2.1 SLOPE DEFLECTION EQUATION

Slope-Deflection Method of Analysis of Indeterminate Structures

2.1.1 INTRODUCTIOIn 1915, George A. Maney introduced the slope-deflection method as one of the classical methods of analysis of indeterminate beams and frames. The method accountsfor flexural deformations, but ignores axial and shear deformations. Thus, th

2.1.2 SIGN CONVENTIONS

An end moment M is considered positive if it tends to rotate the member clockwise and negati ve if it te nds to rotate the member counter-clockwise. The rotation θ of a joint is positive if its tangent turns in a clockwise direction. The rotation of the chord connecting the ends of a member (Δ/L) the displacement of one end of a member relative to the other, is positive if the member turns in a clockwise direction.

2.1.3 SLOPE DEFLECTION EQUATIONS

The slope-deflection equations, consider a beam of length L and of constant flexural rigidity EI loaded. The member experiences the end moments M_FAB & M_FBA at A and B,respectively.And undergoes the deformed shape with the assumption that the right end B of the member settles by an amount Δ. The end moments are the summation of the moments caused by the rotations of the joints at the ends A and B (θ_A and θ_B) of the beam,and it fixed at both ends referred to as fixed end moments (M_FAB & M_FBA)

The slope equation is;

For member AB;

$$MAB = MFAB + 2EI/L(2\theta A+ \theta B+3\delta/L)$$

$$MBA = MFBA + 2EI/L(2\theta B+ \theta A+3\delta/L)$$

Where,

MAB & MBA – Final Moments at members AB and BA.

MFAB & MFBA – Fixed end moments.

E & I – young´s modulus and moment of inertia.

θA & θB – slope angles.

δ – Deflection

L - Length

2.1.4 EQUILIBRIUM CONDITIONS

- Joint equilibrium conditions
- Shear equilibrium conditions

JOINT EQUILIBRIUM

Joint equilibrium conditions imply that each joint with a degree of freedom should have no unbalanced moments i.e. be in equilibrium.Therefore,

Sum of (end moments + fixed end moments) = Sum of external moments directly applied at the joint.

$$MBA + MBC = 0;$$

SHEAR EQUILIBRIUM

When there are chord rotations in a frame, additional equilibrium conditions, namely the shear equilibrium conditions need to be taken into account.

2.1.5 ANALYSIS OF INDETERMINATE BEAMS

The procedure for the analysis of indeterminate beams by the slope-deflection method is summarized below.

Procedure for Analysis of Indeterminate Beams and Non-Sway Frames by the Slope-Deflection Method

• Determine the fixed-end moments for the members of the beam.

• Determine the rotations of the chord if there is any support settlement.

• Write the slope-deflection equation for the members' end moments in terms of unknown rotations.

• Write the equilibrium equations at each joint that is free to rotate in terms of the end moments of members connected at that joint.

• Solve the system of equations obtained simultaneously to determine the unknown joint rotations.

• Substitute the computed joint rotations into the equations obtained in step 3 to determine the members' end moments.

• Draw a free-body diagram of the indeterminate beams indicating the end moments at the joint.

• Draw the shearing force diagrams of the beam by considering the freebody diagram of each span of the beam in the case of a multi-span structure.

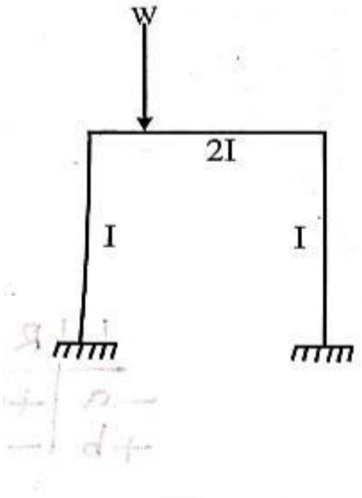
2.1.6 ANALYSIS OF INDETERMINATE FRAMES

Indeterminate frames are categorized as frames with or without side-sway. A frame with side-sway is one that permits a lateral moment or a swaying to one side due to the asymmetrical nature of its structure or loading. The analysis of frames without side-sway is similar to the analysis of beams considered in the preceding section, while the analysis of frames with side-sway requires taking into consideration the effect of the lateral movement of the structure.

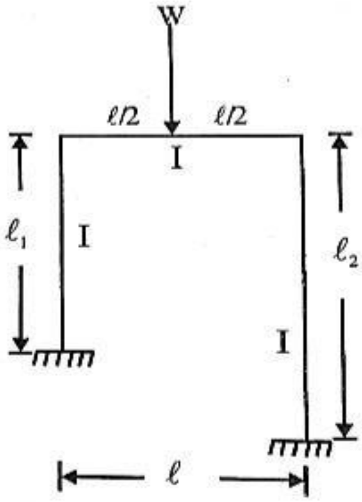
2.1.7. RIGID FRAMES WITH SWAY IN SLOPE DEFLECTION METHOD.

Portal frames may sway due to one of the following reasons:

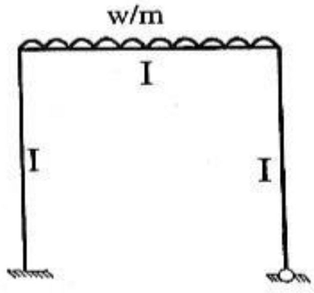
- Eccentric or unsymmetrical loading on the portal frames.
- Unsymmetrical shape of the frames.
- Different end conditions of the columns of the portal frames.
- Non uniform section of the members of the frame.
- Horizontal loading on the columns of the frame.
- Settlement of the supports of the frame.
- A combination of the above.



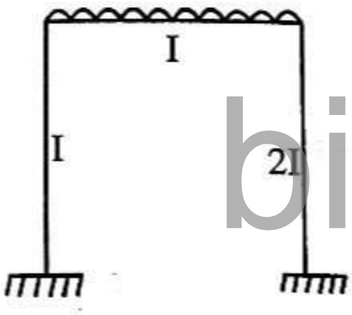
(1)



(2)



(3)



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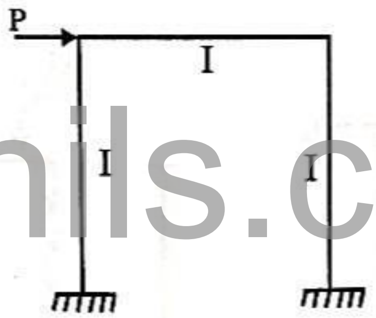

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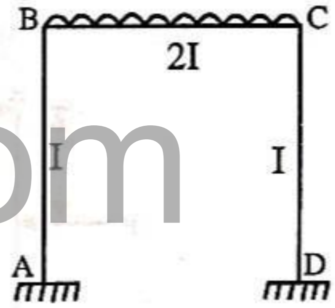
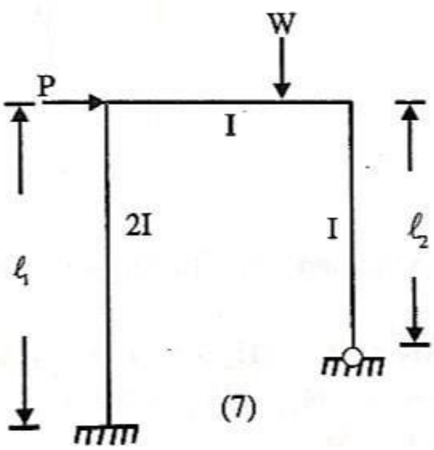


(6)

D Settles down / Sinks by δ



(7)

2.2 ANALYSIS OF CONTINUOUS BEAMS IN SLOPE DEFLECTION METHOD.

2.2.1 NUMERICAL EXAMPLES ON(CONTINUOUS BEAMS):

PROBLEM NO:01

Analysis the continuous beam shown in fig.2.8,Calculate the support moments using slope deflection method.Draw the SF and BM diagrams.

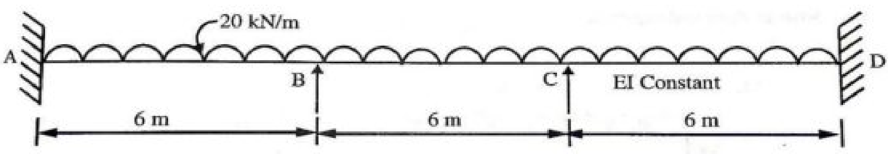


Fig. 2.8

Solutions:

- **Fixed End Moments:**

  MFAB = MFBC = MFCD = $-Wl^2/12$ = - $20x6^2/12$ = - 60 kNm

  MFBA = MFCB = MFDC = $Wl^2/12$ = $20x6^2/12$ = 60 kNm

- **Slope Deflection Equations:**

The structure is symmetrical.So is the loadind.There is no sinking of supports.Hence the following conditions prevail.

- $\theta A = \theta D = 0$
- $\delta = 0$ for all spans
- $\theta B = \theta C$

  Hence there is only one unknown displacement,namely θB.For span AB,the general slope deflection equation is

  MAB = MFAB + $2EI/6(2\theta A + \theta B + 3\delta/l)$

  MAB = - 60 + $2EI/6(\theta B)$ -------(2.1)

Since $\theta A = 0$ and $\delta = 0$

MAB = 60 + $2EI/6(\theta B)$ -------(2.2)

No other slope deflection equation is needed.

Since θB is the only unknown.

For span BC,

$$MBC = MFBC + 2EI/6(2\theta B + \theta C + 3\delta/l)$$

$$MBC = - 60 + 2EI/6(3\theta B) \text{------- } (2.3)$$

- **Joint Equilibrium Equations:**

MAB + MBC = 0

$60 + 2EI\theta B/3 – 60 + EI\theta B = 0$

Hence, θB = 0

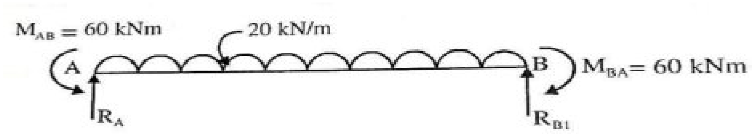
- **Final Moments:**

  MAB = MBC = MCD = - 60 kNm

  MBA = MCB = MDC = 60 kNm

- **Shear Force Diagram:**

  Span AB:



Taking moments about A,on the free body diagram of span AB,

$$- RB1 \times 6 – MAB + MBA + wl^2/2 = 0$$

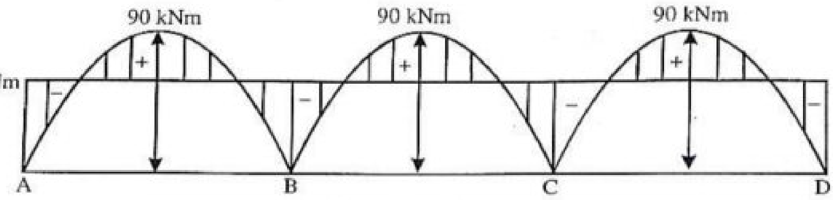
$$- RB1 \times 6 – 60 + 60 + 20 \times 6^2/2 = 0$$

RB1 = 60KN ; RA = 60KN

Similarly in span BC, RB2 = RC1 = 60

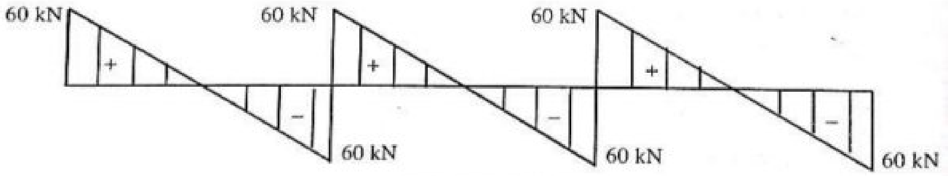
RB = RB1 + RB2 = 120 KN

- **BMD and SFD:**



Bending moment diagram

Simply supported span bending moment = $\frac{wl^2}{8} = \frac{20 \times 6^2}{8} = 90$ kNm



Shear force diagram

PROBLEM NO:02

Analysis the continuous beam shown in fig.2.10,Calculate the support moments using slope deflection method.Support B sinks by 10mm.Take E = 2 x 10^5 N/mm^2,I = 16 x 10^7 mm^4.Sketch the SF and BM diagrams.

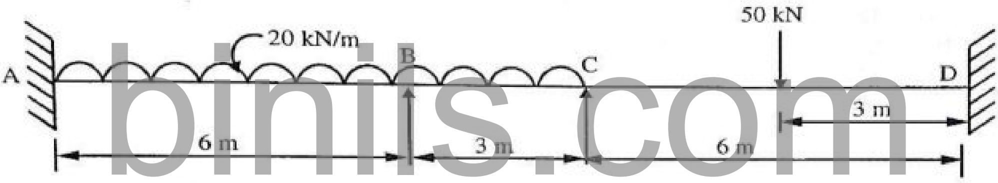


Fig. 2.10

Solution:

- **Fixed End Moments:**

  MFAB = -Wl^2/12 = - 20x6^2/12 = - 60 kNm;

  MFBA = Wl^2/12 = 20x6^2/12 = 60 kNm;

  MFBC = -Wl^2/12 = - 20x3^2/12 = - 15 kNm;

  MFCB = Wl^2/12 = 20x3^2/12 = 15 kNm;

  MFCD = -Wl/8 = - 50x6/8 = - 37.5 kNm;

  MFDC = Wl/8 = 50x6/8 = 37.5 kNm;

- **Slope Deflection Equations:**

  MAB = MFAB + 2EI/6(2θA + θB + 3δ/l)

  = - 60 + EI/3(0 + θB – 1/200) --- (1)

  MBA = MFBA + 2EI/6(2θB + θA + 3δ/l)

= 60 + EI/3(2θB – 3 x 10/6000) --- (2)

MBC = MFBC + 2EI/3(2θB + θC + 3δ/l)

= - 15 + 2EI/3(2θB + θC + 1/100) --- (3)

MCB = MFCB + 2EI/3(2θC + θB + 3δ/l)

= 15 + 2EI/3(2θC + θB + 1/100) --- (4)

MCD = MFCD + 2EI/6(2θC + θD + 3δ/l)

= - 37.5 + EI/3(2θC) --- (5)

MDC = MFDC + 2EI/6(2θD + θC + 3δ/l)

= 37.5 + EI/3(θC) --- (6)

- **Joint Equilibrium Equations:**

Joint B:

MBA + MBC = 0

EI/3(6θB+2θC+3/200) = - 135 --- (7)

Joint C:

MCB + MCD = 0

EI(θB+3θC+1/100) = 33.75 --- (8)

Equvating (7 & 8);we get

θC = - 1/464; θB = - 1/402

- **Final Moments:**

MAB = - 139.843 kNm;

MBA = - 46.354 kNm;

MBC = 46.3 kNm;

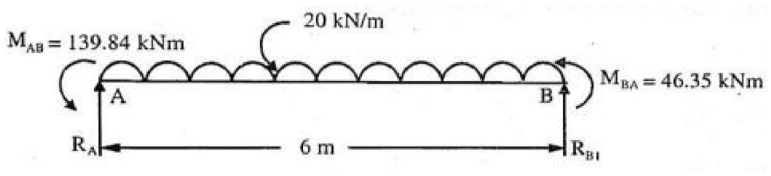
MCB = 83.35 kNm;

MCD = - 83.477 kNm;

MDC = 14.51 kNm;

- **To Draw S.F.D:**

Span AB:

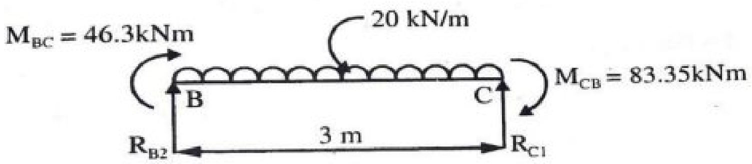


Taking moments about A.

20 x 6^2/2 – 46.35 – 139.84 – RB1(6) = 0; RB1 = 28.97 KN

RA = 20 x 6 – 28.97; RA = 91.03 KN

Span BC:

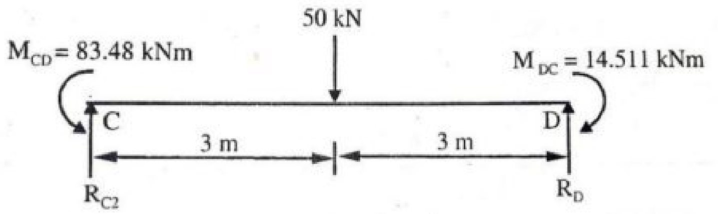


Taking moments about B.

20 x 3^2/2 + 83.35 + 46.3 – RC1(3) = 0; RC1 = 73.22 KN

RB2 = 20 x 3 – 73.22; RB2 = - 13.21 KN

Span CD:



Taking moments about C.

14.511 + 50(3) – 83.48 – RD(6) = 0;

RD = 13.5 KN; RC2 = 36.5 KN

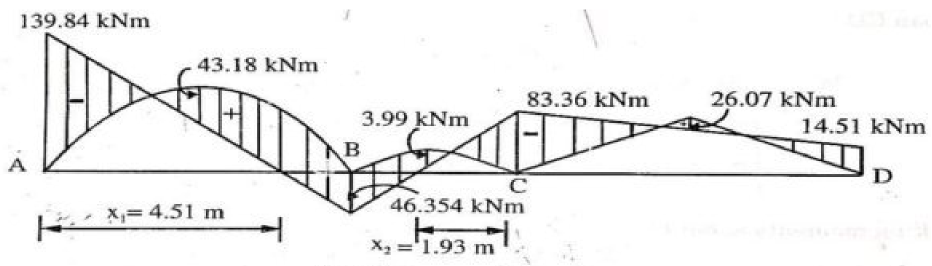
- **Free BMD:**

  MAB = $Wl^2/8 = 20 \times 6^2/8 = 90$ kNm

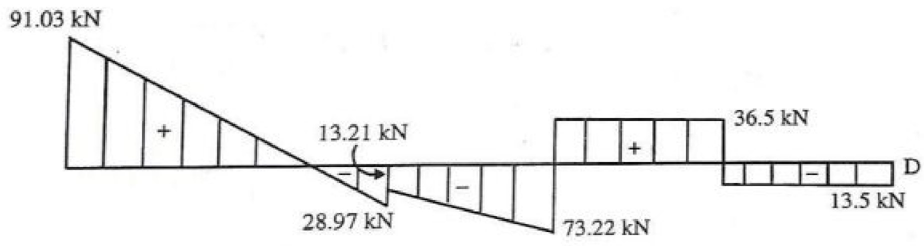
  MBC = $Wl^2/8 = 20 \times 3^2/8 = 22.5$ kNm

  MCD = $Wl/4 = 50 \times 6/4 = 75$ kNm

- **BMD and SFD:**



Bending moment diagram



Shear force diagram

PROBLEM NO:03

Analysis the continuous beam shown in fig.2.3,Calculate the support moments using slope deflection method. Sketch the BM diagrams.

$$2I_{AB} = I_{BC} = 2I_{CD} = 2I$$

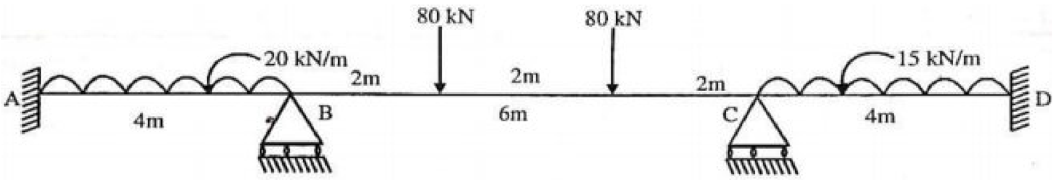


Fig. 2.3

$I_{AB} = I_{CD} = I$, $I_{BC} = 2I$, $\theta_A = \theta_D = 0$ (A and D are fixed)

Solution:

- **Fixed End Moments:**

  MFAB = $-Wl^2/12 = -20\times4^2/12 = -26.67$ kNm;

  MFBA = $Wl^2/12 = 20\times4^2/12 = 26.67$ kNm;

MFBC = -Wa(a + c)/6 = - 80(4 + 2)/6 = - 106.67 kNm;

MFCB = Wa(a + c)/6 = 80(4 + 2)/6 = 106.67 kNm;

MFCD = $-Wl^2/12$ = - $15x4^2/12$ = - 20 kNm;

MFDC = $Wl^2/12$ = $15x4^2/12$ = 20 kNm;

- **Slope Deflection Equations:**

MAB = MFAB + 2EI/6(2θA + θB + 3δ/l)

= - 26.67 + 0.5EIθB --- (1)

MBA = MFBA + 2EI/6(2θB + θA + 3δ/l)

= 26.67 + 0.5EIθB --- (2)

MBC = MFBC + 2EI/3(2θB + θC + 3δ/l)

= - 106.67 + EI (1.332θB + 0.666θC) --- (3)

MCB = MFCB + 2EI/3(2θC + θB + 3δ/l)

= 106.67 + EI (1.332θC + 0.666θB) --- (4)

MCD = MFCD + 2EI/6(2θC + θD + 3δ/l)

= - 20 + EIθC --- (5)

MDC = MFDC + 2EI/6(2θD + θC + 3δ/l)

= 20 + EI (0.5θC) --- (6)

- **Joint Equilibrium Equations:**

Joint B:

MBA + MBC = 0

2.333θB + 0.666θC = 80/EI --- (7)

Joint C:

MCB + MCD = 0

0.666θB + 2.333θC = 86.67/EI --- (8)

Equvating (7 & 8);we get

θC = - 51.11/EI; θB = 48.88/EI;

- **Final Moments:**

MAB = - 2.23 kNm;

MBA = 75.55 kNm;

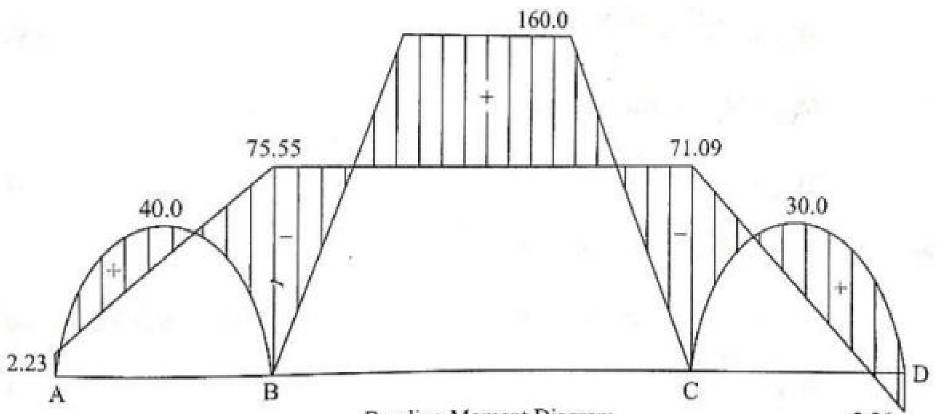
MBC = - 75.55 kNm;

MCB = 71.09 kNm;

MCD = - 71.09 kNm;

MDC = - 5.56 kNm;

- **BMD:**



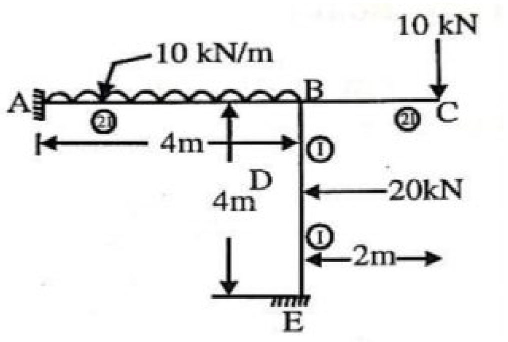
Bending Moment Diagram

2.3 ANALYSIS OF RIGID FRAMES IN SLOPE DEFLECTION METHOD.

2.3.1 NUMERICAL EXAMPLES ON (RIGID FRAMES):

PROBLEM NO:01

Analysis the rigid frame shown in fig.,Calculate the support moments using slope deflection method.Draw the SF and BM diagrams.



Solution:

- **Fixed End Moments:**

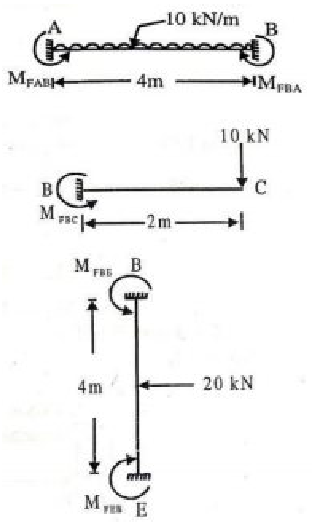
  MFAB = $-Wl^2/12 = -10x4^2/12 = -13.33$ kNm;

  MFBA = $Wl^2/12 = 10x4^2/12 = 13.33$ kNm;

  MFBC = -10 x 2 = - 20 kNm;

  MFBE = -Wl/8 = - 20x4/8 = - 10 kNm;

  MFEB = Wl/8 = 20x4/8 = 10 kNm;



- **Slope Deflection Equations:**

MAB = MFAB + 2E(2I)/4(2θA + θB + 3δ/l)

= - 13.33 + EIθB --- (1)

MBA = MFBA + 2E(2I)/4(2θB + θA + 3δ/l)

= 13.33 + EIθB --- (2)

MBE = MFBE + 2EI/3(2θB + θE + 3δ/l)

= - 10 + EIθB --- (3)

MEB = MFEB + 2EI/3(2θE + θB + 3δ/l)

= 10 + 0.5EIθB --- (4)

- **Joint Equilibrium Equations:**

Joint B, ΣM = 0;

MBA + MBC + MBE = 0

13.33 + 2EIθB – 10 + EIθB – 20 = 0

3EIθB – 16.67 = 0

θB = 5.557/EI;

- **Final Moments:**

MAB = - 7.773 kNm;

MBA = 24.44 kNm;

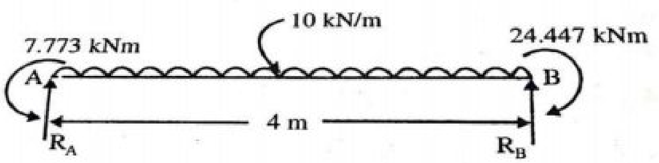
MBC = - 20 kNm;

MCD = - 4.33 kNm;

MDC = 12.78 kNm;

- **To Draw S.F.D:**

Span AB:

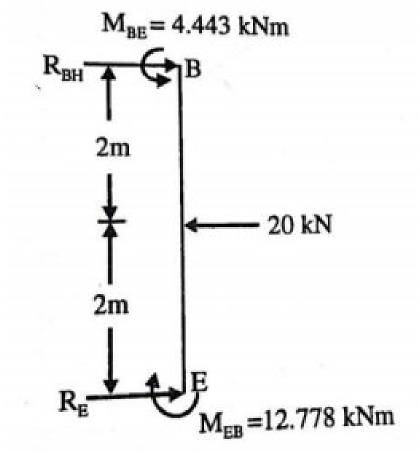


Taking moments about A.

RA x 4 – 10 x 4 x 4/2 – 7.773 + 24.447 = 0; RA = 15.83 KN

RBH = 10 x 4 – RA; RBH = 24.168 KN

Span BE:

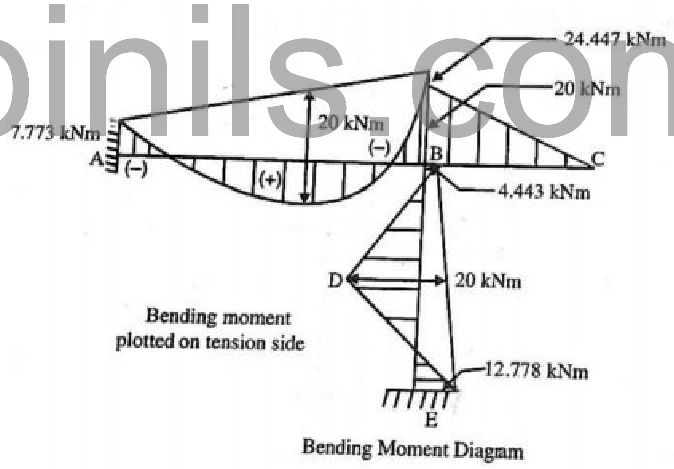


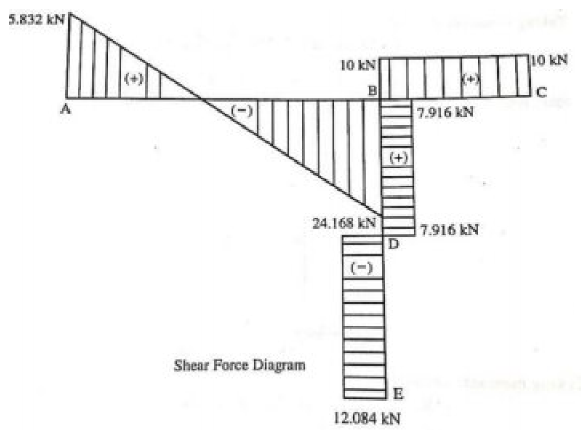
Taking moments about B.

-RE x 4 – 4.443 + 12.778 + 20 x 2 = 0; RE = 12.083 KN

RBH = Total load – RE = 7.916 KN

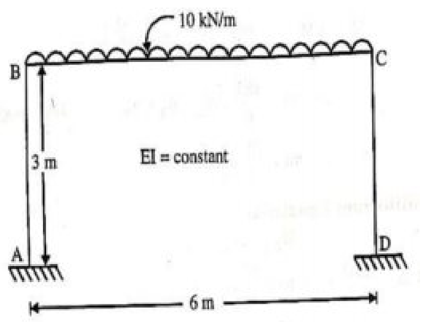
- **BMD and SFD:**





PROBLEM NO:02

Analysis the rigid frame shown in fig.,Calculate the support moments using slope deflection method.Draw the SF and BM diagrams.



Solution:

- **Fixed End Moments:**

  MFAB = MFBA = 0

  MFBC = $-Wl^2/12$ = - $10x6^2/12$ = - 30 kNm

  MFCB = $Wl^2/12$ = $10x6^2/12$ = 30 kNm

  MFCD = MFDC = 0

- **Slope Deflection Equations:**

  MAB = MFAB + 2EI/3(2θA + θB + 3δ/l)

  = 2/3EIθB --- (1)

  MBA = MFBA + 2EI/3(2θB + θA + 3δ/l)

  = 4/3EIθB --- (2)

  MBC = MFBC + 2EI/6(2θB + θC + 3δ/l)

  = - 30 + 1/3EIθB --- (3)

  MCB = MFCB + 2EI/3(2θC + θB + 3δ/l)

  = 30 + 1/3EIθB --- (4)

  MCD = MFCD + 2EI/6(2θC + θD + 3δ/l)

  = 4/3EIθB --- (5)

  MDC = MFDC + 2EI/6(2θD + θC + 3δ/l)

  = 2/3EIθB --- (6)

- **Joint Equilibrium Equations:**

Joint B:

MBA + MBC = 0

4/3θB – 30 + 1/3EIθB = 0 --- (7)

Joint C:

MCB + MCD = 0

4/3θC – 30 + 1/3EIθC = 0 --- (8)

Equvating (7 & 8);we get

θC = - 18/EI; θB = 18/EI;

- **Final Moments:**

MAB = 12 kNm;

MBA = 24 kNm;

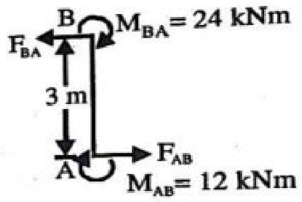
MBC = - 24 kNm;

MCB = 24 kNm;

MCD = - 24 kNm;

MDC = - 12 kNm;

- **To Draw S.F.D:**



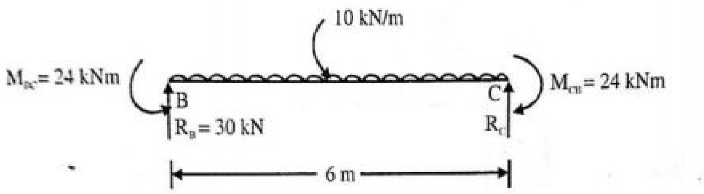
Span AB:

Taking moments about A.

-FBA x 3 + MBA + MAB = 0;

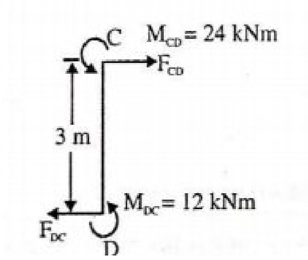
24 + 12 = FBA x 3; FBA = FAB = 12 KN

Span BC:



RB = RC = Total load/2 = 10 x 6/2 = 30 KN

Span CD:

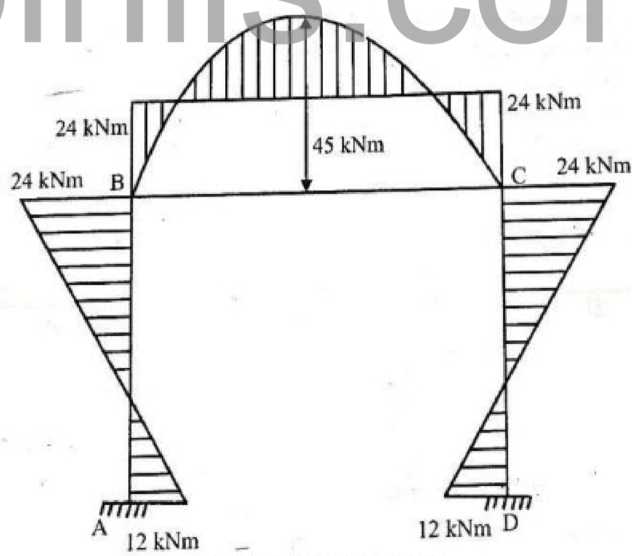


FCD = FDC = 12KN (by symmetry)

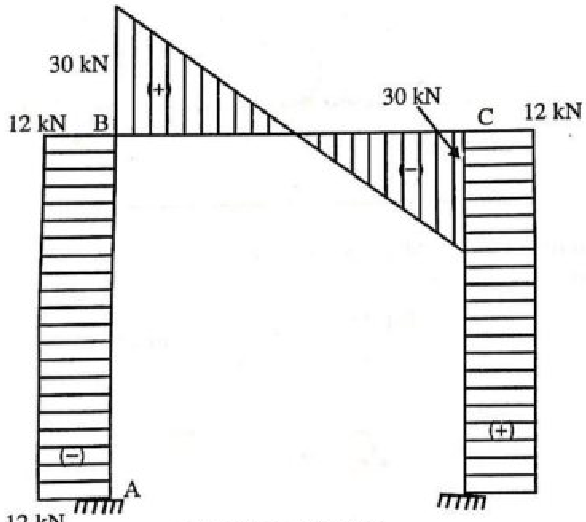
- **Free BMD:**

  MBC = $Wl^2/8 = 10 \times 6^2/8 = 45$ kNm

- **BMD and SFD:**



Bending Moment Diagram

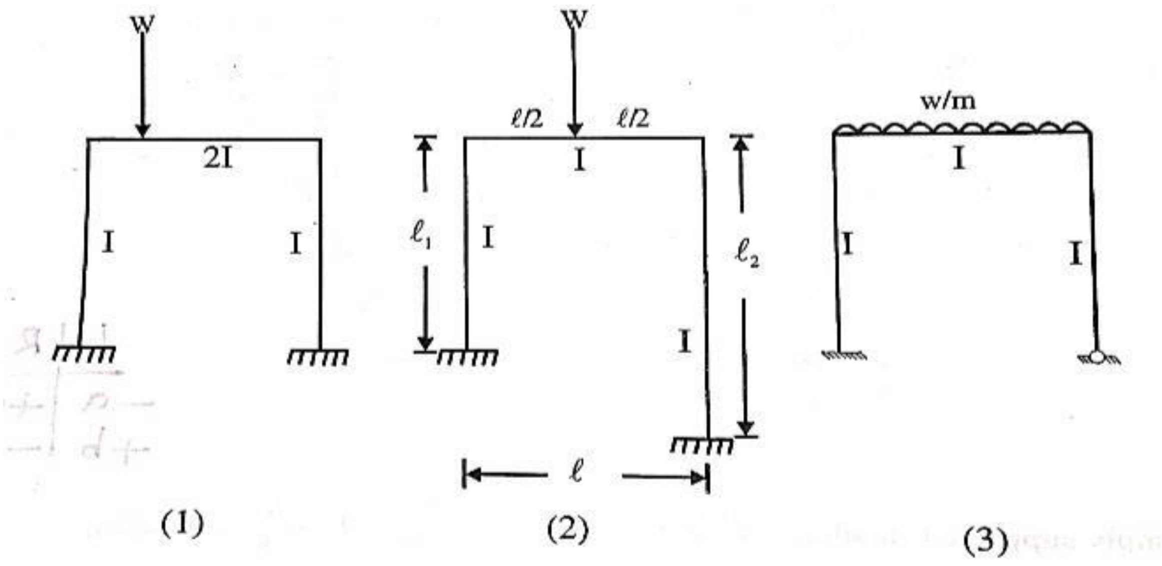


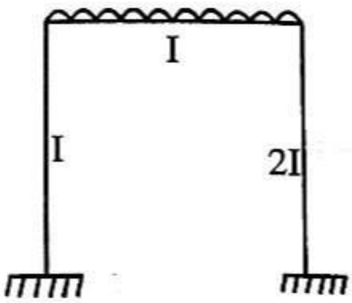
Shear force diagram

2.3.2. RIGID FRAMES WITH SWAY IN SLOPE DEFLECTION METHOD.

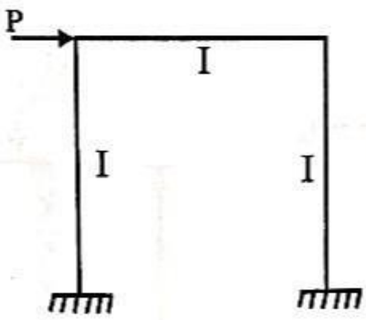
Portal frames may sway due to one of the following reasons:

- Eccentric or unsymmetrical loading on the portal frames.
- Unsymmetrical shape of the frames.
- Different end conditions of the columns of the portal frames.
- Non uniform section of the members of the frame.
- Horizontal loading on the columns of the frame.
- Settlement of the supports of the frame.
- A combination of the above.





(4)



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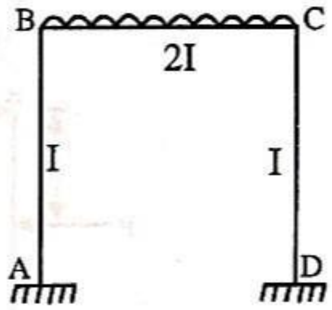
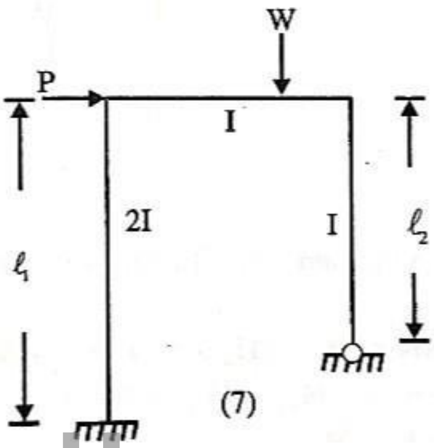


(6)

D Settles down / Sinks by δ



(7)

2.3.3. NUMERICAL EXAMPLES ON (RIGID FRAMES WITH SWAY):

PROBLEM NO:03

Analysis the rigid frame shown in fig.,Calculate the support moments using slope deflection method.Draw the SF and BM diagrams.

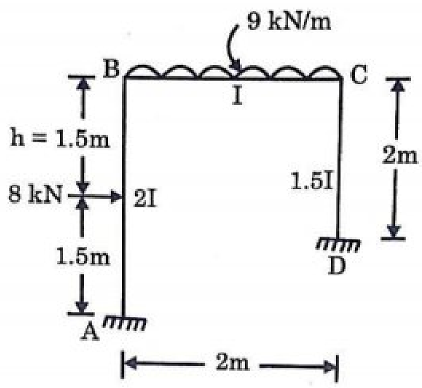


Fig. 2.16

Solution:

- **Fixed End Moments:**

  MFAB = -Wl/8 = - 8x3/8 = - 3 kNm;

  MFBA = Wl/8 = 8x3/8 = 3 kNm;

  MFAB = $-Wl^2/12 = -20x4^2/12 = -26.67$ kNm;

  MFBA = $Wl^2/12 = 20x4^2/12 = 26.67$ kNm;

  MFCD = 0;

  MFDC = 0;

- **Slope Deflection Equations:**

  MAB = MFAB + 2E(2I)/3(2θA + θB + 3δ/l)

  = - 3 + 4/3EI(θB – δ) --- (1)

  MBA = MFBA + 2E(2I)/3(2θB + θA + 3δ/l)

  = 3 + 4/3EI(2θB – δ) --- (2)

  MBC = MFBC + 2EI/2(2θB + θC + 3δ/l)

  = - 3 + EI(2θB + θC) --- (3)

  MCB = MFCB + 2EI/2(2θC + θB + 3δ/l)

  = 3 + EI(2θC + θB) --- (4)

  MCD = MFCD + 2E(1.5I)/2(2θC + θD + 3δ/l)

  = 1.5EI(2θC - 3δ/2) --- (5)

  MDC = MFDC + 2EI/6(2θD + θC + 3δ/l)

  = 1.5EI(2θC - 3δ/2) --- (6)

- **Equilibrium and Shear Equations:**

  MBA + MBC = 0

  14 θB - 4δ + 3θC = 0 ---(7)

  MCB + MCD = 0

  θB - 2.25δ + 5θC = 0 ---(8)

  Using Shear Equations,we get;

  MAB + MBA – Ph/l +MCD + MDC/l + P = 0

  (θC = - 0.044/EI; θB = 0.414/EI).

- **Final Moments:**

  MAB = - 4.34 kNm;

  MBA = 2.24 kNm;

  MBC = - 2.24 kNm;

  MCB = 3.33 kNm;

  MCD = - 3.33 kNm;

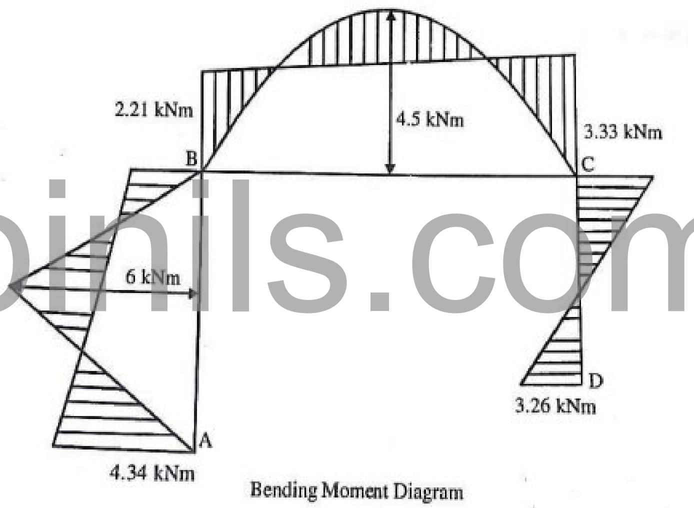
  MDC = - 3.26 kNm;

- **Free Bending Moments:**

  AB = Wl/4 = 8 x 3/4 = 6 kNm

  BC = $Wl^2/8$ = 9 x $2^2/8$ = 4.5 kNm

- **BMD:**



Bending Moment Diagram

2.4. SUPPORT SETTLEMENTS IN SLOPE DEFLECTION METHOD.

2.4.1 SUPPORT SETTLEMENT IN STRUCTURAL ANALYSIS:

Support settlements may be caused by **soil erosion**, dynamic soil effects during earthquakes, or by partial failure or settlement of supporting structural elements. Supports could also potentially heave due to frost effects (this could be considered a negative settlement).

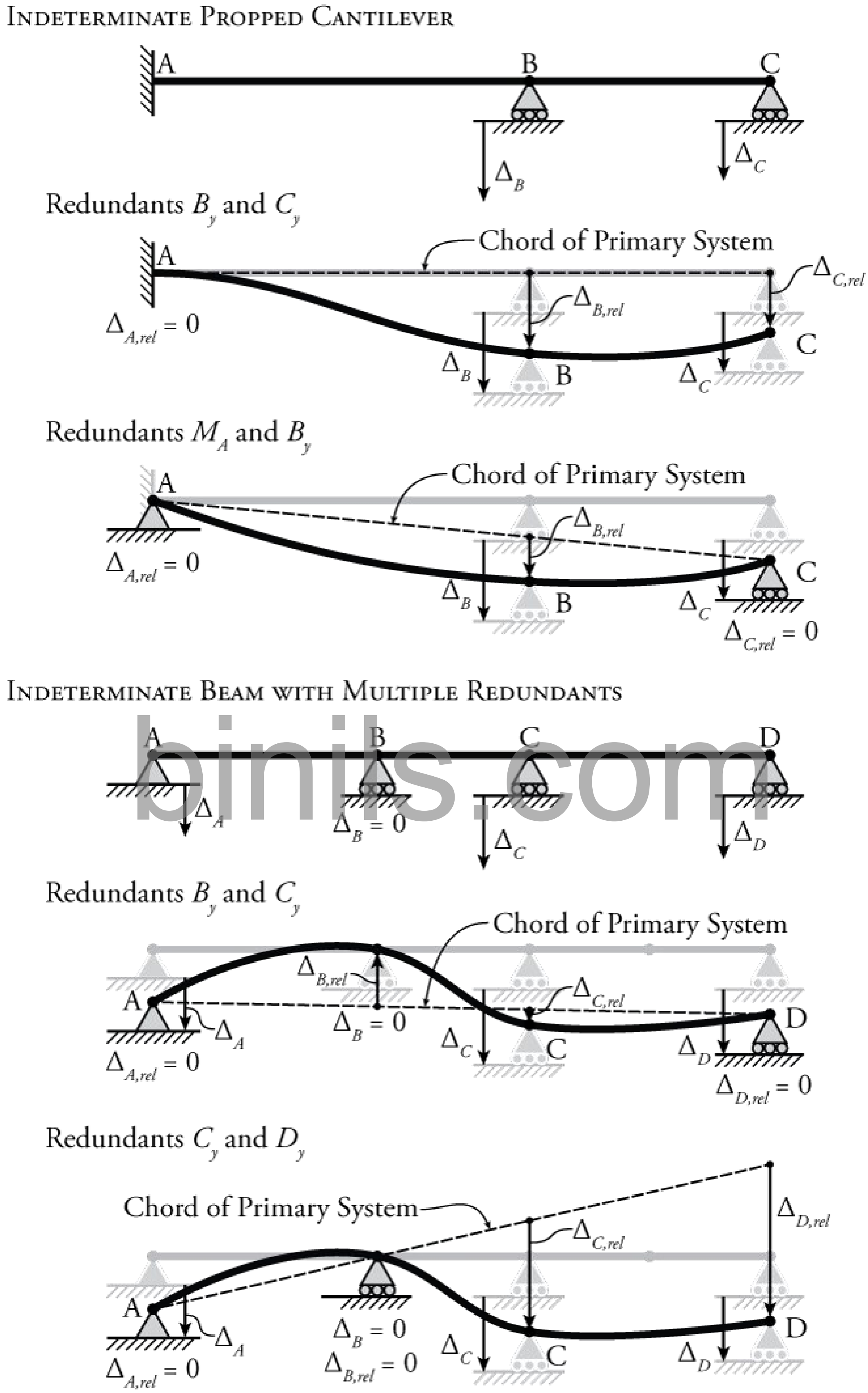
2.4.2. INTRODUCTION:

In the last lesson, the force method of analysis of statically indeterminate beams subjected to external loads was discussed. It is however, assumed in the analysis that the supports are unyielding and the temperature remains constant. In the design of indeterminate structure, it is required to make necessary provision for future unequal vertical settlement of supports or probable rotation of supports. It may be observed here that, in case of determinate structures no stresses are developed due to settlement of supports. The whole structure displaces as a rigid body. Hence, construction of determinate structures is easier than indeterminate structures.

The statically determinate structure changes their shape due to support settlement and this would in turn induce reactions and stresses in the system. Since, there is no external force system acting on the structures, these forces form a balanced force system by themselves and the structure would be in equilibrium. The effect of temperature changes, support settlement can also be easily included in the force method of analysis. In this lesson few problems, concerning the effect of support settlement are solved to illustrate the procedure.

2.4.3. SUPPORT DISPLACEMENTS:

The whole structure displaces as a rigid body. Hence, construction of determinate structures is easier than indeterminate structures. **The statically determinate structure changes their shape due to support settlement** and this would in turn induce reactions and stresses in the system.



Sup port settlements in continuous beams

2.4.4. NUMERICAL EXAMPLES ON(CONTINUOUS BEAMS):

PROBLEM NO:01

Analysis the continuous beam shown in fig.2.10,Calculate the support moments using slope deflection method.Support B settlememts by 10mm.Take E = 2 x 10^5 N/mm^2,I = 16 x 10^7 mm^4.Sketch the SF and BM diagrams.

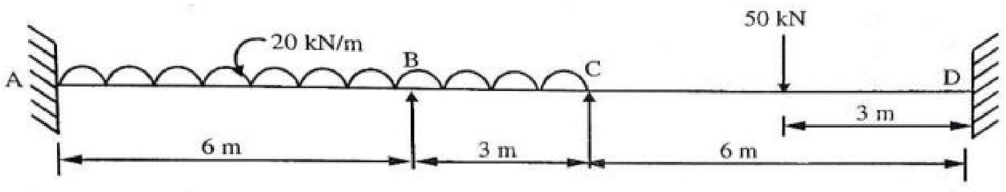


Fig. 2.10

Solution:

- **Fixed End Moments:**

MFAB = -Wl^2/12 = - 20x6^2/12 = - 60 kNm;

MFBA = Wl^2/12 = 20x6^2/12 = 60 kNm;

MFBC = -Wl^2/12 = - 20x3^2/12 = - 15 kNm;

MFCB = Wl^2/12 = 20x3^2/12 = 15 kNm;

MFCD = -Wl/8 = - 50x6/8 = - 37.5 kNm;

MFDC = Wl/8 = 50x6/8 = 37.5 kNm;

- **Slope Deflection Equations:**

MAB = MFAB + 2EI/6(2θA + θB + 3δ/l)

= - 60 + EI/3(0 + θB – 1/200) --- (1)

MBA = MFBA + 2EI/6(2θB + θA + 3δ/l)

= 60 = EI/3(2θB – 3 x 10/6000) --- (2)

MBC = MFBC + 2EI/3(2θB + θC + 3δ/l)

= - 15 + 2EI/3(2θB + θC + 1/100) --- (3)

MCB = MFCB + 2EI/3(2θC + θB + 3δ/l)

= 15 + 2EI/3(2θC + θB + 1/100) --- (4)

MCD = MFCD + 2EI/6(2θC + θD + 3δ/l)

= - 37.5 + EI/3(2θC) --- (5)

MDC = MFDC + 2EI/6(2θD + θC + 3δ/l)

= 37.5 + EI/3(θC) --- (6)

- **Joint Equilibrium Equations:**

Joint B:

MBA + MBC = 0

EI/3(6θB+2θC+3/200) = - 135 --- (7)

Joint C:

MCB + MCD = 0

EI(θB+3θC+1/100) = 33.75 --- (8)

Equvating (7 & 8);we get

θC = - 1/464; θB = - 1/402

- **Final Moments:**

MAB = - 139.843 kNm;

MBA = - 46.354 kNm;

MBC = 46.3 kNm;

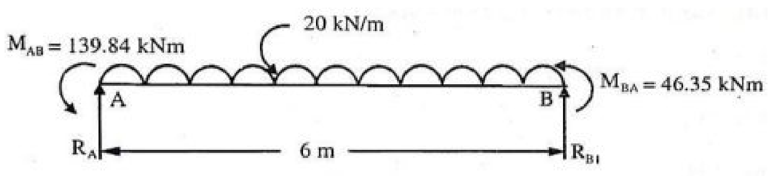
MCB = 83.35 kNm;

MCD = - 83.477 kNm;

MDC = 14.51 kNm;

- **To Draw S.F.D:**

Span AB:

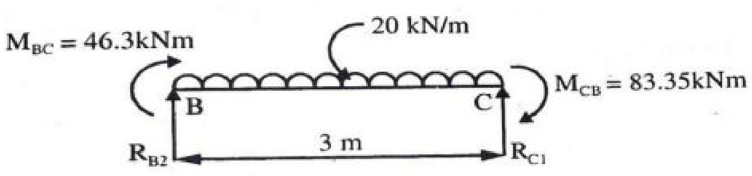


Taking moments about A.

$20 \times 6^2/2 - 46.35 - 139.84 - RB1(6) = 0$; $RB1 = 28.97$ KN

$RA = 20 \times 6 - 28.97$; $RA = 91.03$ KN

Span BC:

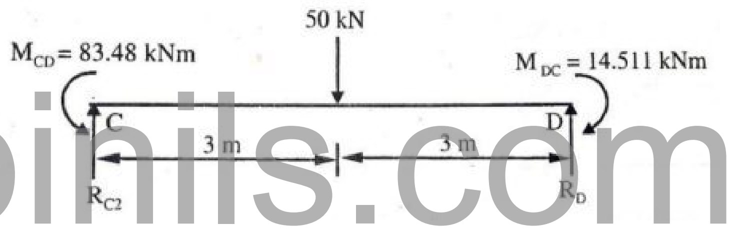


Taking moments about B.

$20 \times 3^2/2 + 83.35 + 46.3 - RC1(3) = 0$; $RC1 = 73.22$ KN

$RB2 = 20 \times 3 - 73.22$; $RB2 = - 13.21$ KN

Span CD:



Taking moments about C.

$14.511 + 50(3) - 83.48 - RD(6) = 0$;

$RD = 13.5$ KN; $RC2 = 36.5$ KN

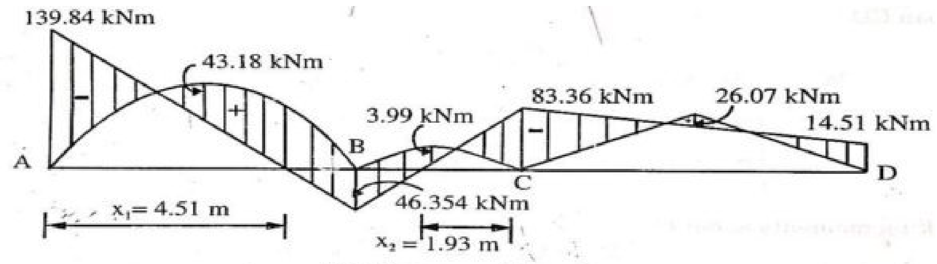
- **Free BMD:**

  $MAB = Wl^2/8 = 20 \times 6^2/8 = 90$ kNm

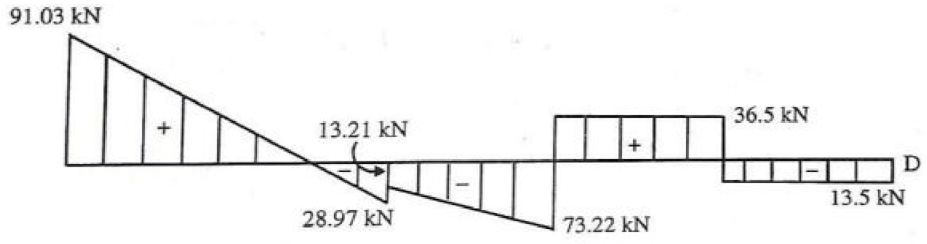
  $MBC = Wl^2/8 = 20 \times 3^2/8 = 22.5$ kNm

  $MCD = Wl/4 = 50 \times 6/4 = 75$ kNm

- **BMD and SFD:**



Bending moment diagram

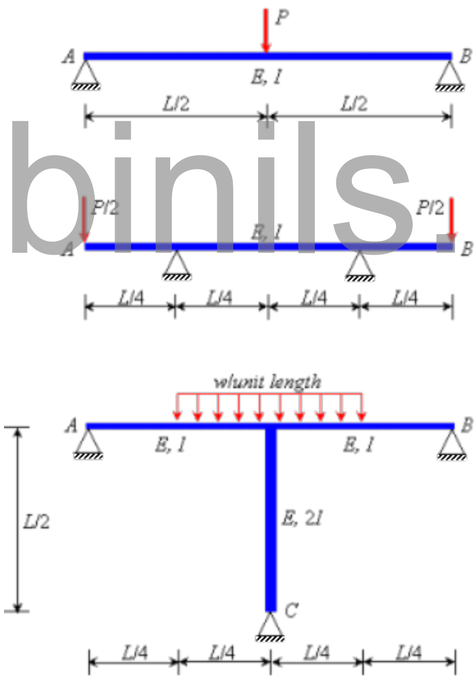


Shear force diagram

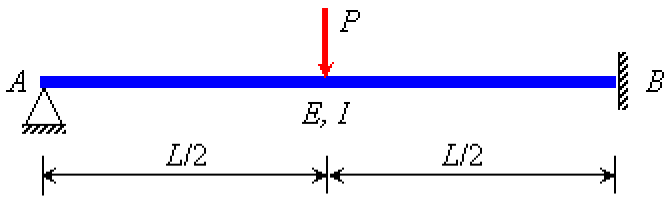
2.5 SYMMETRIC FRAMES WITH SYMMETRIC AND SKEW-SYMMETRIC LOADINGS

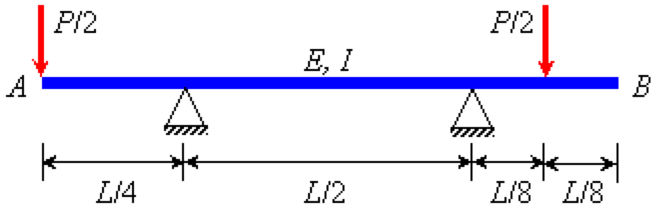
2.5.1 SYMMETRY AND ANTISYMMETRY

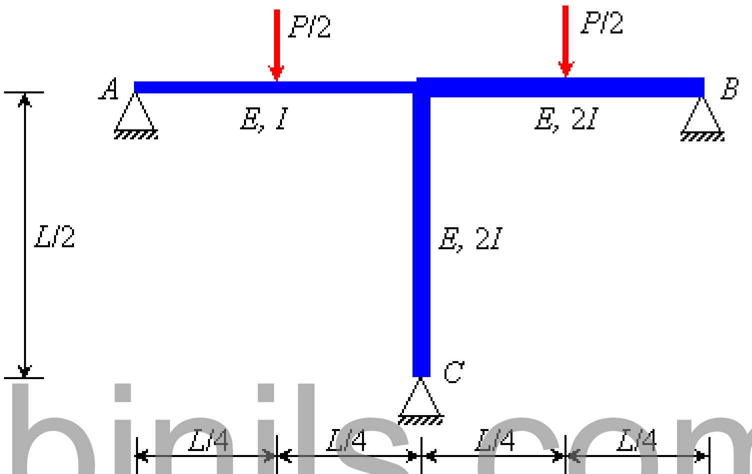
Symmetry or antisymmetry in a structural system can be effectively exploited for the purpose of analyzing structural systems. Symmetry and anti-symmetry can be found in many real-life structural systems (or, in the idealized model of a real-life structural system). It is very important to remember that when we say symmetry in a structural system, it implies the existence of symmetry both in the structure itself including the support conditions and also in the loading on that structure. The systems shown in Fig. are symmetric because, for each individual case, the structure is symmetric and the loading is symmetric as well. However, the systems shown in Fig. are not symmetric because either the structure or the loading is not symmetric.



1.1. Symmetric structural systems



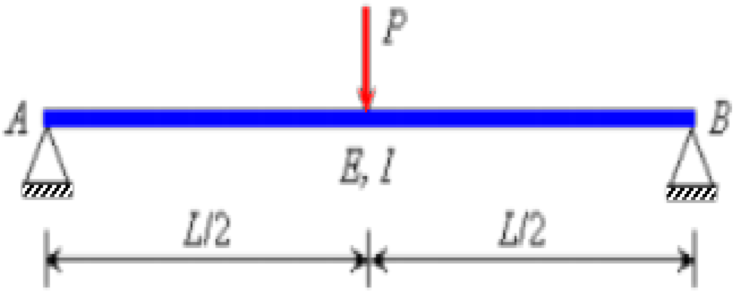


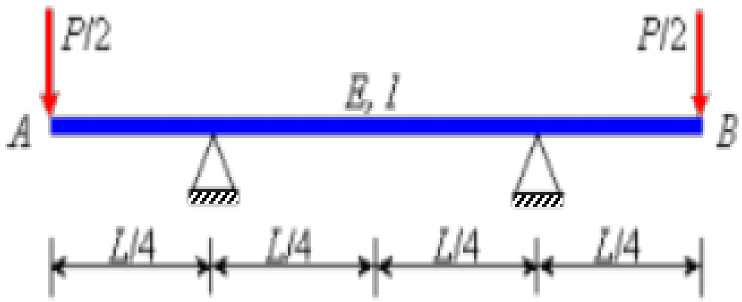


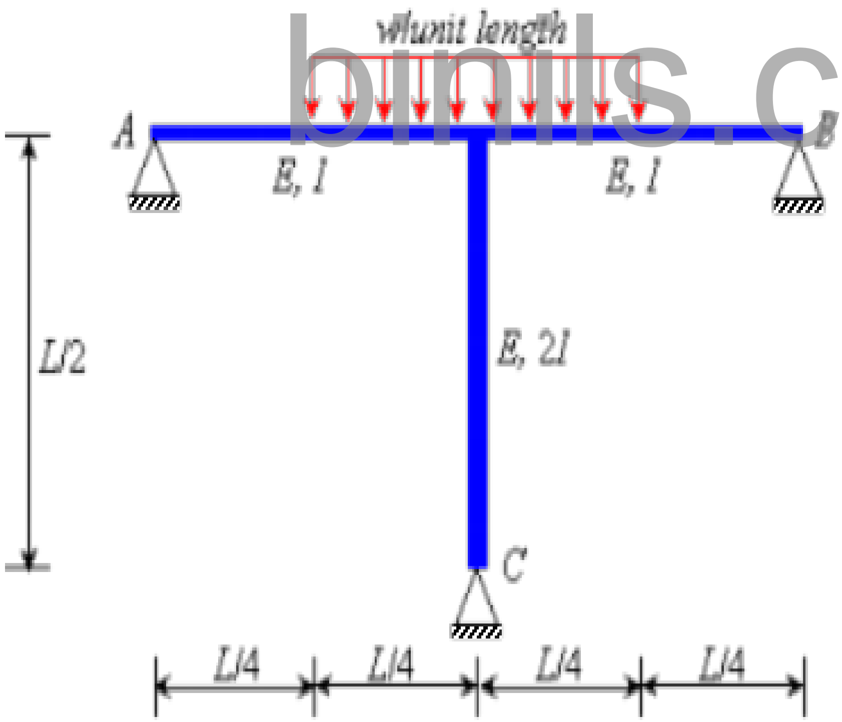
1.2. Non-symmetric (asymmetric) structural systems

For an antisymmetric system the structure (including support conditions) remains symmetric,however, the loading is antisymmetric.The fig.1.2 ,shows the example of antisymmetric structural systems.

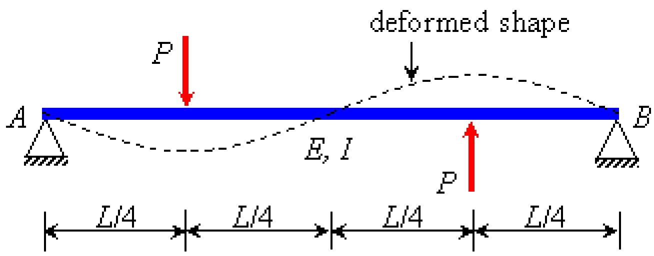
It is not difficult to see that the deformation for a symmetric structure will be symmetric about the same line of symmetry. This fact is illustrated in Fig. 1.3, where we can see that every symmetric structure undergoes symmetric deformation. It can be proved using the rules of structural mechanics (namely, equilibrium conditions, compatibility conditions and constitutive relations), that deformation for a symmetric system is always symmetric. Similarly, we always get antisymmetric deformation for antisymmetric structural systems

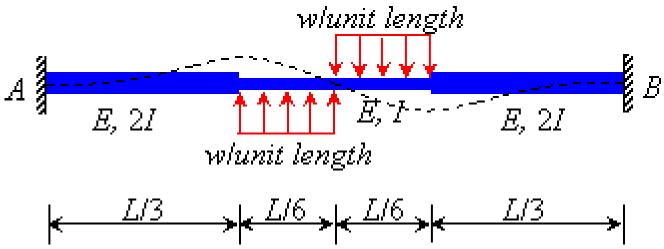


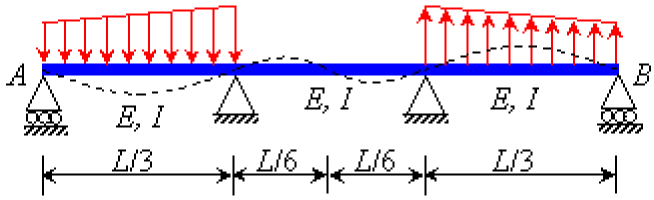




Deformation in s ymmetric systems

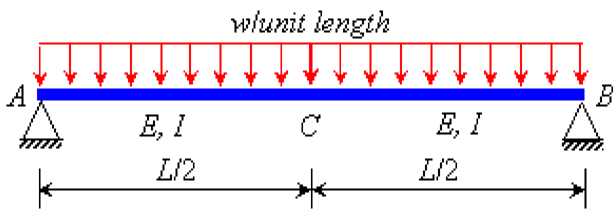




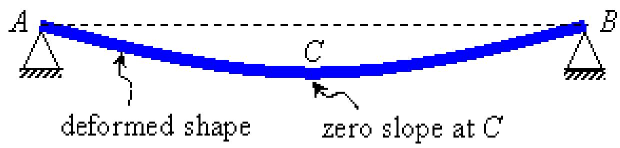


1.3. Deformation in antisymmetric systems

Let us look at beam AB in Fig. 1.20(a), which is symmetric about point C. The deformed shape of the structure will be symmetric as well (Fig. 1.20(b)). So, if we



(a)



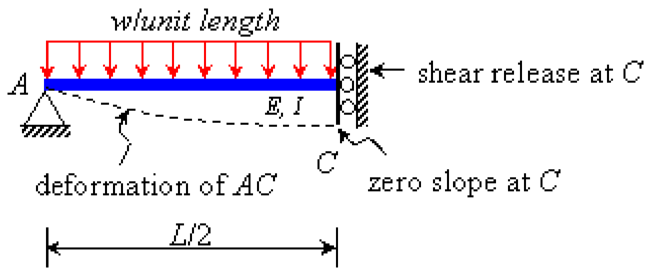
(b)

solve for the forces and deformations in part AC of the beam, we do not need to solve for part CB separately. The symmetry (or antisymmetry) in deformation gives us additional information prior to analyzing the structure and these information can be used to reduce the size of the structure that needs to be considered for analysis.

To elaborate on this fact, we need to look at the deformation condition at the point/line of symmetry (or antisymmetry) in a system. The following general rules about deformation can be deduced looking at the examples in Fig. 1.3 and Fig. 1.4:

- For a symmetric structure: slope at the point/line of symmetry is zero.
- For an antisymmetric structure: deflection at the point/line of symmetry is zero.

These information have to be incorporated when we reduce a symmetric (or antisymmetric) structure to a smaller one. If we want to reduce the symmetric beam in Fig. 1.20 to its one symmetric half AC , we have to integrate the fact the slope at point



1.6.Reduced system AC is adopted for analysis for beam AB

C for the reduced system AC will have to be zero. This will be a necessary boundary condition for the reduced system AC . We can achieve this by providing a support at C, which restricts any rotation, but allows vertical displacement, as shown in Fig. 1.6 (Note: this specific type of support is known as a “shear-release” or “shear-hinge”). Everything else (loading, other support conditions) remains unchanged in the reduced system. We can use this system AC for our analysis instead of the whole beam AB .

3.5.2 INTERNAL FORCE DIAGRAMS FOR A) A SYMMETRIC SYSTEM, AND B) AN ANTISYMMETRIC SYSTEM

Having a priory knowledge about symmetry/antisymmetry in the structural system and in its deformed shape helps us know about symmetry/antisymmetry in internal forces in that system. (Symmetry in the system implies symmetry in equilibrium and constitutive relations, while symmetry in deformed shape implies symmetry in geometric compatibility.) Internal forces in a symmetric system are also symmetric about the same axis and similarly antisymmetric systems have antisymmetric internal forces. Detailed discussion on different types of internal forces in various structural systems and on internal force diagrams are provided in the next module (Module 2: Analysis of Statically Determinate Structures). Once we know about these diagrams we can easily see the following:

- A symmetric beam-column system has a symmetric bending moment diagram.
- A symmetric beam-column system has an antisymmetric shear force diagram.
- An antisymmetric beam-column system has an antisymmetric bending moment diagram.
- An antisymmetric beam-column system has a symmetric shear force diagram.