3.1 MOMENT DISTRIBUTION METHOD

Objectives:

Definition of stiffness, carry over factor, distribution factor. Analysis of continuous beams without support yielding – Analysis of continuous beams with support yielding – Analysis of portal frames – Naylor's method of cantilever moment distribution – Analysis of inclined frames – Analysis of Gable frames.

3.1.1 INTRODUCTION

The end moments of a redundant framed structure are determined by using the classical methods, viz. Clapeyron's theorem of three moments, strain energy method and slope deflection method. These methods of analysis require a solution of set of simultaneous equations. Solving equations is a laborious task if the unknown quantities are more than three in number. In such situations, the moment distribution method developed by Professor Hardy Cross is useful. This method is essentially balancing the moments at a joint or junction. It can be described as a method which gives solution by successive approximations of slope deflection equations.

In the moment distribution method, initially the structure is rigidly fixed at every joint or support. The fixed end moments are calculated for any loading under consideration. Subsequently, one joint at a time is then released. When the moment is released at the joint, the joint moment becomes unbalanced. The equilibrium at this joint is maintained by distributing the unbalanced moment. This joint is temporarily fixed again until all other joints have been released and restrained in the new position. This procedure of fixing the moment and releasing them is repeated several times until the desired accuracy is obtained. The experience of designers points that about five cycles of moment distribution lead to satisfactory converging results.

Basically, in the slope deflection method, the end moments are computed using the slopes and deflection at the ends. Contrarily in the moment distribution method, as a first step — the slopes at the ends are made zero. This is done by fixing the joints. Then with successive release and balancing the joint moments, the state of equilibrium is obtained. The release-balance cycles can be carried out using the following theorems.

In Conclusion,when a positive moment M is applied to the hinged end of a beam and a positive moment of M will be transformed to the fixed end.

3.1.2 IMPORTANT FACTORS

- Carry over moment carry over factor
- Relative stiffness or stiffness factor
- Distribution moment and distribution factor.

3.1.3 BASIC DEFINITIONS OF TERMS IN THE MOMENT DISTRIBUTION METHOD

(a) Stiffness

Rotational stiffness can be defined as the moment required to rotate through a unit angle (radian) without translation of either end.

(b) Stiffness Factor

(i) It is the moment that must be applied at one end of a constant section member (which is unyielding supports at both ends) to produce a unit rotation of that end when the other end is fixed, i.e. $k = 4EI/l$.

(ii) It is the moment required to rotate the near end of a prismatic member through a unit angle without translation, the far end being hinged is $k = 3EI/l$.

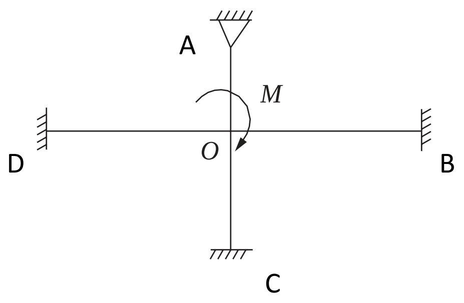
(c) Carry over factor

It is the ratio of induced moment to the applied moment (Theorem 1). The carry over factor is always (1/2) for members of constant moment of inertia (prismatic section). If the end is hinged/pin connected, the carry over factor is zero. It should be mentioned here that carry over factors values differ for non-prismatic members. For non-prismatic beams (beams with variable moment of inertia); the carry over factor is not half and is different for both ends.

Carry over factor = Carry over moment / Applied moment

(d) Distribution Factors

Consider a frame with members *OA*, *OB*, *OC* and *OD* rigidly connected at *O* as shown in Fig. 2.6. Let *M* be the applied moment at joint *O* in the clockwise direction. Let the joint rotate through an angle θ. The members *OA*,*OB*,*OC* and *OD* also rotate by the same angle θ.



3.1.4 SIGN CONVENTION

Clockwise moments are considered positive and anticlockwise moments negative.

3.1.5 BASIC PROCEDURES IN THE MOMENT DISTRIBUTION METHOD

- Assuming all the members as fixed at the both ends.
- Calculate fixed end moments due to external loads.
- At hinged supports (or) simply supported supports,release the points by applying equal and opposite moment.
- Calculate stiffness factor at each joint.
- Unbalanced moment at a point,is distributed to the adjacent spans according to their distribution factors.
- Proceed this process to get the required degree of precision.

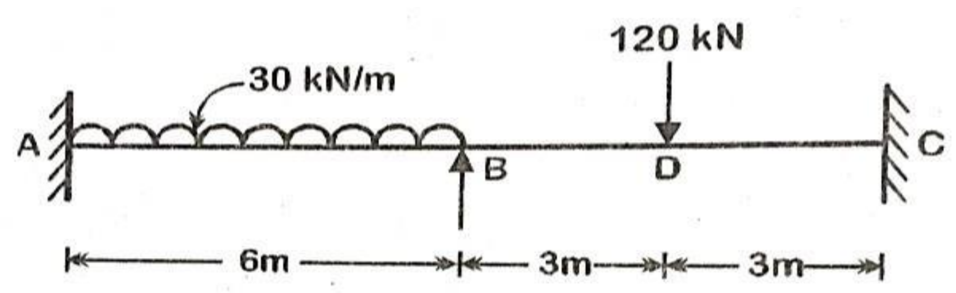
3.2 ANALYSIS OF CONTINUOUS BEAMS IN MOMENT DISTRIBUTION METHOD.

3.2.1 NUMERICAL EXAMPLES ON(CONTINUOUS BEAMS):

PROBLEM NO:01

For the continuous beam shown in figure,Calculate the support moments distribution method.Draw the SF and BM diagrams.

Solutions:



- **Fixed End Moments:**

MFAB = $-Wl^2/12 = -30x6^2/12 = -90$ kNm

MFBA = $Wl^2/12 = 30x6^2/12 = 90$ kNm

MFBC = -Wl/8 = -120X6/8 = -90 kNm

MFCB = Wl/8 = 120X6/8 = 90 kNm

- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	4EI/l = 4EI/6	4EI/3	0.5
	BC	4EI/l = 4EI/6		0.5

- **Reactions:**

Moment About B,

MB = RAx6 - 30x6x6/2 + MA

-90 = 6RA – 540 – 90

6RA = 540; RA = 90KN

Moment About B,

MB = RC x 6 – 120 x 3 + MC

-90 = 6RC – 360 – 90

6RC = 360; RC = 60KN

RB = Total load – (RA + RC) = 30 x 6 + 120 – (90 + 60)

= 300 – 150 = 150KN

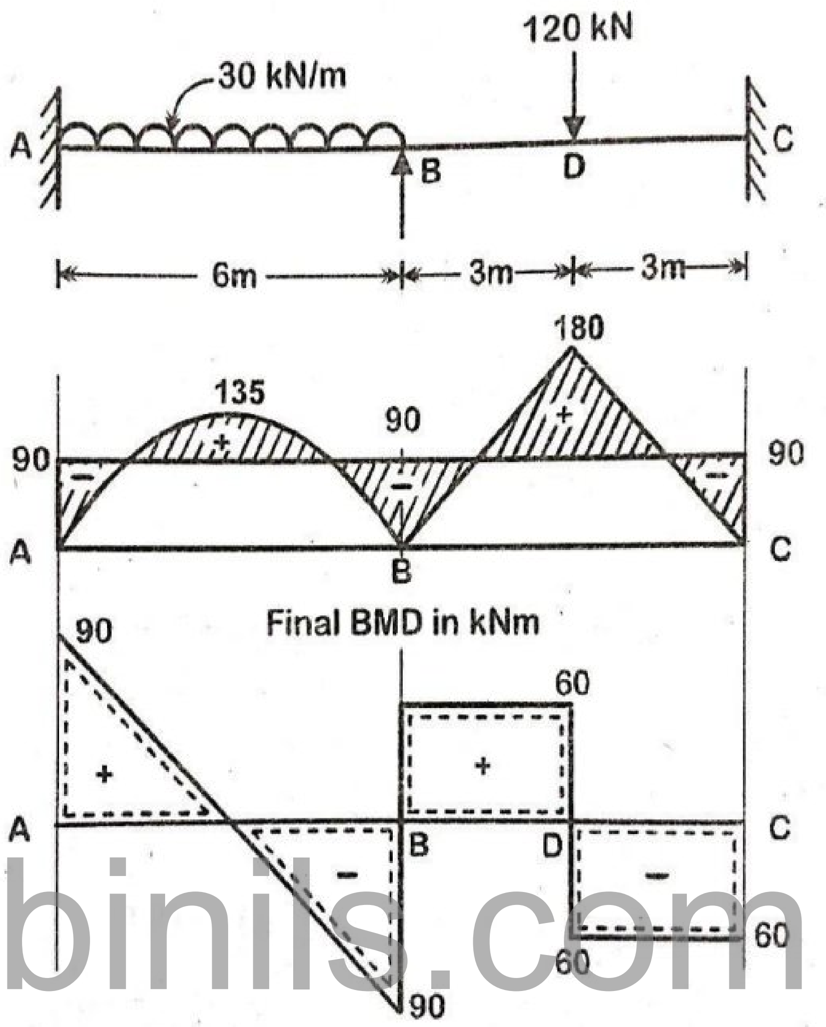
- **Moment Distribution Table**:

Joint	A	B		C
Member	AB	BA	BC	CB
D.F	-	0.5	0.5	-
F.E.M	-90	90	-90	90
Distribute	-	0	0	-
Final Moments	-90	90	-90	90
Conventional Moments	-90	-90	90	-90

- **Free BMD:**

MAB = $Wl^2/8 = 30 \times 6^2/8 = 135$kNm

MBC = Wl/4 = 120 x 6/4 = 180kNm



- **Final Bending Moments:**

MA = -90kNm; MB = -90kNm; MC = -90kNm

PROBLEM NO:02

For the continuous beam as shown in fig; Find the support moment carry out two cycles of distribution.

Solutions:

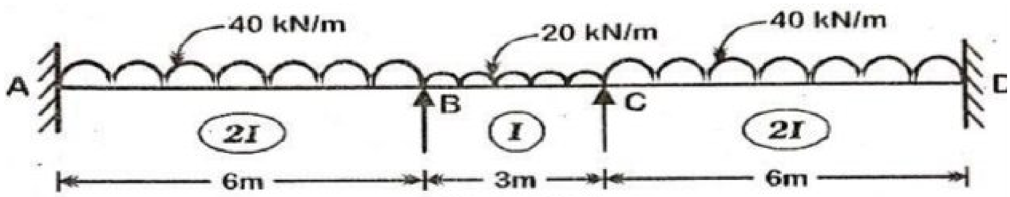


Fig. 3.9(a)

- **Fixed End Moments:**

MFAB = $-Wl^2/12 = -40x6^2/12 = -120$ kNm;

MFBA = $Wl^2/12 = 40x6^2/12 = 120$ kNm;

MFBC = $-Wl^2/12 = -20x3^2/12 = -15$ kNm;

MFBC = $Wl^2/12 = 20x3^2/12 = 15$ kNm;

MFCD = $-Wl^2/12 = -40x6^2/12 = -120$ kNm;

MFDC = $Wl^2/12 = 40x6^2/12 = 120$ kNm;

- **Distribution Factor Table:**

<table>
<tr><th>Joint</th><th>Member</th><th>k</th><th>Σk</th><th>Distribution factor (k/Σk)</th></tr>
<tr><td rowspan="2">B</td><td>BA</td><td>4E(2I)/l = 4EI/3</td><td rowspan="2">7EI/3</td><td>0.57</td></tr>
<tr><td>BC</td><td>3EI/l = 3EI/3</td><td>0.43</td></tr>
<tr><td rowspan="2">C</td><td>CB</td><td>3EI/l = 3EI/3</td><td rowspan="2">7EI/3</td><td>0.43</td></tr>
<tr><td>CD</td><td>4E(2I)/l = 4EI/3</td><td>0.57</td></tr>
</table>

- **Free BMD:**

MAB = $Wl^2/8 = 40 \times 6^2/8 = 180$kNm;

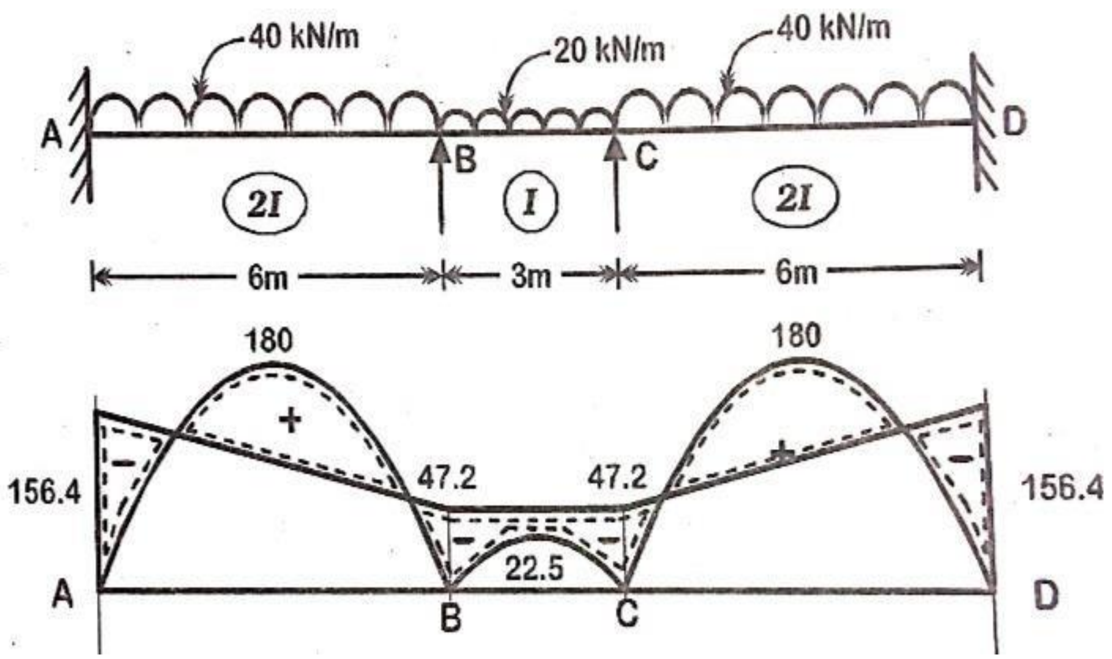
MBC = $Wl^2/8 = 20 \times 3^2/8 = 22$kNm;

MCD = $Wl^2/8 = 40 \times 6^2/8 = 180$kNm.

- **Moment Distribution Table:**

Joint	A	B		C		D
Member	AB	BA	CB	BC	CD	DC
D.F	-	0.57	0.43	0.43	0.57	-
F.E.M	-120	120	-15	15	-120	120
Distribute		-59.9	-45.1	45.1	59.9	
Carry over	-29.95		22.6	-22.6		29.95
Distribute		-12.9	-9.7	9.7	12.9	
Carry over	-6.45					6.45
Final Moments	-156.4	47.2	-47.2	47.2	-47.2	156.4
Conventional Moments	-156.4	-47.2	-47.2	-47.2	-47.2	-156.4

- **Moment Diagram:**



- **Final Moments:**

  MA = -156.40kNm; MB = -47.20kNm; MC = -156.40kNm

PROBLEM NO:03

A continuous beam ABCD, simply supported at A,B,C and D is loaded as shown in fig. EI is constsnt.

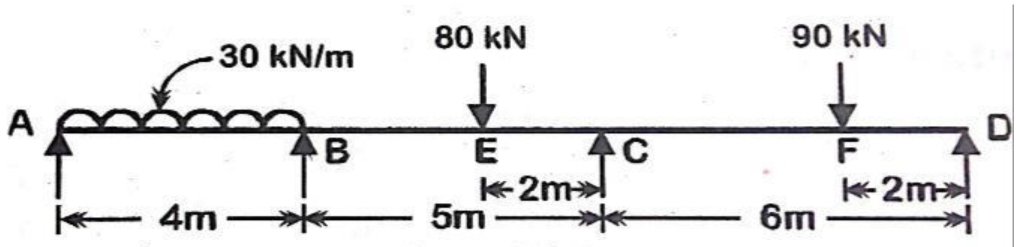


Fig. 3.11(a)

Solutions:

- **Fixed End Moments:**

  MFAB = $-Wl^2/12 = -30x4^2/12 = -40$ kNm;

  MFBA = $Wl^2/12 = 30x4^2/12 = 40$ kNm;

  MFBC = $-Wab^2/l^2 = -80x3x2^2/5^2 = -38.4$ kNm;

  MFBC = $Wa^2b/l^2 = 20x3^2x2/5^2 = 57.6$ kNm;

  MFCD = $-Wab^2/l^2 = -90x4x2^2/6^2 = -40$ kNm;

  MFDC = $Wa^2b/l^2 = 90x2^2x4/6^2 = 80$ kNm.

- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	3EI/l = 3EI/4	27EI/20	0.56
	BC	3EI/l = 3EI/5		0.44
C	CB	3EI/l = 3EI/5	11EI/10	0.5
	CD	3EI/l = 3EI/6		0.5

- **Free BMD:**

  MAB = $Wl^2/8$ = 30 x $4^2/8$ = 60kNm;

  MBC = Wab/l = 80 x 2 x 3/5 =

  96kNm;MCD = Wab/l = 90 x 4 x 2/5 =

  120kNm

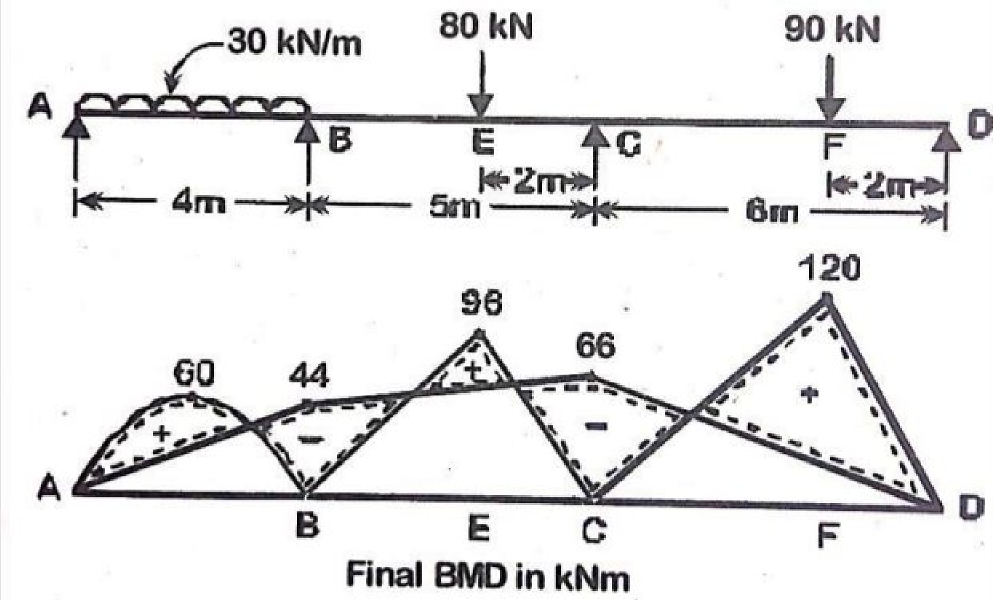
- **Moment Distribution Table:**

Joint	A	B		C		D
Member	AB	BA	CB	BC	CD	DC
D.F	-	0.56	0.44	0.5	0.5	-
F.E.M	-40	40	-38.4	57.6	-40	80
Release C & D	40					-80
carry over		20			-40	
Initial moments	0	60	-38.4	57.6	-80	0
Distribute		-12.096	-9.504	11.2	11.2	
Carry over		0	5.6	-4.552	0	
Distribute		-3.136	-2.464	2.376	2.376	
Carry over		0	1.188	-1.232	0	
Distribute		-0.66	-0.52	0.61	0.61	
Carry over			0.308	-0.26		
Final Moments	0	44	-44	66	-66	0
Conventional Moments	0	-44	-44	-66	-66	0

- **Final Moments:**

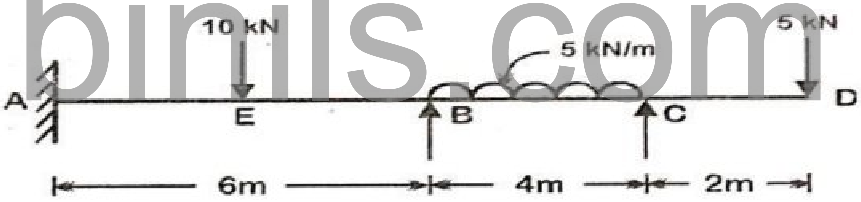
  MA = 0kNm; MB = -44kNm; MC = -66kNm; MD = 0kNm.

- **Final Moments Diagram:**



PROBLEM NO:04

Find the support moments for the continuous beam using moment distribution method. EI is constant.



Solutions:

- **Fixed End Moments:**

  MFAB = -Wl/8 = -10X6/8 = -7.5 kNm

  MFBA = Wl/8 = 10X6/8 = 7.5 kNm

  MFBC = $-Wl^2/12 = -5x4^2/12$ = -6.67 kNm

  MFCB = $Wl^2/12 = 5x4^2/12$ = 6.67 kNm

  MFCD = -Wxl= -5X2 = -10 kNm

- **Free BMD:**

  MAB = Wl/4 = 10 x 6/4 = 15kNm

  MBC = $Wl^2/8 = 5 \times 4^2/8$ = 10kNm

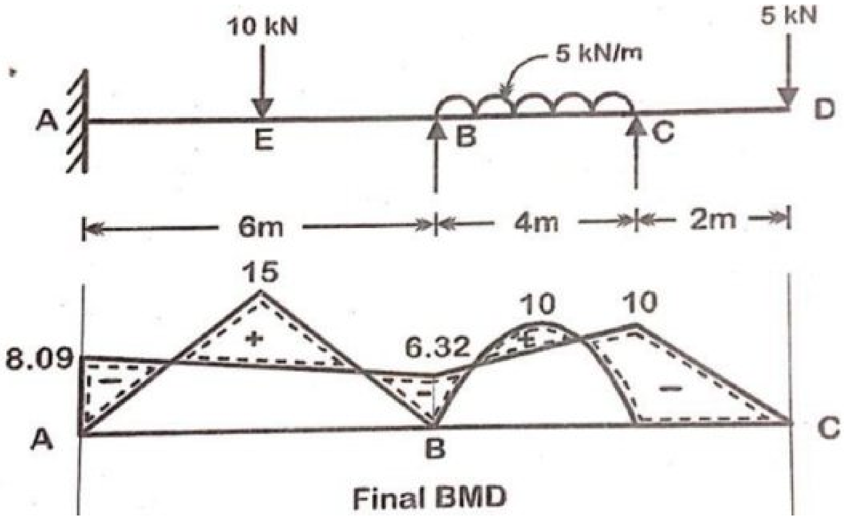
- **Distribution factor table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	4EI/l = 4EI/6	17EI/12	0.47
	BC	3EI/l = 3EI/4		0.53
C	CB	4EI/l = 3EI/4	3EI/4	1
	CD	0		0

- **Moment Distribution table:**

Joint	A	B		C	
Member	AB	BA	BC	CB	CD
D.F	-	0.47	0.53	1	0
F.E.M	-7.5	7.5	-6.67	6.67	-10
Release C & carry over	-	-	1.67	3.33	0
Initial moments	-7.5	7.5	-5	10	-10
Distribute	-	-1.18	-1.32	-	-
Carry over	-0.59	-	-	-	-
Final Moments	-8.09	6.32	-6.32	10	-10
Conventional Moments	-8.09	-6.32	-6.32	-10	-10

- **Final Moments Diagram:**



- **Final Moments:**

MA = -8.09kNm;   MB = -6.32kNm;   MC = -10kNm

3.3. ANALYSIS OF PLANE RIGID FRAMES WITH AND WITHOUT SWAY IN MOMENT DISTRIBUTION METHOD.

3.3.1. INTRODUCTION

PLANE RIGID FRAMES

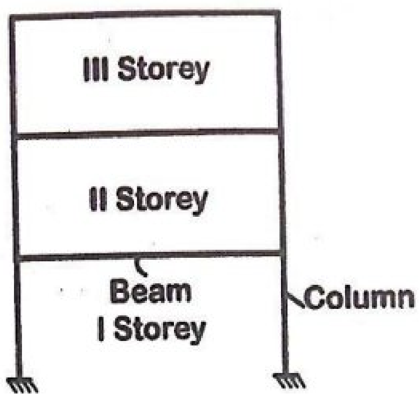
It is an indeterminate structure consisting of horizontal and inclined beams resting over columns.The nodes (or) joints of beams and columns behave like rigid joints.

Depending on the number of bays and number of storeys these are classified as;

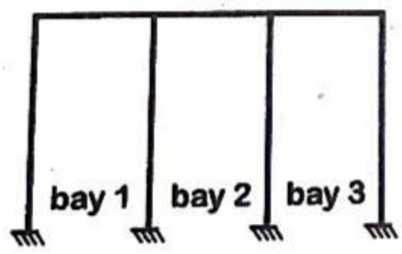
- Single bay single storey rigid frames
- Single bay multistorey rigid frames
- Multi bay single storey rigid frames and
- Multistorey rigid frames

SKETCHES OF RIGID FRAMES

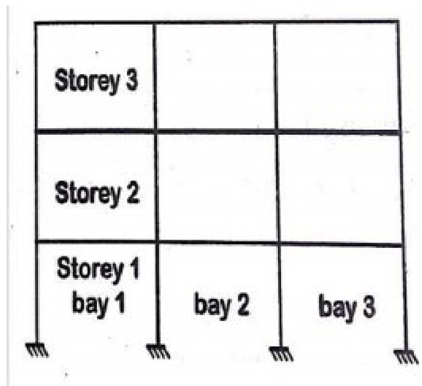
(a) Single bay single storey rigid frames



(b) Single bay multistorey rigid frames



(c) Multi bay single storey rigid frames

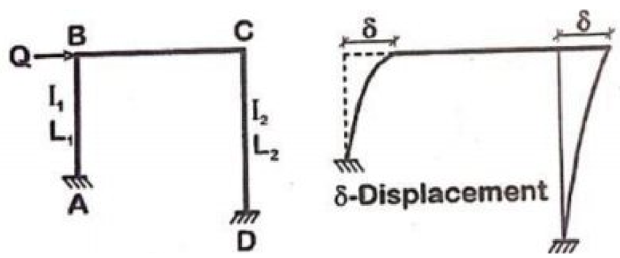


(d) Multistorey rigid frames

3.3.2. SWAY AND NON-SWAY FRAMES

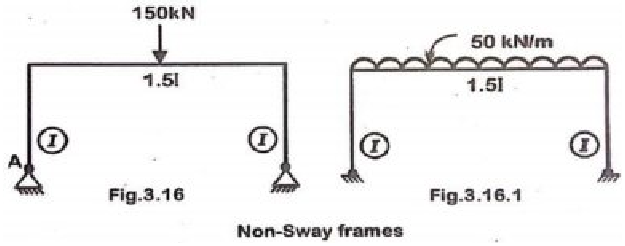
With Sway Frames:

A portal frames is said to be a sway frame when it is subjected to unbalanced and instrument horizontal forces.



Without Sway Frames:

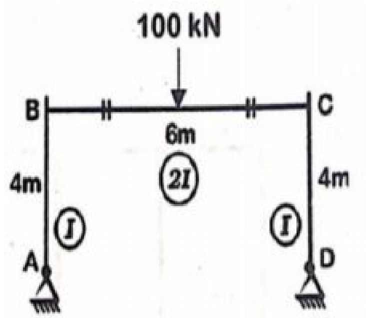
A portal frames is said to be a non-sway frame or without sway when it is subjected to vertical forces.



3.3.3. NUMERICAL EXAMPLES ON(PLANE RIGID FRAMES):

PROBLEM NO:01

For the portal rigid frame compute the bending moments and draw the BMD



Solution:

- **Fixed End Moments:**

  MFAB = MFBA = 0

  MFBC = Wl/8 = 100X6/8 = -75 kNm

  MFCB = -Wl/8 = 100X6/8 = 75 kNm

  MFCD = MFDC = 0

- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	3EI/l = 3EI/4	7EI/4	0.43
	BC	3E(2I)/l = 6EI/6		0.57
C	CB	3E(2I)/l = 6EI/6	7EI/4	0.57
	CD	3EI/l = 3EI/4		0.43

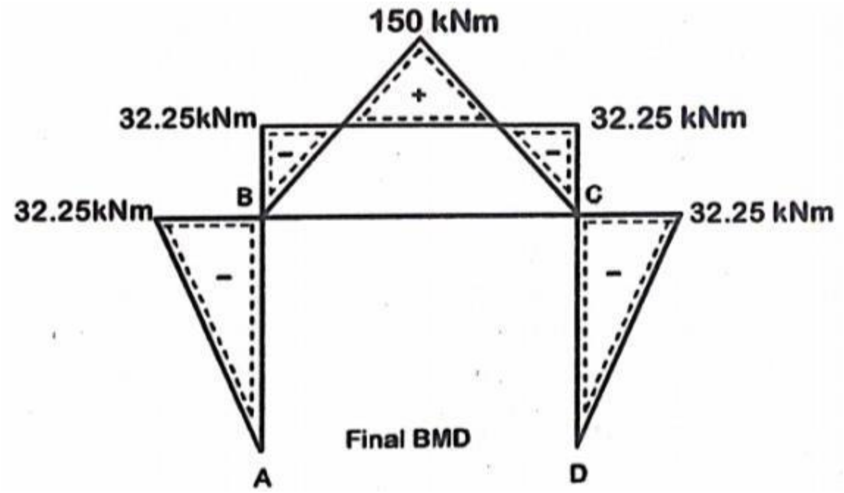
- **Free BMD:**

  MBC = Wl/4 = 100 x 6/4 = 150kNm

Moment Distribution Table:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	-	0.43	0.57	0.57	0.43	-
F.E.M	0	0	-75	75	0	0
Distribute		32.25	42.75	-42.75	-32.25	
Final Moments	0	32.25	-32.25	32.25	-32.25	0
Conventional Moments	0	-32.25	-32.25	-32.25	-32.25	0

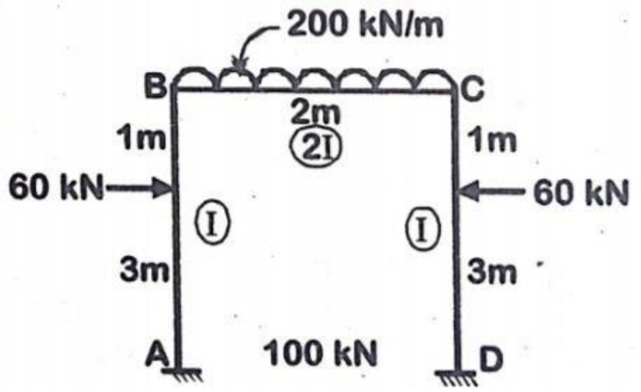
- **Moment Diagram:**



PROBLEM NO:02

Analyse the portal frames shown in fig by moment distribution method and draw the BMD.

Solution:



- **Fixed End Moments:**

  MFAB = $-Wab^2/l^2 = -60x3x1^2/4^2 = -11.25$ kNm;

  MFBA = $Wa^2b/l^2 = 60x3^2x1/4^2 = 33.76$ kNm;

  MFBC = $-Wl^2/12 = -200x2^2/12 = -66.67$ kNm;

  MFCB = $Wl^2/12 = 200x2^2/12 = 66.67$ kNm;

  MFCD = $-Wab^2/l^2 = -60x3x1^2/6^2 = -33.75$ kNm;

  MFDC = $Wa^2b/l^2 = 60x3^2x1/4^2 = 11.25$ kNm.

- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	4EI/l = 4EI/4	4EI	0.25
	BC	3E(2I)/l = 6EI/2		0.75
C	CB	3E(2I)/l = 6EI/2	4EI	0.75
	CD	4EI/l = 4EI/4		0.25

- **Free BMD:**

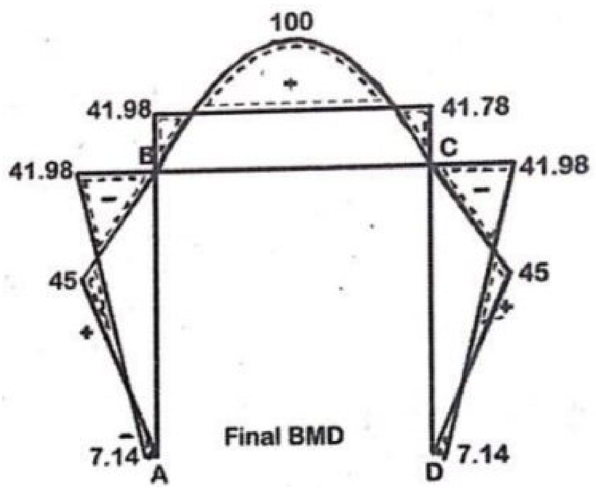
MAB = MCD = Wab/l= 60x3x1/4 = 45 kNm;

MBC = $Wl^2/8 = 200x2^2/8$ = 100 kNm.

- **Moment Distribution Table**:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	-	0.25	0.75	0.75	0.25	-
F.E.M	-11.25	33.75	-66.67	66.67	-33.75	11.25
Distribute		8.23	24.69	-24.69	-8.23	
Carry over	4.11					-4.11
Final Moments	-7.14	41.98	-41.98	41.98	-41.98	7.14
Conventional Moments	-7.14	-41.98	-41.98	-41.98	-41.98	-7.14

- **Moment Diagram:**

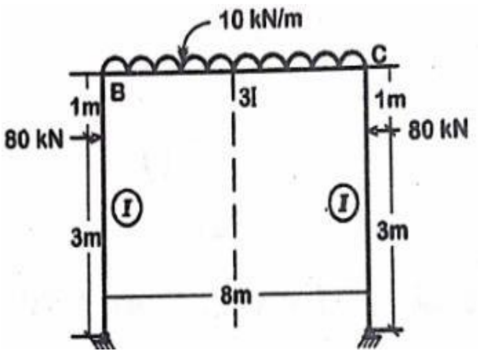


- **Result:**

MA = MD = -7.14 kNm; MB = MC = -41.98 kNm

PROBLEM NO:03

Analyze the symmetrical portal rigid frame shown below by moment distribution method and draw BMD



Solution:

- **Fixed End Moments:**

MFAB = $-Wab^2/l^2 = -80x3x1^2/4^2 = -15$ kNm;

MFBA = $Wa^2b/l^2 = 80x3^2x1/4^2 = 45$ kNm;

MFBC = $-Wl^2/12 = -10x8^2/12 = -53.33$ kNm;

MFCB = $Wl^2/12 = 10x8^2/12 = 53.33$ kNm;

MFCD = $-Wab^2/l^2 = -80x1x3^2/4^2 = -45$ kNm;

MFDC = $Wa^2b/l^2 = 80x1^2x3/4^2 = 15$ kNm;

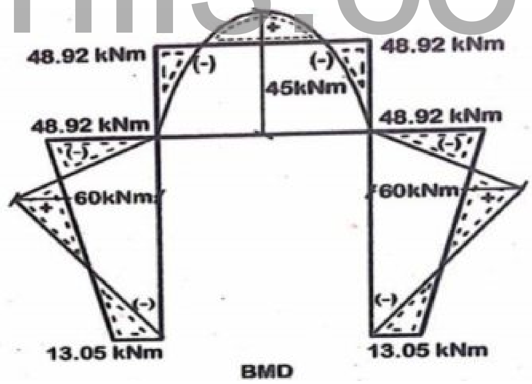
- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	4EI/l = 4EI/4	17EI/8	0.47
	BC	3E(3I)/l = 9EI/8		0.53
C	CB	3E(3I)/l = 9EI/8	17EI/8	0.53
	CD	4EI/l = 4EI/4		0.47

- **Moment Distribution Table**:

Joint	A	B		C		D
Member	AB	BA	BC	CB	CD	DC
D.F	-	0.47	0.53	0.53	0.47	-
F.E.M	-15	45	-53.33	53.33	-45	15
Distribute		3.92	4.41	-4.41	-3.92	
Carry over	1.95					-1.95
Final Moments	-13.05	48.92	-48.92	48.92	-48.92	13.05
Conventional Moments	-13.05	-48.92	-48.92	-48.92	-48.92	-13.05

- **Moment Diagram:**



- **Free BMD:**

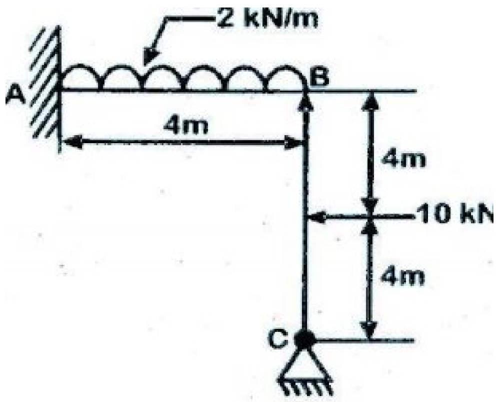
MAB = MCD = Wab/l= 80x3x1/4 = 60 kNm;
MBC = $Wl^2/8$ = $10x6^2/8$ = 45 kNm.

- **Result:**

MA = MD = -13.05 kNm; MB = MC = -48.92 kNm

PROBLEM NO:03

Analyze the symmetrical portal rigid frame shown below by moment distribution method and draw BMD



Solution:

- **Fixed End Moments:**

  MFAB = $-Wl^2/12 = -2x4^2/12 = -2.67$ kNm;

  MFBA = $Wl^2/12 = 2x4^2/12 = 2.67$ kNm;

  MFBC = -Wl/8 = -10X8/8 = -10 kNm

  MFCB = Wl/8 = 10X8/8 = 10 kNm

- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	4EI/l = EI	11EI/8	0.73
	BC	3EI/l = 3EI/8		0.27

- **Free BMD:**

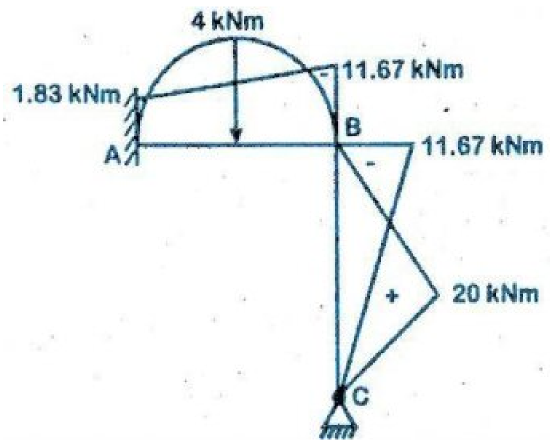
  MAB = $Wl^2/8 = 2x4^2/8 = 4$ kNm.

  MBC = Wl/4= 10x8/4 = 20 kNm;

- **Moment Distribution Table**:

Joint	A	B		C
Member	AB	BA	BC	CB
D.F	-	0.73	0.27	-
F.E.M	-2.67	2.67	-10	10
Release C and carry over	-	0	-5	-10
Initial Moments	-2.67	2.67	-15	0
Distribute	-90	9.00	3.329	-90
Carry over	4.5			
Final Moments	1.83	11.67	-11.67	0
Conventional Moments	-1.83	-11.67	-11.67	0

- **Moment Diagram:**



- **Result:**

MA = -1.83 kNm; MB = -11.67 kNm; MC = 0.

3.4. SUPPORT SETTLEMENTS IN MOMENT DISTRIBUTION METHOD.

3.4.1 SUPPORT SETTLEMENT IN STRUCTURAL ANALYSIS:

Support settlements may be caused by **soil erosion**, dynamic soil effects during earthquakes, or by partial failure or settlement of supporting structural elements. Supports could also potentially heave due to frost effects (this could be considered a negative settlement).

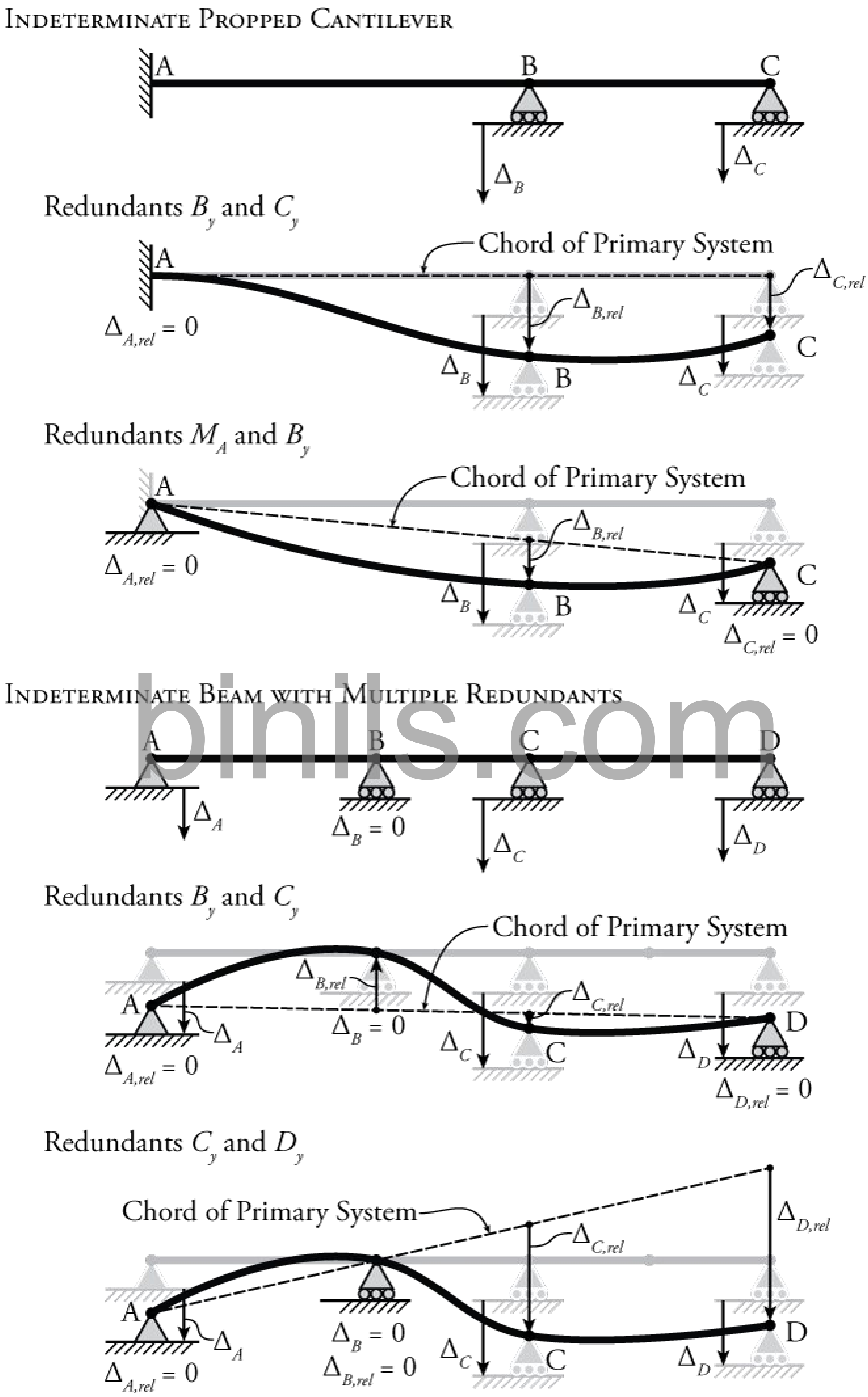
3.4.2. INTRODUCTION:

In the last lesson, the force method of analysis of statically indeterminate beams subjected to external loads was discussed. It is however, assumed in the analysis that the supports are unyielding and the temperature remains constant. In the design of indeterminate structure, it is required to make necessary provision for future unequal vertical settlement of supports or probable rotation of supports. It may be observed here that, in case of determinate structures no stresses are developed due to settlement of supports. The whole structure displaces as a rigid body. Hence, construction of determinate structures is easier than indeterminate structures.

The statically determinate structure changes their shape due to support settlement and this would in turn induce reactions and stresses in the system. Since, there is no external force system acting on the structures, these forces form a balanced force system by themselves and the structure would be in equilibrium. The effect of temperature changes, support settlement can also be easily included in the force method of analysis. In this lesson few problems, concerning the effect of support settlement are solved to illustrate the procedure.

3.4.3. SUPPORT DISPLACEMENTS:

The whole structure displaces as a rigid body. Hence, construction of determinate structures is easier than indeterminate structures. The statically determinate structure changes their shape due to support settlement and this would in turn induce reactions and stresses in the system.



Support settlements in continuous beams

3.4.4. NUMERICAL EXAMPLES ON(CONTINUOUS BEAMS):

PROBLEM NO:01

Analysis the continuous beam shown in fig.2.10,Calculate the support moments using moment distribution method.Support B settlememts by 10mm below the levels of A,C and D.Take E = 2 x 10^5 N/mm^2,I = 132 x 10^6 mm^4.Sketch the SF and BM diagrams.

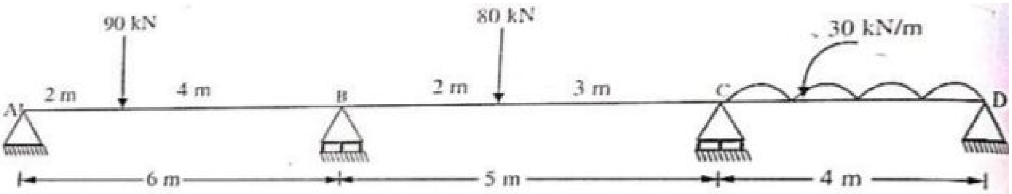
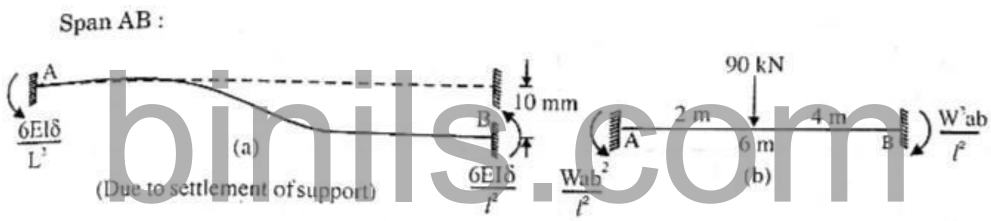


Fig. 3.42

Solution:

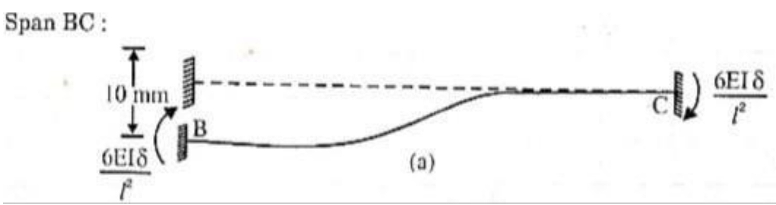
- **Fixed End Moments:**



MFAB = -6EIδ/l^2 - Wab^2/l^2= -6x26400x10x10^{-3}/6^2-90x2x4^2/6^2 = -124 kNm;

MFBA = -6EIδ/l^2 + Wa^2b/l^2 = -6x26400x10x10^{-3}/6^2+90x2x4^2/6^2 = - 4 kNm.

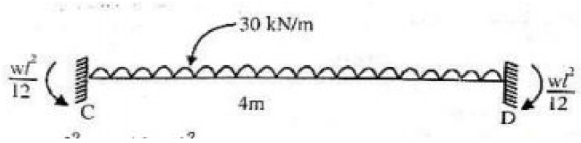
Span BC:



MFBC = 6EIδ/l^2 - Wab^2/l^2= 6x26400x10x10^{-3}/5^2-80x2x3^2/5^2 = 5.76 kNm;

MFCB = 6EIδ/l^2 + Wa^2b/l^2 = 6x26400x10x10^{-3}/5^2+80x2^2x3/5^2 = 101.76 kNm.

Span CD:



MFCD = -Wl^2/12 = -30x4^2/12 = -40 kNm;

MFDC = Wl^2/12 = 30x4^2/12 = 40 kNm;

- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	3/4xl/6 = I/8	13I/40	0.385
	BC	l/5 = I/5		0.615
C	CB	l/5 = I/5	31I/80	0.516
	CD	3/4xl/4 = 3I/16		0.484

- **Free BMD:**

MAB = MCD = Wab/l= 90x2x4/6= 120 kNm;

MBC = MCD = Wab/l= 80x2x3/4 = 96 kNm;

MCD = Wl^2/8 = 30x4^2/8 = 60 kNm.

- **Final Moments:**

MAB = 0

MBA = 35.841 kNm

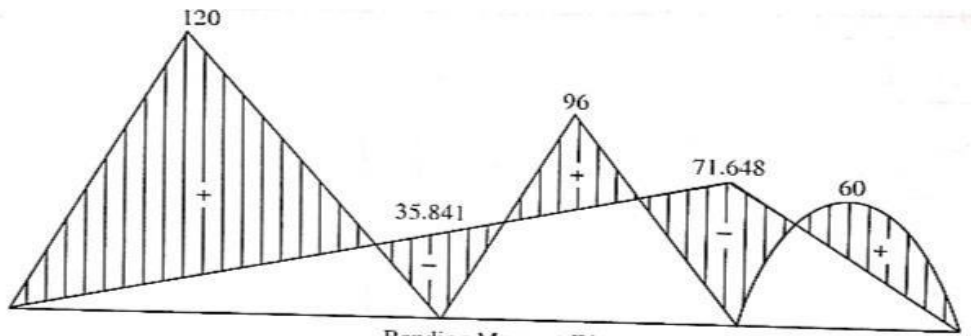
MBC = - 35.841 kNm

MCB = 71.648 kNm

MCD = - 71.648 kNm

MDC = 0

- **Bending Moment Diagram**:



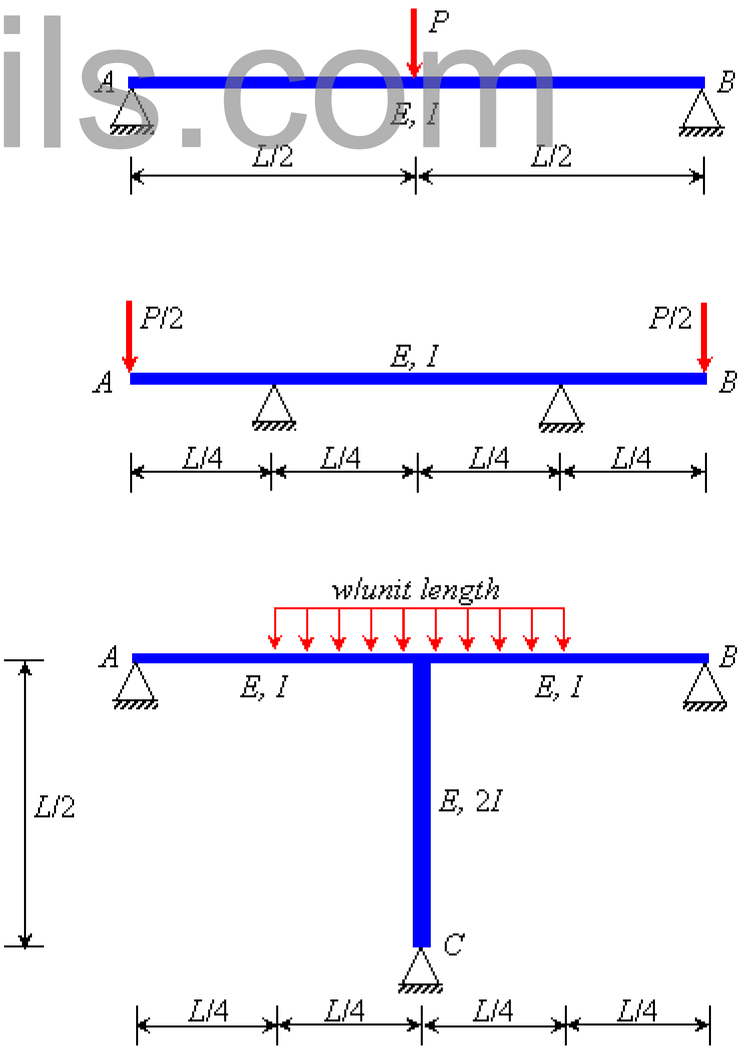
Bending Moment Diagram

3.5 SYMMETRIC FRAMES WITH SYMMETRIC AND SKEW-SYMMETRIC LOADINGS

3.5.1 SYMMETRY AND ANTISYMMETRY

Symmetry or antisymmetry in a structural system can be effectively exploited for the purpose of analyzing structural systems. Symmetry and antisymmetry can be found in many real-life structural systems (or, in the idealized model of a real-life structural system). It is very important to remember that when we say symmetry in a structural system, it implies the existence of symmetry both in the structure itself including the support conditions and also in the loading on that structure. The systems shown in Fig. are symmetric because, for each individual case, the structure is symmetric and the loading is symmetric as well. However, the systems shown in Fig. are not symmetric because either the structure or the loading is not symmetric.

1.1. Symmetr



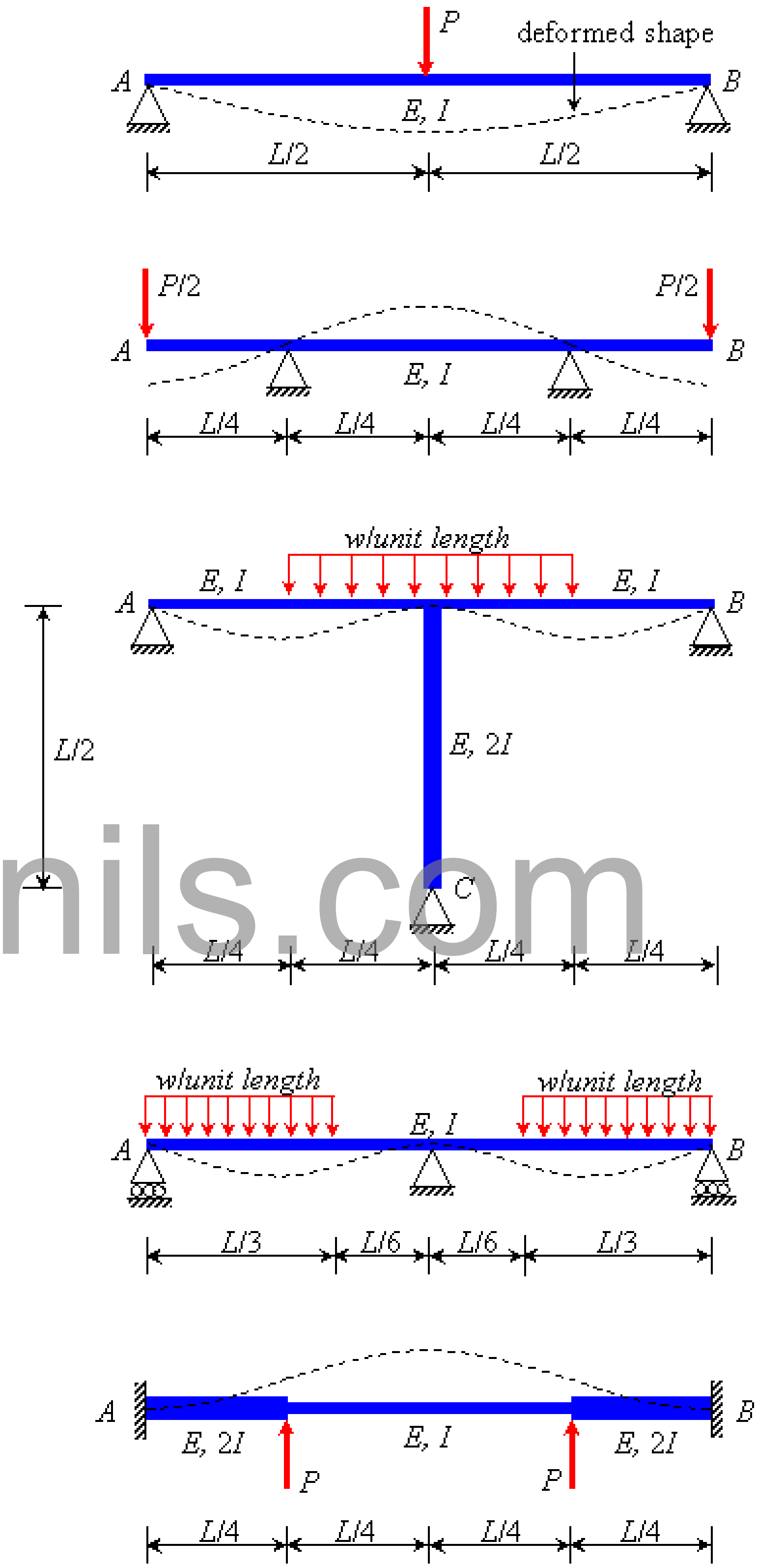
ic

structural systems

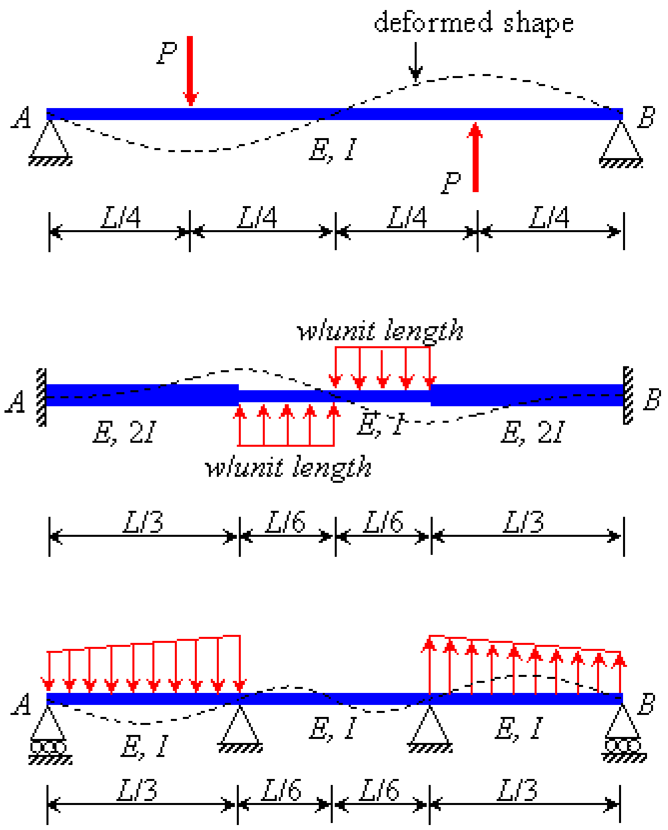
1.2. Non-symmetric (asymmetric) structural systems

For an antisymmetric system the structure (including support conditions) remains symmetric,however, the loading is antisymmetric.The fig.1.2 ,shows the example of antisymmetric structural systems.

It is not difficult to see that the deformation for a symmetric structure will be symmetric about the same line of symmetry. This fact is illustrated in Fig. 1.3, where we can see that every symmetric structure undergoes symmetric deformation. It can be proved using the rules of structural mechanics (namely, equilibrium conditions, compatibility conditions and constitutive relations), that deformation for a symmetric system is always symmetric.

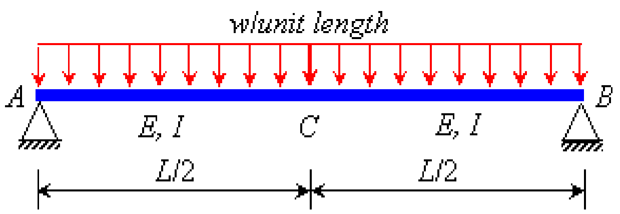


1.3. Deformation in symmetric systems

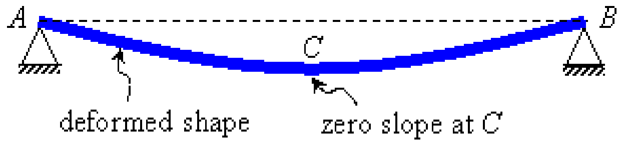


1.4. Deformation in antisymmetric systems

Let us look at beam *AB* in Fig. 1.20(a), which is symmetric about point C. The deformed shape of the structure will be symmetric as well (Fig. 1.20(b)). So, if we solve for the forces and deformations in part *AC* of the beam, we do not need to solve for part *CB* separately. The symmetry (or antisymmetry) in deformation gives us additional information prior to analyzing the structure and these information can be used to reduce the size of the st ructure that needs to be considered for analysis.



(a)

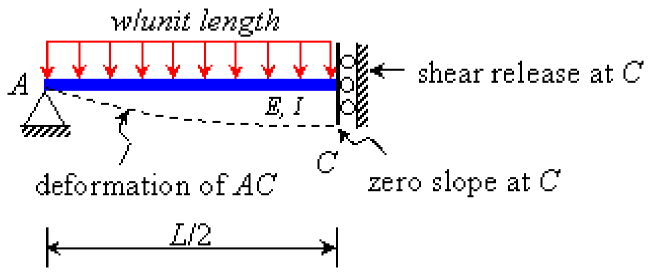


(b)

To elaborate on this fact, we need to look at the deformation condition at the point/line of symmetry (or antisymmetry) in a system. The following general rules about deformation can be deduced looking at the examples in Fig. 1.3 and Fig. 1.4:

- For a symmetric structure: slope at the point/line of symmetry is zero.
- For an antisymmetric structure: deflection at the point/line of symmetry is zero.

These information have to be incorporated when we reduce a symmetric (or antisymmetric) structure to a smaller one. If we want to reduce the symmetric beam in Fig. 1.20 to its one symmetric half *AC* , we have to integrate the fact the slope at point



1.6.Reduced system AC is adopted for analysis for beam AB

C for the reduced system AC will have to be zero. This will be a necessary boundary condition for the reduced system AC . We can achieve this by providing a support at C, which restricts any rotation, but allows vertical displacement, as shown in Fig. 1.6 (Note: this specific type of support is known as a “shear-release” or “shear-hinge”). Everything else (loading, other support conditions) remains unchanged in the reduced system. We can use this system AC for our analysis instead of the whole beam AB .

3.5.2 INTERNAL FORCE DIAGRAMS FOR A) A SYMMETRIC SYSTEM, AND B) AN ANTISYMMETRIC SYSTEM

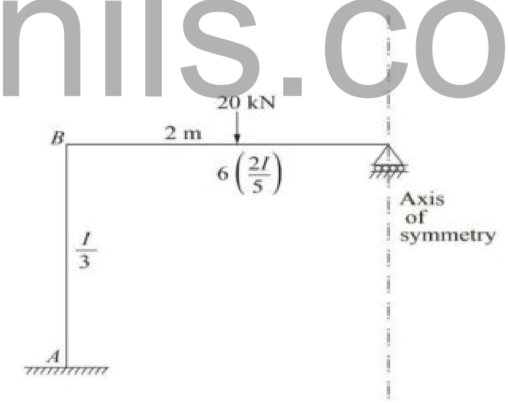
Having a priory knowledge about symmetry/antisymmetry in the structural system and in its deformed shape helps us know about symmetry/antisymmetry in internal forces in that system. (Symmetry in the system implies symmetry in equilibrium and constitutive relations, while symmetry in deformed shape implies symmetry in geometric compatibility.) Internal forces in a symmetric system are also symmetric about the same axis and similarly antisymmetric systems have antisymmetric internal forces. Detailed discussion on different types of internal forces in various structural systems and on internal force diagrams are provided in the next module (Module 2: Analysis of Statically Determinate Structures). Once we know about these diagrams we can easily see the following:

- A symmetric beam-column system has a symmetric bending moment diagram.
- A symmetric beam-column system has an antisymmetric shear force diagram.
- An antisymmetric beam-column system has an antisymmetric bending moment diagram.
- An antisymmetric beam-column system has a symmetric shear force diagram.

3.5.3 NUMERICAL EXAMPLES ON(SYMMETRIC AND SKEW-SYMMETRIC FRAMES):

PROBLEM NO:01

For the portal rigid frame compute the bending moments and draw the BMD



Solution:

- **Fixed end moments in skew symmetric case**

  MFBC = $-Wab^2/l^2 = -20x2x3^2/5^2 = -4.8$ kNm;

  MFCB = $Wa^2b/l^2 = 20x2^2x3/5^2 = 9.6$ kNm;

- **Distribution Factor Table for skews symmetric case**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	I/3 = 0.33I	2.73I	0.12
	BC	6(2I)/5 = 2.4I		0.88

- **Moment Distribution Table**:

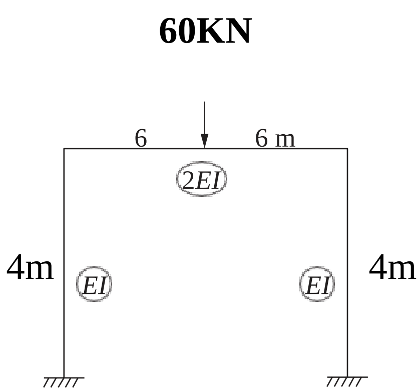
Joint	A	B	
Members	AB	BA	BC
DF	0	0.12	0.88
FEM			– 4.80
Balance		+0.58	+4.22
Carry over	– 0.58		
Final moment	– 0.58	+0.5	– 0.58

- **Result:**

MAB = -0.58 kNm; MBA = 0.5kNm; MBC = -0.58 kNm

PROBLEM NO:02

For the portal rigid frame compute the bending moments and draw the BMD



- **Fixed end moments in skew symmetric case**

  MFAB = MFBA = 0

  MFBC = -Wl/8 = -60X12/8 = -90 kNm

  MFCB = Wl/8 = 60X12/8 = 90 kNm

  MFCD = MFDC = 0

- **Distribution Factor Table:**

Joint	Member	k	Σk	Distribution factor (k/Σk)
B	BA	3E(2I)/l = 6EI/12	EI	0.43
	BC	3E(2I)/l = 6EI/12		0.57

- **Moment Distribution Table**:

Joint	A	B	
Members	AB	BA	BC
	0	0.12	0.88
FEM		-90	90
Balance		+0.58	+4.22
Carry over	– 0.58		
Final moment	– 0.58	+0.5	– 0.58

- **Result:**

  MAB = -0.58 kNm; MBA = 0.5kNm; MBC = -0.58 kNm