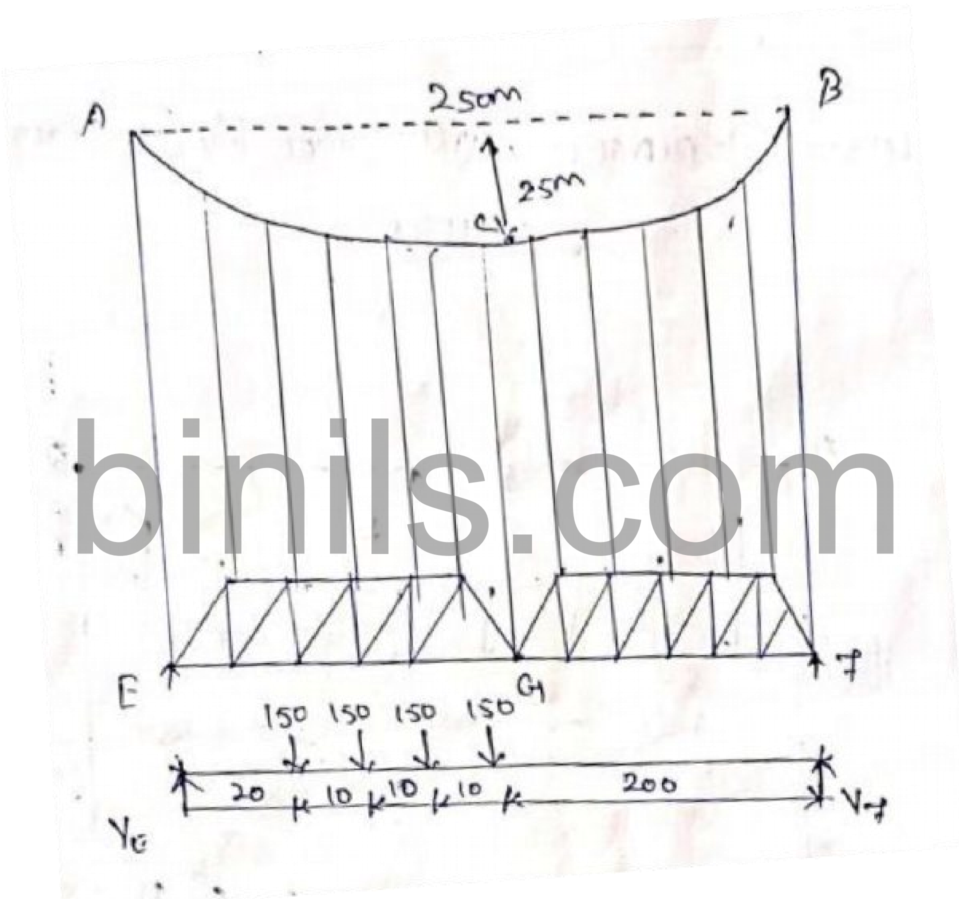
a. Shear forces
b. Torsional moments

Types of significant cable structures

Linear structures, Suspension bridges ,Cable-stayed beams or trusses , Cable trusses ,Straight tensioned cables ,Three-dimensional structures , 3D cable trusses

Example :

A suspension bridge of 250m span has two nos. of three hinged stiffening girders supported by cables with a central dip of 25m. If 4 point loads of 300KN each are placed at the centre line of the road way at 20m, 30m, 40m, 50m from the left hand hinge. Find the shear force and bending moment in each girder at 62.5m from each end. Calculate the max tension in the cable



Given:

span = 250m

'd' = 25m

To find :

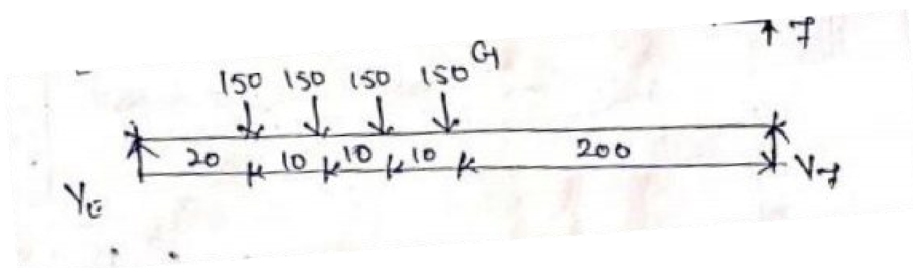
(i) Shear force and bending moment in each girder at 62.5m from each end

(ii) Max tension

solution :

Find v_E of v_f

Taking moment about "f"



$= \quad v_E \times 250 - 150 \times 200 - 150 \times 210 - 150 \times 220 - 150 \times 230 = 0$

= 516 KN

Total load = $V_E + V_F$

600 = 516 + VF

V_F = 84KN

Horizontal pull

H = μ_c / d

μ_c = Vf x c

$= \dfrac{V_F \times 125}{25}$

$= \frac{84 \times 125}{25}$

= 420KN

a) Bending Moment

BM @ 62.5m from left hinge

$= V_F \times 187.5 - H \times y$

$y = \frac{4d}{l^2} X(l - X)$

$= \frac{4 \times 25}{250^2} \times 62.5 \times 187.5$

= 18.75m

BM@ 62.5

= 84×187.5-18.75×420

= 787.5KNM

BM@62.5 from right hand Hinge

= VF × 62.5 – H × y

= 84 × 62.5 –420 ×18.75

= - 2625KNM

b) Shear force

SF @62.5 from left hand hinge

$V = V_b - \tan\theta\, H$

$$\tan\theta = \frac{4d}{l^2}(l - 2X)$$

$$= \frac{4\times 25}{250^2}(250 - 2\times 62.5) = 0.2$$

$$\tan\theta = 0.2$$

V_b = V_E - 4×150

= 516 -600

= - 84 KN

V_b = V_F

= - 84 KN

$V_{62.5}$ = -84 – 420×0.2

= - 168KN

SF @62.5 from right side

$V_{187.5}$ = - V + H tanθ

= - 84 + 420 × 0.2

= 0

c) Vertical pull on the cable

$$H = \frac{Pl^2}{8d}$$

$$420 = \frac{p\times 250^2}{8\times 25}$$

p = 1.34 KN/m

d) Max tension in cable

$$T = \sqrt{X_n^2 + H^2}$$

VA = VB

$$= \frac{PL^2}{2}$$

$$= \frac{1.34 \times 250}{2} = 168KN$$

$$T = \sqrt{168^2 + 420^2} = 452.35KN$$

Example :

Determine the span of steel parabolic cable suspended between to supports at the same level. The limiting value of the central dip is 1/12th of the span and the permissible stress in the cable is 125 N/mm^2.

Given data

Central dip , d =l/12

Permissible stress, σ_p = 125 N/mm^2

Solution

Length of the cable is given by the expression

S = l + 8/3.d^2/l

= l + l^2/144 x l

= l + 8/432 l

$S = 55/54\ l$

Assume the density of steel ,

$\rho = 78 \times 10^{-6}$ N/ mm^2

W = Area x Length x Density

$= A \times 55/54 \times l \times 78 \times 10^{-6}$

$= 715/9\ Al \times 10^{-6}$

Horizontal force ,

$H = W \times l/8d$

$= 715/9\ Al \times l \times 10^{-6} / 8 \times (l/2)$

$\mathbf{= 715 / 6\ A.\ l \times 10^{-6}}$

Vertical reaction

$V = W/2$

$= 715/18\ .A\ l \times 10^{-6}$

Maximum tension ,

$T_{max} = \sqrt{V^2 + H^2}$

$= \sqrt{(715/18\ .Al \times 10^{-6}) + (715 / 6\ A.\ l \times 10^{-6})}$

$= Al \times 10^{-6}\sqrt{(715^2/18^2 + 715^2/6^2)}$

$= 125.613 \times 10^{-6} A.l$

$T_{max} = \sigma_p \times A$

$125.613 \times 10^{-6} A.l = 125 \times A$

$l = 125 / 125.613 \times 10^{-6}$

$l = 0.99512 \times 10^{6}$ mm

length of the cable,

$S = 55/54 \times 995.12$

$S = 1013.548$ m

Example :

A suspension bridge has a span 50m with a 15m wide runway. It is subjected to a load of 30KN/m including self weight .the bridge is supported by a pair of cables having a central dip of 4m.find the cross sectional area of the cable necessary if the maximum permissible stress is not to exceed 600 Mpa.

GIVEN

Span of bridge ,L =50m

Load on the bridge W =30 KN/m

Dip of the cables, d =4m

Permissible stresses = 600 Mpa

$=600 \text{ N/mm}^2$

To find

Cross sectional area

Solution

Horizontal reaction of tension

H $=w L^2 /8d$

$=30 \times 50 \times 50 /8 \times 4$

$=2343.75$ KN

Vertical reaction is given by ,

V $=w L/2$

$=30 \times 50 /2$

$=750$ KN

Maximum tension is

T $= \sqrt{V^2 + H^2}$

$=\sqrt{2343.75^2 + 750^2}$

$=2460.825$KN

Area of the cable is

A = T /permissible stress

$= 2460.825 \times 10^3 / 600 = 4.101 \times 10^3 \text{ mm}^2$

A $= 41.01 \text{ cm}^2$

Example :

A suspension bridge has a span 60m with a 15m wide runway. It is subjected to a load of 35KN/m including self weight .the bridge is supported by a pair of cables having a central dip of 6m.find the cross sectional area of the cable necessary if the maximum permissible stress is not to exceed 650 Mpa.

GIVEN

Span of bridge ,L =60m

Load on the bridge W =35 KN/m

Dip of the cables, d =6m

Permissible stresses = 650 Mpa

$=650 \text{ N/mm}^2$

To find

Cross sectional area

Solution

Horizontal reaction of tension

H $= w L^2 / 8d$

$= 35 \times 60 \times 60 / 8 \times 6$

=2625 KN

Vertical reaction is given by ,

V =w L/2

=35 x 60 /2

=1050 KN

Maximum tension is'

T $= \sqrt{V^2 + H^2}$

$=\sqrt{2625^2 + 1050^2}$

=2827.8 KN

Area of the cable is

A = T /permissible stress

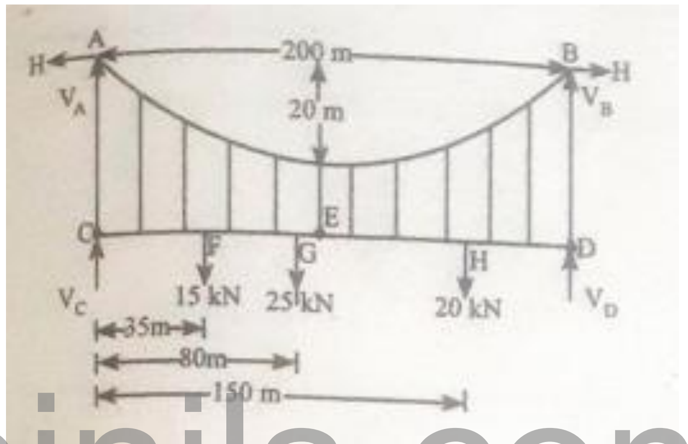
$= 2827.21 \times 10^3 / 650$

$= 4.349 \times 10^3$ mm^2

A = 43.49 cm^2

Example :

A three hinged suspension girder bridge has a span of 200m over the supports at same level. It has a central dip of 20m.the girder carries three point loads of 15KN, 25KN,and 20KN.acting at 35m,80m, and 150m respectively from the left end. Draw the B.M.D



GIVEN

Length of span l = 200m

Central dip (yc)=20 m

To find

Draw the BMD

Solution

i)SUPPORTS REACTION

$V_C + V_D$ =15+ 25+20

=620 KN

Taking moments at C,

V_D x 200 = 15 x 35 +25 x80 +20 x 150

V_D x 200 =5525

V_D = 5525 / 200

=27.63 KN

V_C = 60 – 27.63

=32.37 KN

Bending moment at F,

M_F = V_C x 35

=32.37 x 35

= 1132.95 KN.m

Moment about G,

M_G =V_C x80 – 15x 45

=32.37 x 80 – 15x45

=1914.6 KN.m

Moment about H

M_H = 32.37 x 150 – 15 x 115 – 25x 70

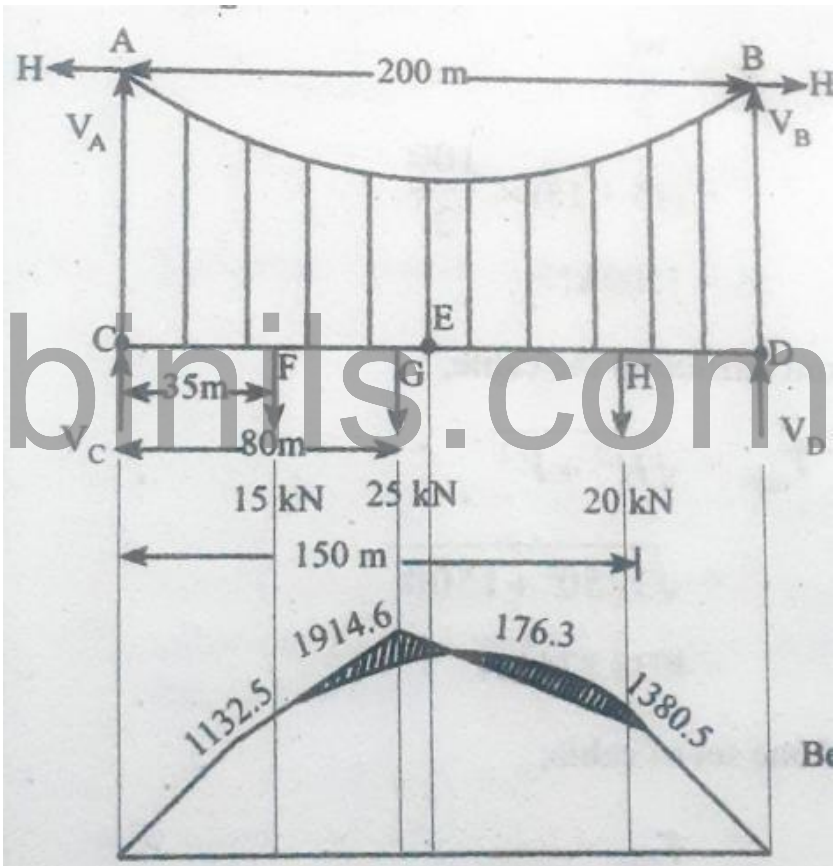
=1380.5 KN.m

Moment about E,

M_E = V_D x 100 – 20 x50

=27.63 x 100-20x50

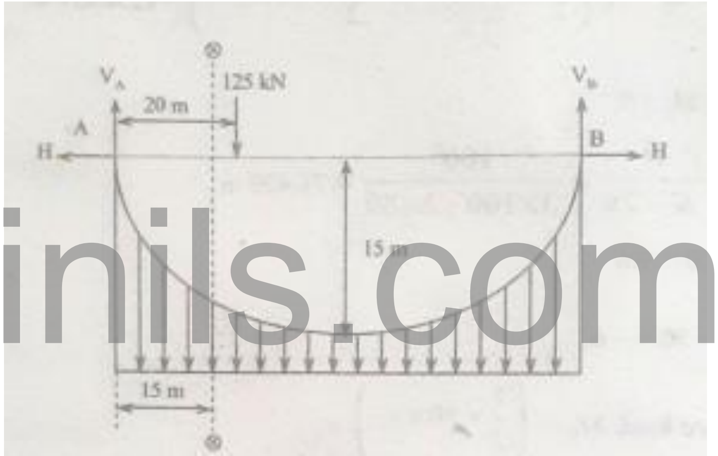
=1763 KN.m



INFLUENCE LINES FOR THREE HINGED STIFFENING GIRDERS

Example :

A suspension cable of 100m span and 15m dip is stiffened by a three hinged stiffening girder. It is subjected to a concentrated load of 125KN at 20m from the left end is addition to a dead load of 10KN/m .find the maximum tension in the cable and the shear force and the bending moment in the girder at 15m from the left.



GIVEN

Span of the cable =100m

Dip of cable =15m

To find

Maximum tension in the cable

Shear force and the bending moment in the girder at 15m from the left

Solution

Total load =125 +10 x 100

=1125KN

FLD for bending moment due to H

=wx(l – x)/l

= 1 x 15 x85

=12.75 KNm

Ordinate of BM under total load (165KN)

=12.75 x 20 /15 - 6.375

=17 – 6.375

=10.625 KN.m

Maximum + ve BM at given section is

BM_{MAX} = 1125 X 10.625

=11.953 KN.m

ILD for shear force

Ordinate of shear force for 165KN

= - 15/100 x 20 – (0.35)

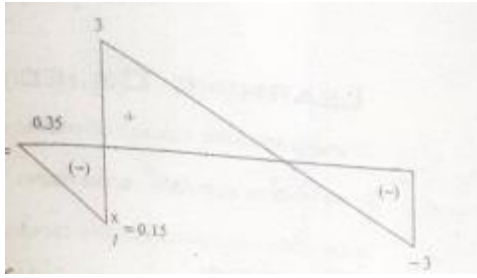
= - 3 – 0.35

= - 3.35 m

Net shear force

=1125 x (- 3.35)

= - 3768.75 KN



Maximum tension in the cable

The load acting over the beam are converted into equivalent UDL

W_e = 1125 / 100

=11.25 KN/m

Horizontal reaction

H =$W_e l^2$ /8.4c

=11.25 x 100^2 /8 x 15

=937.5 KN

Vertical reaction ,

V =W_e l /2

= 11.25 x 100 / 2

=562.5 KN

Maximum tension

T_{MAX} =$\sqrt{V^2 + H^2}$

=$\sqrt{562.5^2 + 937.5^2}$

=10930.351 KN

T_{MAX} = 1094KN