STRAIN ENERGY METHOD

(Introduction)

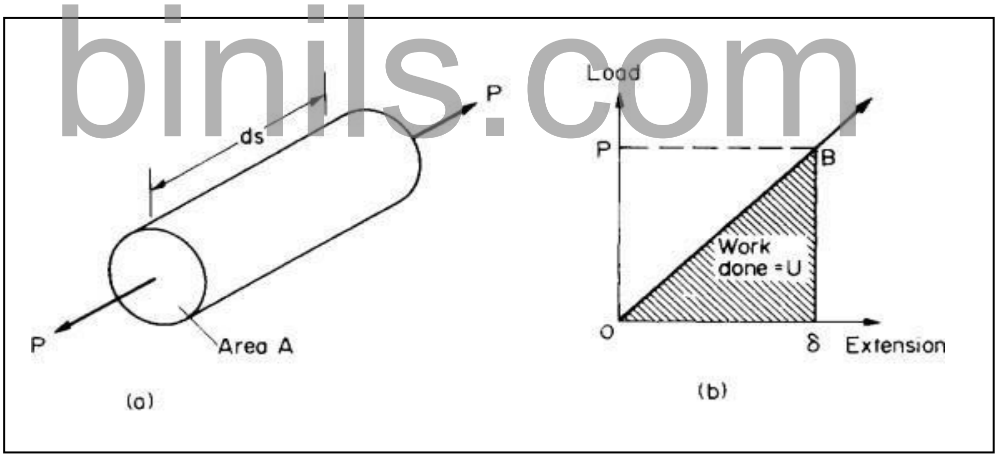
Strain Energy

Strain energy is as the energy which is stored within a material when work has been done on the material. Here it is assumed that the material remains elastic whilst work is done on it so that all the energy is recoverable and no permanent deformation occurs due to yielding of the material,

Strain energy U = work done

Thus for a gradually applied load the work done in straining the material will be given by the shaded area under the load-extension graph of Fig.7.1

$U = P\delta$



Work done by a gradually applied load.

The unshaded area above the line OB of Fig. 7.1 is called the complementary energy, a quantity which is utilized in some advanced energy methods of solution and is not considered within the terms of reference of this text.

Strain Energy (Tension or Compression)

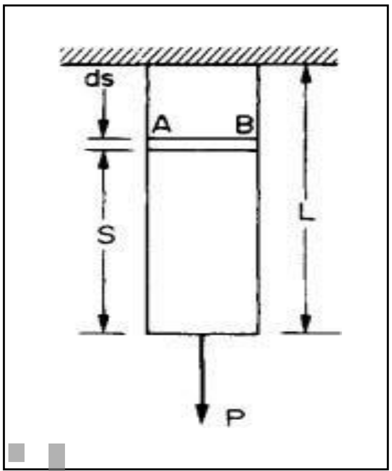
(a) Neglecting the weight of the bar: -

Consider a small element of a bar, length ds, shown in Fig. 7.1. If a graph is drawn of load against elastic extension the shaded area under the graph gives the work done and hence the strain energy,

Strain energy; $U=P^2L / 2AE$

(b) Including the weight of the bar: -

Consider now a bar of length L mounted vertically, as shown in Fig. 7.2. At any section A B the total load on the section will be the external load P together with the weight of the bar material below AB.

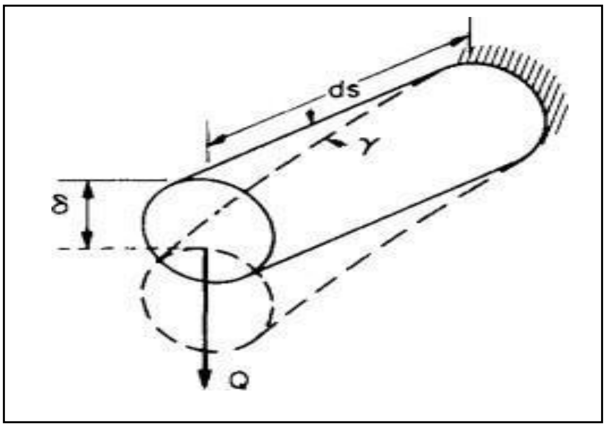


Direct load - tension or compression.

Strain energy; $U=P^2L/2AE + P\rho gL^2/2E + (\rho g)^2 AL^3/6E$

Strain Energy : Shear

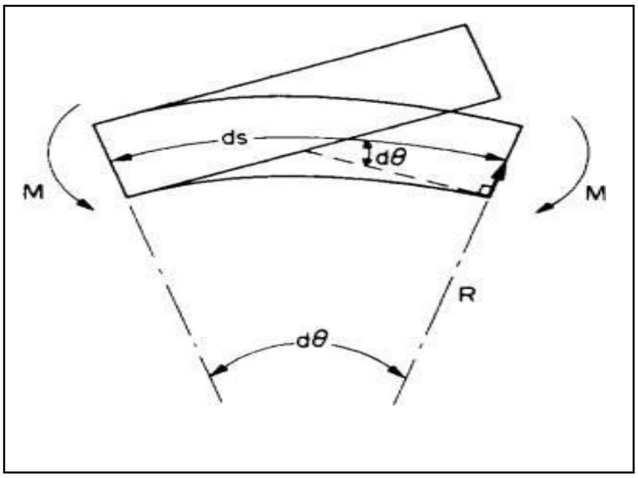
Consider the elemental bar now subjected to a shear load Q at one end causing deformation through the angle y (the shear strain) and a shear deflection 6, as shown in Fig.



Strain energy; $U=Q^2L / 2AG$

Strain Energy – Bending

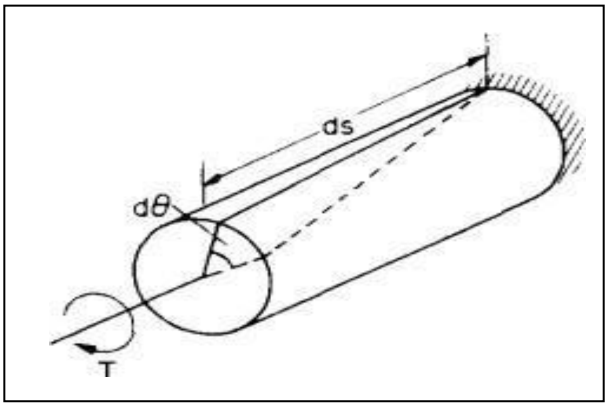
Let the element now be subjected to a constant bending moment M causing it to bend into an arc of radius R and subtending an angle $d\Theta$ at the center (Fig. 7.4). The beam will also have moved through an angle $d\Theta$.



Strain energy; $\mathbf{U= M^2 L / 2EI}$

Strain Energy - Torsion

The element is now considered subjected to a torque T as shown in Fig. producing an angle of twist $d\Theta$ radians.



Strain energy; $\mathbf{U=T^2 L / 2GJ}$

Note: - It should be noted that in the four types of loading case considered above the strain energy expressions are all identical in form,

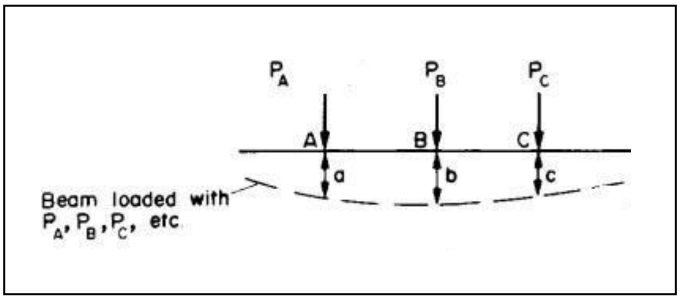
U = (unit applied load)^{2}x L / 2(product of two related constants)

Castigliano's first theorem assumption for deflection: -

If the total strain energy of a body or framework is expressed in terms of the external loads and is partially differentiated with respect to one of the loads the result is the deflection of the point of application of that load and in the direction of that load,

i.e, a=ΔU/ΔPa , b=ΔU/ΔPb and c=ΔU/Δpc

Where a,b and c are deflections of a beam under loads P_a, P_b and P_c etc. as shown in fig 7.6.



In most beam applications the strain energy, and hence the deflection, resulting from end loads and shear forces are taken to be negligible in comparison with the strain energy resulting from bending (torsion not normally being present),

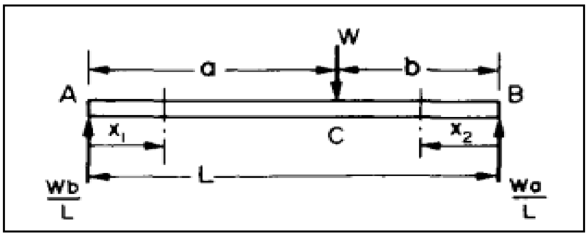
Therefore deflection; **Δ = ρv / ρp**

Application of Castigliano's theorem to angular movements:

If the total strain energy, expressed in terms of the external moments, be partially differentiated with respect to one of the moments, the result is the angular deflection (in radians) of the point of application of that moment and in its direction.

Example 1: -

Using Castigliano's first theorem, obtain the expressions for (a) the deflection under a single concentrated load applied to a simply supported beam as shown in Figure below, (b) the deflection at the center of a simply supported beam carrying a uniformly distributed load.



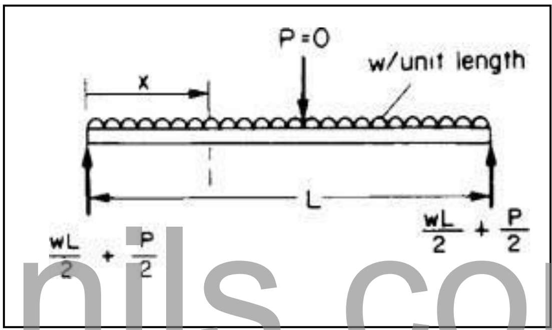
$$\delta = \int_B^A \frac{M}{EI}\frac{\partial M}{\partial W} ds$$

$$= \int_A^C \frac{M}{EI}\frac{\partial M}{\partial W} ds + \int_C^B \frac{M}{EI}\frac{\partial M}{\partial W} ds$$

$$= \frac{1}{EI}\int_0^a \frac{Wbx_1}{L} \times \frac{bx_1}{L} \times dx_1 + \frac{1}{EI}\int_0^b \frac{Wax_2}{L} \times \frac{ax_2}{L} \times dx_2$$

$$= \frac{Wb^2}{L^2EI}\int_0^a x_1^2 dx_1 + \frac{Wa^2}{L^2EI}\int_0^b x_2^2 dx_2$$

$$= \frac{Wb^2a^3}{3L^2EI} + \frac{Wa^2b^3}{3L^2EI} = \frac{Wa^2b^2}{3L^2EI}(a+b) = \frac{Wa^2b^2}{3LEI}$$



$$\delta = \int_0^L \frac{Mm}{EI} ds = 2\int_0^{L/2} \frac{Mm}{EI} ds$$

$$M = \frac{wL}{2} - \frac{wx^2}{2} \quad \text{and} \quad m = \frac{x}{2}$$

$$\delta = \frac{2}{EI}\int_0^{L/2} \left(\frac{wLx}{2} - \frac{wx^2}{2}\right)\frac{x}{2} dx$$

$$= \frac{1}{2EI}\int_0^{L/2} (wLx^2 - wx^3) dx$$

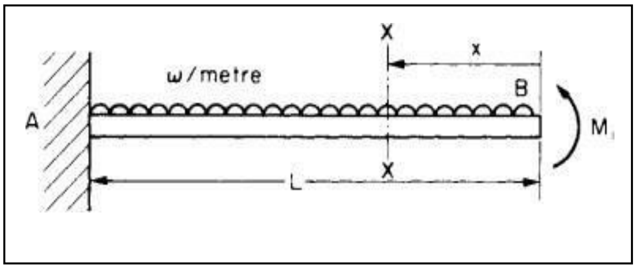
$$\delta = \frac{w}{2EI}\left[\frac{Lx^3}{3} - \frac{x^4}{4}\right]_0^{L/2}$$

$$= \frac{wL^4}{2EI}\left[\frac{1}{24} - \frac{1}{64}\right]$$

$$= \frac{wL^4}{2EI}\left[\frac{8-3}{192}\right] = \frac{5WL^4}{384EI}$$

Example 2: -

Derive the equation for the slope at the free end of a cantilever carrying a uniformly distributed load over its full length.



$$M = M_i - \frac{wx^2}{2}$$

$$\frac{\partial M}{\partial M_i} = 1$$

$$\theta = \int_0^L \frac{M}{EI}\cdot\frac{\partial M}{\partial M_i}\cdot dx$$

$$= \frac{1}{EI}\int_0^L \left(M_i - \frac{wx^2}{2}\right)(1)\,dx$$

$$\theta = \frac{-w}{2EI}\int_0^L x^2 . dx = \frac{wL^3}{6EI} \text{ radian}$$

1.2 DEGREE OF STATIC & KINEMATIC INDETERMINACY OF STRUCTURES

A structural system that can be analyzed by using the equation of static equilibrium only is called statically determinate structure i.e. reaction components and internal stresses can be calculated using static equilibrium equations only. Eg: Simply supported beam. If it cannot be analyzed by the equation of static equilibrium alone, then it is called a statically indeterminate structure. Eg: Fixed beam. A structural system is said to be kinematically indeterminate if the displacement components of its joints cannot be determined by the compatibility equation alone. Eg: Simply supported beam. If those unknown quantities can be found by using compatibility equations alone then the structure is called kinematically determinate structure. Eg: Fixed beam. But before calculating the degree of indeterminacy of a structure, it is good to know about its stability. If a structure is unstable, then it doesn't matter whether it is statically determinate or indeterminate. In all cases, such types of structures should be avoided in practice.

1.2.1 STABILITY OF A STRUCTURE

The stability of a structure is of the following types:

1. External Stability

2. Internal Stability

EXTERNAL STABILITY

For a structure to be externally stable, the reactive forces should be non-parallel & non-concurrent.

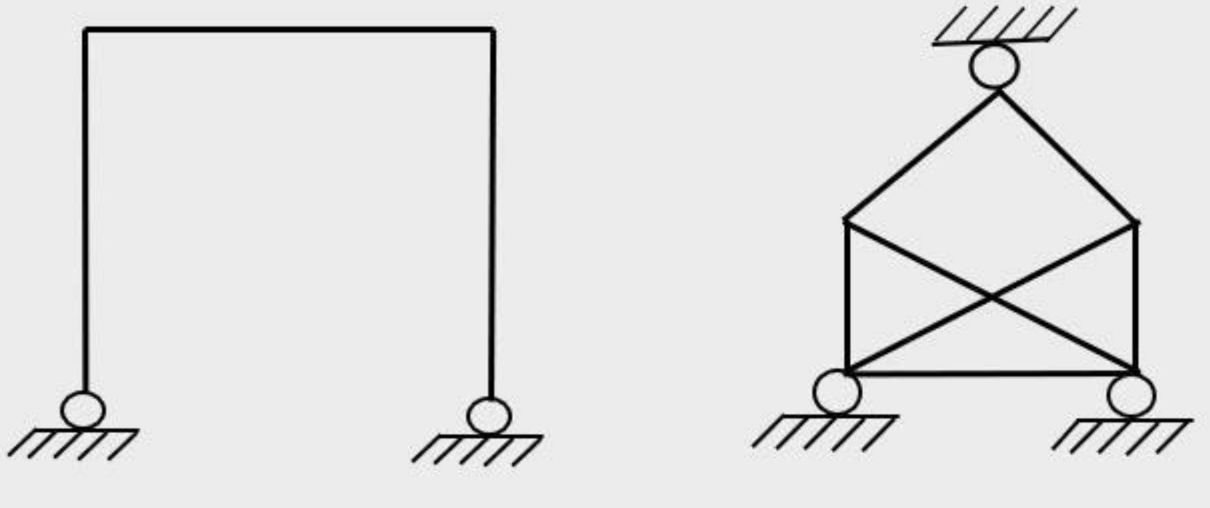


Figure: Externally Unstable System

INTERNAL STABILITY

An internally unstable system can change its shape without any deformation of its members.

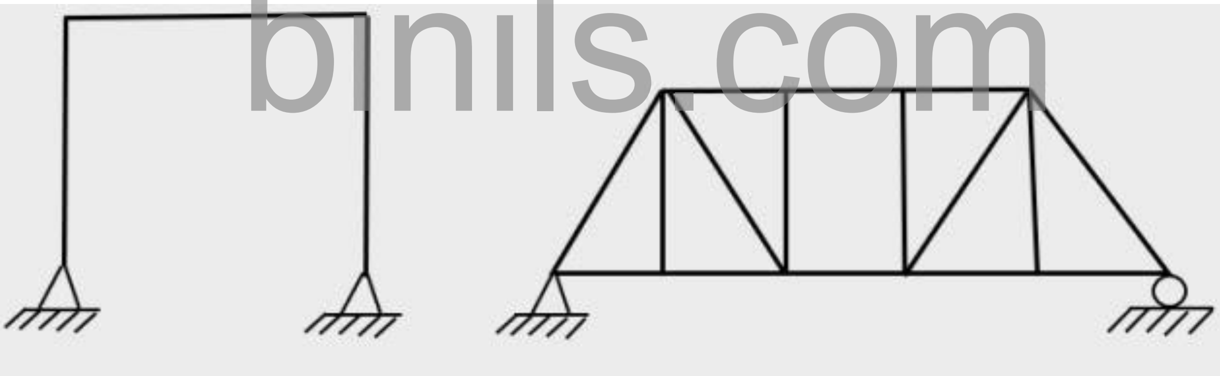
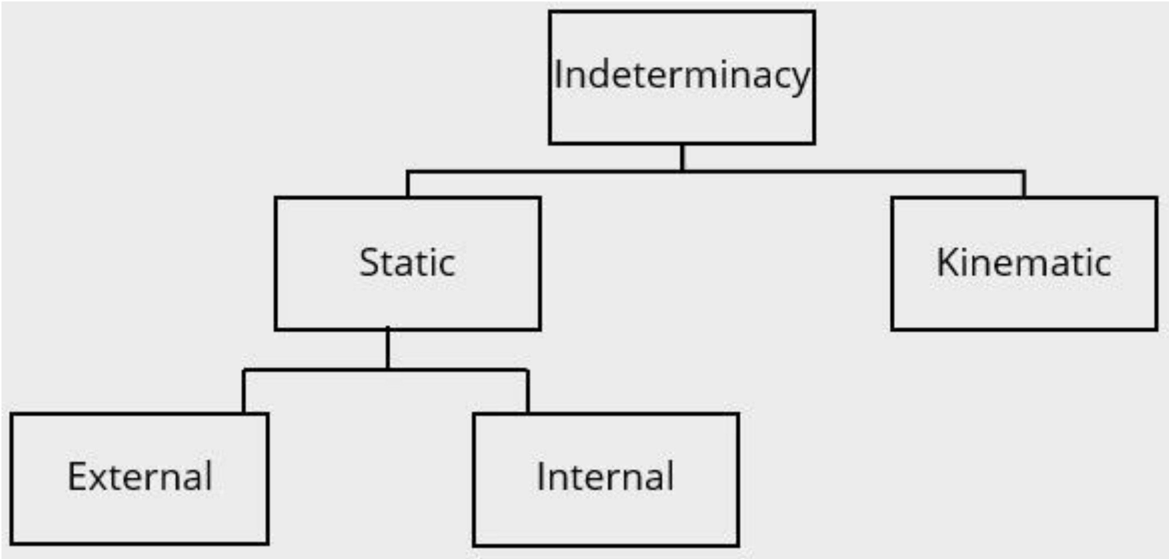


Figure: Internally Unstable System

INDETERMINACY OF A STRUCTURE

The degree of static indeterminacy or degree of redundancy shows the number of additional equations required to solve the system. External indeterminacy is related to the support system of the structure whereas internal indeterminacy is related to the internal stress in the members of the structure. The degree of kinematic indeterminacy shows the number of unknown joint deformations.



The degree of static indeterminacy or degree of redundancy shows the number of additional equations required to solve the system. External indeterminacy is related to the support system of the structure whereas internal indeterminacy is related to the internal stress in the members of the structure. The degree of kinematic indeterminacy shows the number of unknown joint deformations.

1.2.2 DEGREE OF STATIC INDETERMINACY OF PIN JOINTED PLANE FRAMES (PLANE TRUSS)

The simplest form of a stable plane truss is a triangle having three members & three joints. Then, for each additional joint, two extra members are required. In this way, a stable & internally determinate truss system can be formed. Thus, considering all the joints except the first three joints (that form the basic triangle), 2(j-3) number of members will be required. And then adding the first three members that form the basic triangle, the total number of members in a truss can be formulated as below:

$m = 2(j-3) + 3$

or, $m = 2(j-6) + 3$

or, $m = 2(j-3)$

where,

m = number of members

j = number of joints

Hence, if a plane truss is formed with 2j - 3 members (since, m = 2j -3) & if they are arranged in a suitable manner that the system cannot change its shape without deformation of its members, then a stable & internally determinate plane truss is formed.

Thus, the Degree of internal indeterminacy for a plane truss is m - (2j - 3).

The equations of static equilibrium available in order to find reaction forces in a plane truss is 3 (H = 0, V = 0, M= 0). If 'r' be the number of unknown reaction forces, then the degree of external indeterminacy related to the support system is
r - 3.

Thus, the Degree of external indeterminacy for a plane truss is r - 3.

Finally, the total degree of static indeterminacy is the summation of the degree of internal indeterminacy & the degree of external indeterminacy.

An alternative approach for the determination of the degree of indeterminacy of a structure is to take a unified view without considering external and internal indeterminacies separately. In a plane truss, the total number of unknowns will be (m + r) as there will be a single unknown force (either tensile or compressive) in each member & the remaining unknowns will be reaction forces. Also, 2j independent equations will be available as two equations of static equilibrium (H = 0, V = 0) are applicable in each joint. Thus, the total degree of static indeterminacy for a pin-jointed plane frame is (m + r) - 2j.

CONCLUSION

Degree of external indeterminacy: r - 3

Degree of internal indeterminacy: m - (2j -3)

Degree of static indeterminacy: Degree of external Indeterminacy + Degree of internal indeterminacy

if m < 2j - 3, internally unstable

if m = 2j - 3, stable/unstable, statically determinate internally if stable internally

if m > 2j - 3, overstiff, statically indeterminate internally

Alternatively,

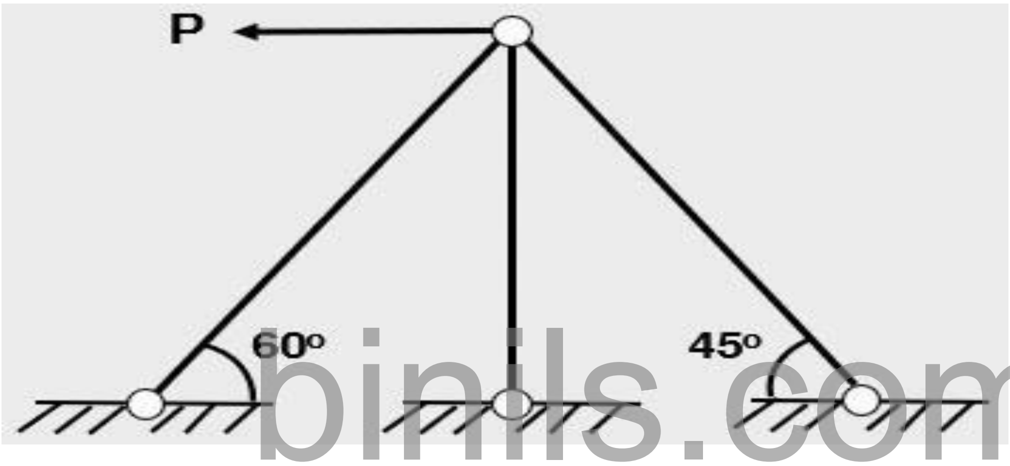
Degree of static indeterminacy: (m + r) - 2j

if (m + r) < 2j, unstable

if (m + r) = 2j, stable, statically determinate

if (m + r) > 2j, stable, statically indeterminate

Exampe;



Degree of external indeterminacy = r - 3 = 6 - 3 = 3

Degree of internal indeterminacy = m - (2j - 3) = 3 - (2*4 - 3) = -2

Degree of static indeterminacy = External + Internal = 3 -2 = 1

Alternatively,

Degree of static indeterminacy = (m + r) - 2j = (3 + 6) - 2*4 = 1

1.2.3 Degree of Static Indeterminacy of Rigid Jointed Plane Frame(Plane Frame)

In a plane frame, the total number of unknowns will be (3m + r) as there will be three unknowns (axial force, shear force & bending moment) in each member & the remaining unknowns will be reaction forces. Also, (3j + c) independent equations will be available as three equations of static equilibrium (H = 0, V = 0, M = 0) is applicable in each joints plus 'c' equations of condition will also be available in some cases (eg: internal hinge, internal roller etc).

Thus, the total degree of static indeterminacy for a rigid jointed plane frame is (3m + r) - (3j + c).

Here, m, r & j have their usual meaning & are easy to determine but the equation of condition 'c' needs to be understood well before moving on. The equation of condition provides extra equations in addition to the three equations of static equilibrium. Let's say, an internal hinge is provided at the center of the prop cantilever beam, then the moment about the internal hinge is zero from either side of the internal hinge.

This means, two equations can be written by equating total moments to zero from the left side of the internal hinge & from the right side of the internal hinge. But these two equations are not independent because if the moment of all the forces left of the internal hinge is zero then the moment of forces on the right of the internal hinge is zero by default since the summation of the moment at an internal hinge is always zero.

Thus, only one equation of condition (i.e. c = 1) will be available in such a case. If there are three members meeting at a single internal hinge (like in complex frames), then three equations can be written equating moments to zero. But there will be only two independent equations (i.e. c = 2) available since if two members at a hinge provide zero moments then the moment of the third member is automatically zero as the sum of moments at an internal hinge is zero. Now, while calculating indeterminacies, the value of 'c' can be determined by the simple formulation as below:

Equation of Condition(c) = Number of members connected to an internal hinge – 1

CONCLUSION

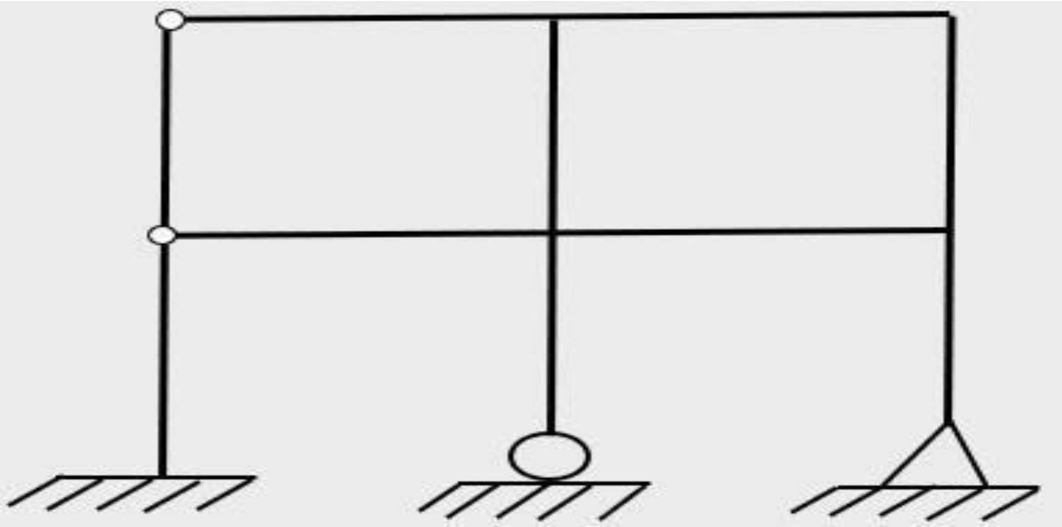
Degree of static indeterminacy: (3m + r) - (3j + c)

if (3m + r) < (3j + c), unstable

if (3m + r) = (3j + c), stable/unstable, statically determinate if stable

if (3m + r) > (3j + c), stable/unstable, statically indeterminate

Examble;



Degree of static indeterminacy = (3m+r) - (3j+c)

= (3*10+6) - (3*9+3) since, c = (3-1) + (2-1)

= 36 - 30

= 6

1.2.4 DEGREE OF KINEMATIC INDETERMINACY OF PIN JOINTED PLANE FRAME (PLANE TRUSS)

Each joints of a plane truss has two independent displacement components as they can have translation along x & y direction. The degree of kinematic indeterminacy (or degree of freedom) for a plane truss can be written as: 2j - e

Where;

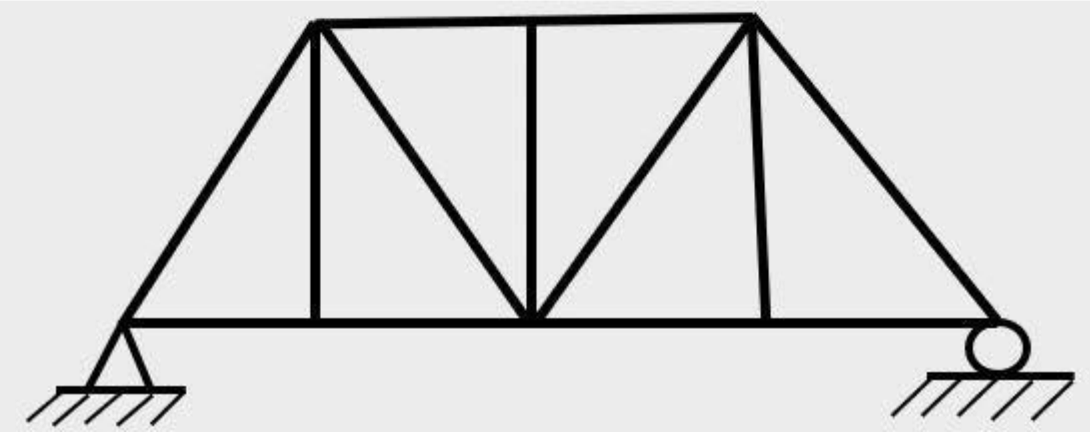
e - number of compatibility (boundary) conditions known.

As all the members of a truss system are considered to be extensible (or axial deformation is considered for every truss), the boundary conditions(value of e) will be obtained only from support conditions & will be equal to the number of reaction forces (r). Hence, the degree of kinematic indeterminacy for a plane truss is equal to 2j - r.

Conclusion

Degree of kinematic indeterminacy for plane truss: 2j – r

Examble;



Degree of kinematic indeterminacy = 2j - r = 2*8 - 3 = 13

1.2.5 DEGREE OF KINEMATIC INDETERMINACY OF RIGID JOINTED PLANE FRAME (PLANE FRAME)

Each joint of a plane frame has three independent displacement components as they can have translation along x & y-direction as well as rotation about the z-axis. So,the degree of kinematic indeterminacy for a plane frame can be written as: 3j - e

If the members of the plane frame are considered to be extensible, then the external supports only will provide the boundary conditions & the number of compatibility equations(e) will be equal to the number of reaction forces (r). The degree of kinematic indeterminacy for a plane frame is then equal to 3j - r.

But if the members of the plane are considered to be inextensible, then boundary conditions will also be obtained from inextensibility in addition to support conditions. The number of additional boundary conditions due to inextensibility in an unbraced plane frame is equal to the number of members in the frame system.

And hence, the degree of kinematic indeterminacy for the unbraced plane frame is equal to 3j - (r + m). This is valid only for an unbraced plane frame system. In the case of a plane braced frame, it is convenient to count the degree of freedom for each joint by visual inspection rather than using a formula.

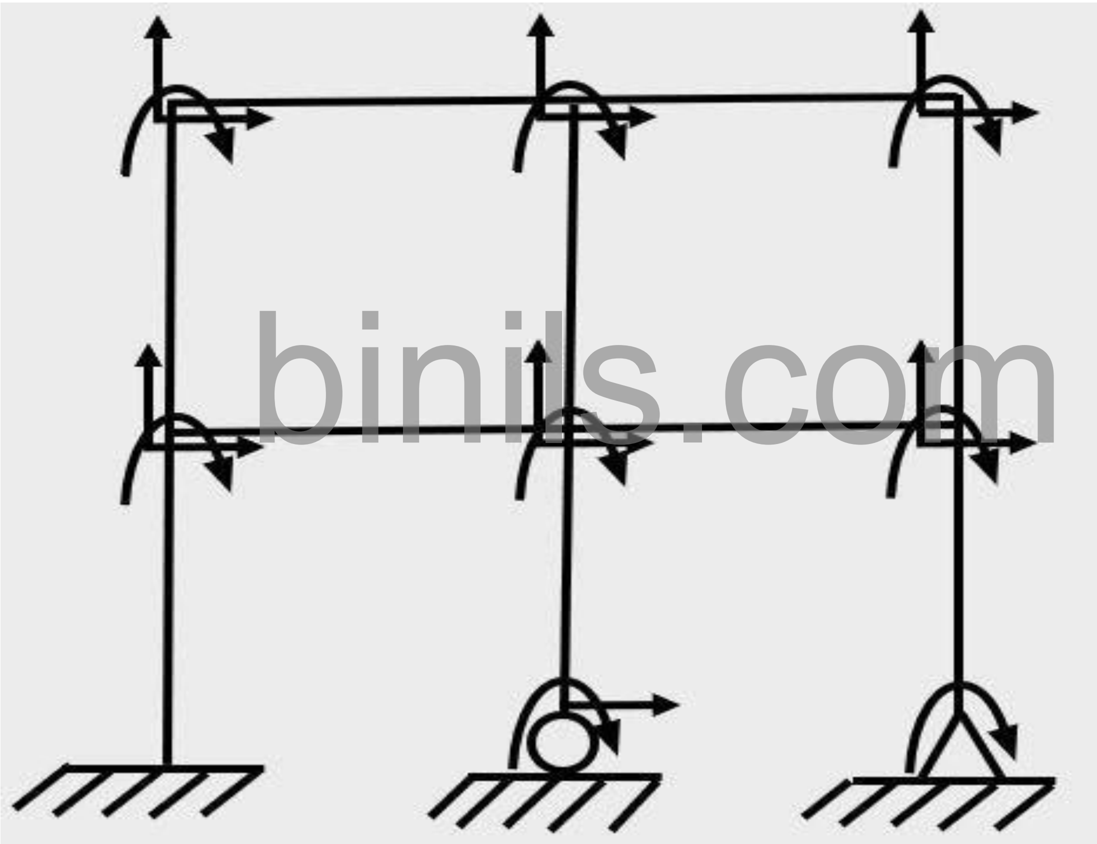
CONCLUSION

If plane frame is extensible, degree of kinematic indeterminacy: 3j - r

If plane frame is inextensible & unbraced, Degree of kinematic indeterminacy: 3j - (r + m)

If plane frame is inextensible & braced, count the degree of freedom by visual inspection.

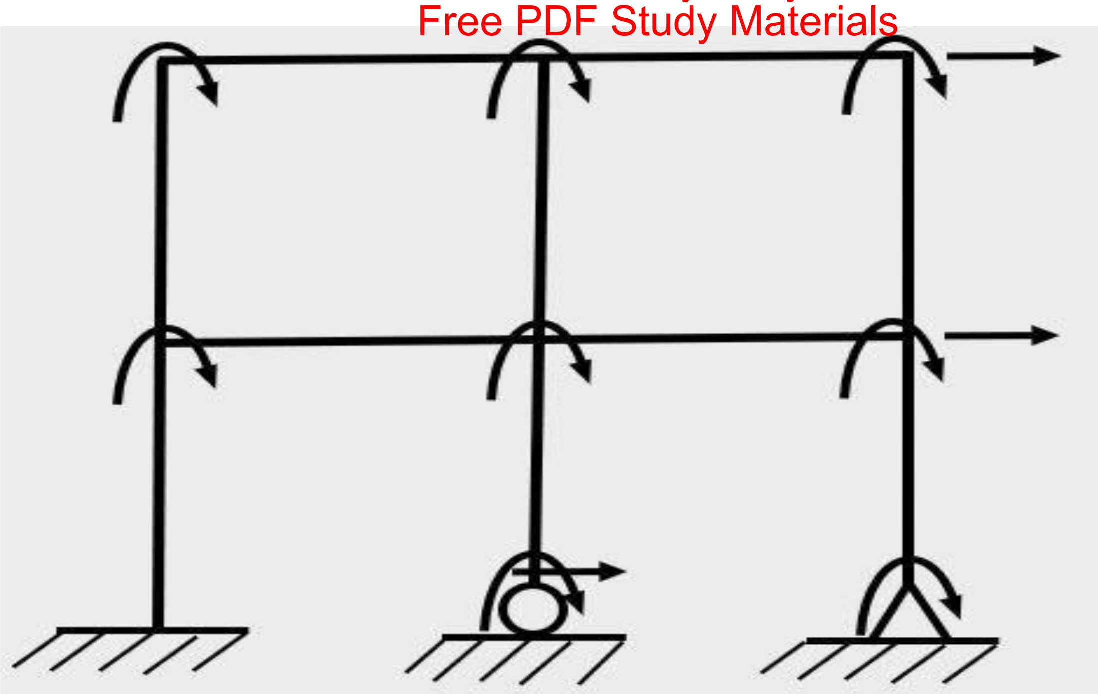
Example 1; (Unbraced Extensible Frame)



Assuming frame to be extensible,

Degree of kinematic indeterminacy = 3j - r = 3*9 - 6 = 21

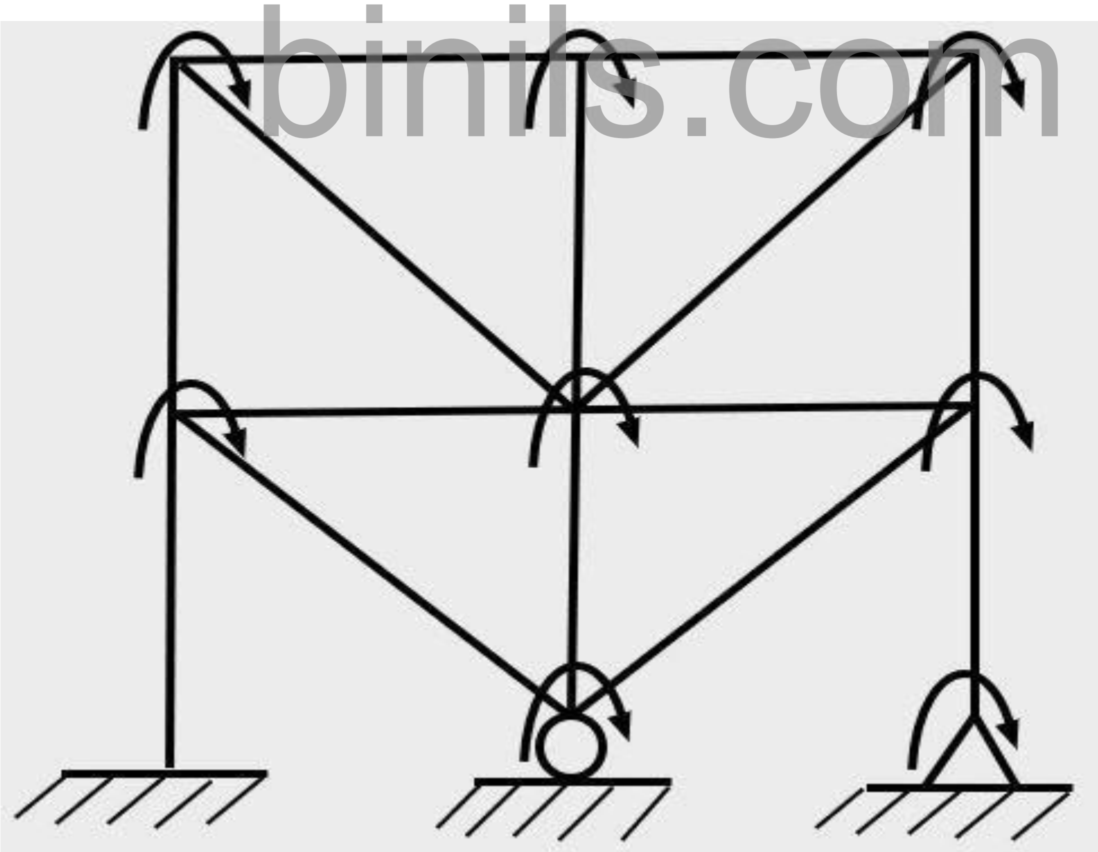
Example 2; (Unbraced inextensible)



Assuming frame to be inextensible,

Degree of kinematic indeterminacy = 3j - (r + m) = 3*9 - (6+10) = 11

Example 3; (Braced Frame)



By visual inspection,

Degree of kinematic indeterminacy = 8

1.3 ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES BY STRAIN ENERGY METHOD

(CONTINUOUS BEAMS)

Problem No:01

Determine the support reactions of the continuous beam as shown in Figure 5.24(a) if the beam is assumed to be subjected to a linear temperature gradient such that the top surface of the beam is at temperature T_t and lower at T_b .The beam is uniform havingflexural rigidity as EI and depth *d* . The coefficient of thermal expansion for beam material is α .

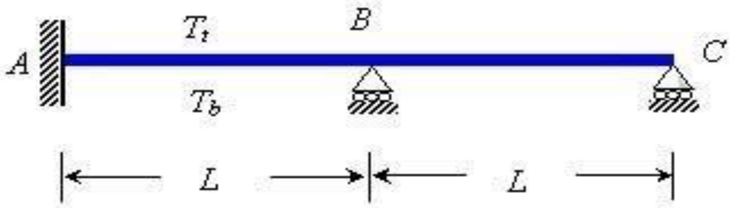


Figure 5.24(a)

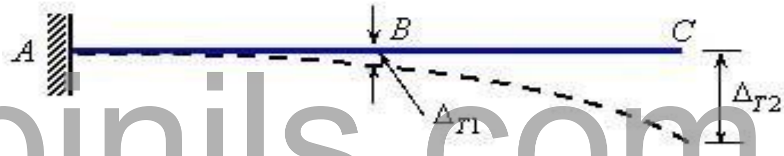


Figure 5.24(b)

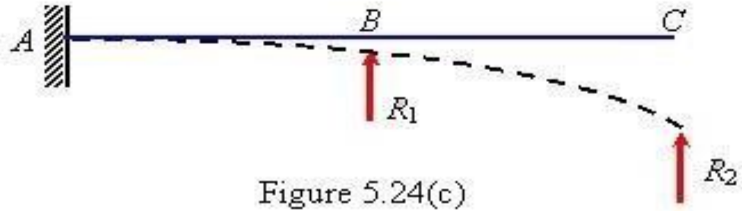


Figure 5.24(c)

[Source: "Structural Analysis – I,Vol – 1" by Bhavikatti]

Solution:

The degree of static indeterminacy =2. Remove the supports at B and C and allow the beam to deflect freely under the temperature variation. The deflection of the points B and C of the beam due to temperature variation

$$\Delta_{T1} = \frac{\alpha(T_t - T_b)L^2}{2d} \quad \text{----} \quad (1)$$

$$\Delta_{T2} = \frac{2\alpha(T_t - T_b)L^2}{d} \qquad ---- \quad (2)$$

Apply the forces R_1 and R_2 at point B and C , respectively. According to Castigliano's theorem

$$\frac{\partial U}{\partial R_1} = \Delta_{T1} = \frac{\alpha(T_t - T_b)L^2}{2d}$$

$$\frac{\partial U}{\partial R_2} = \Delta_{T2} = \frac{2\alpha(T_t - T_b)L^2}{d}$$

Consider BC : (x measured from C)

$$M_x = R_2 x, \quad \frac{\partial M_x}{\partial R_1} = 0, \qquad \frac{\partial M_x}{\partial R_2} = x$$

$$\frac{\partial U_{BC}}{\partial R_1} = 0$$

$$\frac{\partial U_{BC}}{\partial R_2} = \frac{1}{EI}\int_0^L R_2 x.x.dx = \frac{R_2 L^3}{3EI}$$

Consider AB : (x measured from B)

$$M_x = R_1 x + R_2(L + x)$$

$$\frac{\partial M_x}{\partial R_1} = x, \qquad \frac{\partial M_x}{\partial R_2} = (L + x)$$

$$\frac{\partial U_{AB}}{\partial R_1} = \frac{1}{EI}\int_0^L \{R_1 x + R_2(L + x)\} x dx = \frac{R_1 L^3}{3EI} + \frac{5R_2 L^3}{6EI}$$

$$\frac{\partial U_{AB}}{\partial R_2} = \frac{1}{EI}\int_0^L \{R_1 x + R_2(L + x)\}(L + x)dx = \frac{5R_1 L^3}{6EI} + \frac{7R_2 L^3}{3EI}$$

Thus,

$$\frac{\partial U}{\partial R_1} = \frac{R_1 L^3}{3EI} + \frac{5R_2 L^3}{6EI} = \frac{\alpha(T_t - T_b)L^2}{2d}$$

or $$2R_1 + 5R_2 = \frac{3EI\alpha(T_t - T_b)}{dL} \qquad \text{(V)}$$

Similarly,

$$\frac{\partial U}{\partial R_2}=\frac{5R_1L^3}{6EI}+\frac{7R_2L^3}{3EI}+\frac{R_2L^3}{3EI}=\frac{2\alpha(T_t-T_b)L^2}{d}$$

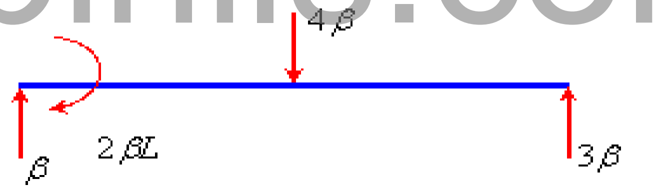
or

$$5R_1+16R_2=\frac{12EI\alpha(T_t-T_b)}{dL} \quad \text{----- (VI)}$$

Solving eqs. (v) and (vi)

$$R_1=-\frac{12EI\alpha(T_t-T_b)}{7dL} \quad \text{and} \quad R_2=\frac{9EI\alpha(T_t-T_b)}{7dL}$$

The reactions of the beam are shown in Figure 5.24(d).



$$\beta=\frac{3EI\alpha(T_t-T_b)}{7dL}$$

Figure 5.24(d) Reactions of the beam

[Source: "Structural Analysis – I,Vol – 1" by Bhavikatti]

Problem No:02

A beam is suspended by three springs as shown in Figure 5.17(a). The flexibility of the springs AD,BE and *CF* are f_1, f_2 and f_3 respectively. The beam carries a load W at the middle of DE. Determine the force in the spring BE assuming (i) the beam to be stiff in comparison to the springs and (ii) flexible with flexural rigidity EI .

Solution:

The degree of static indeterminacy = 3–2 = 1. Let the force in the spring *BE* be *R* as shown in Figure 5.17(b). Taking moment about point *F* , we have

$$F_{AD} \times 2L = W \times \frac{3L}{2} - RL$$

Therefore;

$$F_{AD} = \frac{3W}{4} - \frac{R}{2}$$

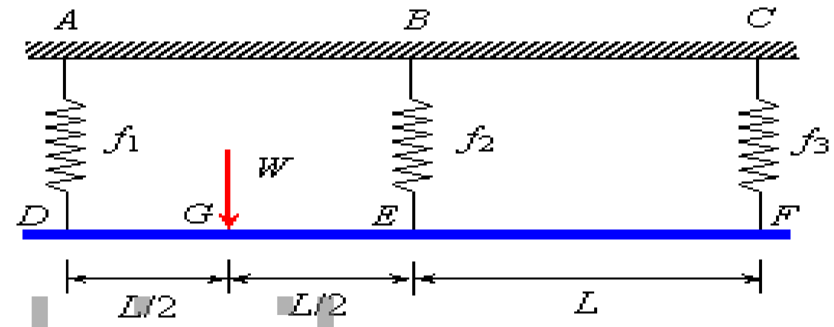


Figure 5.17(a)

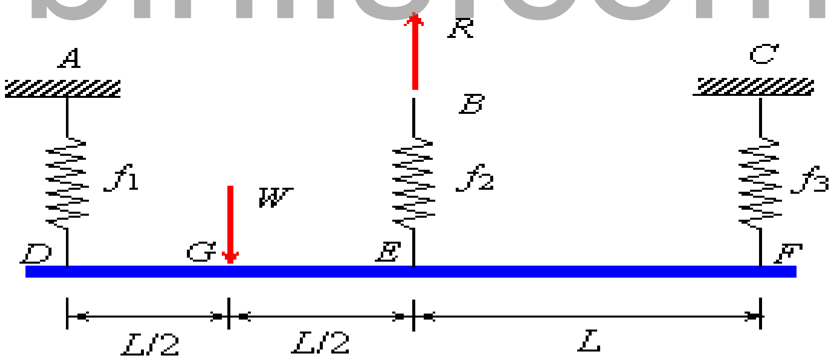


Figure 5.17(b)

[Source: "Structural Analysis – I,Vol – 1" by Bhavikatti]

Similarly, taking moment about point D , we have

Therefore;

$$F_{CF} \times 2L = W \times \frac{L}{2} - RL$$

$$F_{CF} = \frac{W}{4} - \frac{R}{2}$$

(i) When beam is rigid;

Total energy stored in the system is due to springs only as the beam is rigid. Thus,

$$U_s = \frac{1}{2} f_1 \left(\frac{3W}{4} - \frac{R}{2}\right)^2 + \frac{1}{2} f_2 R^2 + \frac{1}{2} f_3 \left(\frac{W}{4} - \frac{R}{2}\right)^2$$

Since the displacement of point *E* is zero in the vertical direction implying that

$$\frac{\partial U_s}{\partial R} = f_1 \left(\frac{3W}{4} - \frac{R}{2}\right)\left(-\frac{1}{2}\right) + f_2 R + f_3 \left(\frac{W}{4} - \frac{R}{2}\right)\left(-\frac{1}{2}\right)$$

or

$$0 = -\frac{W}{8}(3f_1 + f_3) + \frac{R}{4}(f_1 + 4f_2 + f_3)$$

Therefore;

$$R = \frac{W}{2}\left(\frac{3f_1 + f_3}{f_1 + 4f_2 + f_3}\right)$$

(ii) When beam is flexible;

The total energy stored in the beam

$$U_b = U_{DG} + U_{GE} + U_{EF}$$

Span DG **:** (*x* measured from D)

$$\frac{\partial M_x}{\partial R} = -\frac{x}{2} \qquad M_x = \left(\frac{3W}{4} - \frac{R}{2}\right)x$$

$$\frac{\partial U_{DG}}{\partial R} = \frac{1}{EI}\int_0^{L/2}\left[\frac{3W}{4} - \frac{R}{2}\right]\left(-\frac{x^2}{2}\right)dx = \frac{1}{EI}\left[\frac{3W}{4} - \frac{R}{2}\right]\left[\frac{-L^3}{48}\right]$$

Span GE : (x measured from D)

$$M_x = \left(\frac{3W}{4} - \frac{R}{2}\right)x - W\left(x - \frac{L}{2}\right)$$

$$\frac{\partial M_x}{\partial R} = -\frac{x}{2}$$

$$\frac{\partial U_{GE}}{\partial R} = \frac{1}{EI}\int_{L/2}^{L}\left[\left(\frac{3W}{4} - \frac{R}{2}\right)\left(-\frac{x^2}{2}\right) + W\left(x - \frac{L}{2}\right)\left(\frac{x}{2}\right)\right]dx$$

$$\frac{\partial U_{GE}}{\partial R} = \frac{1}{EI}\left[\left(\frac{3W}{4} - \frac{R}{2}\right)\left(\frac{-L^3}{6} + \frac{L^3}{48}\right) + W\left(\frac{L^3}{6} - \frac{L^3}{8} - \frac{L^3}{48} + \frac{L^3}{32}\right)\right] = \frac{1}{EI}\left[\left(\frac{3W}{4} - \frac{R}{2}\right)\left(\frac{-7L^3}{48}\right) + W\left(\frac{5L^3}{96}\right)\right]$$

Span EF **:** (x measured from F)

$$M_x = \left(\frac{W}{4} - \frac{R}{2}\right)x$$

$$\frac{\partial M_x}{\partial R} = -\frac{x}{2}$$

$$\frac{\partial U_{BC}}{\partial R} = \frac{1}{EI}\int_0^L\left[\frac{W}{4} - \frac{R}{2}\right]\left(-\frac{x^2}{2}\right)dx = \frac{1}{EI}\left[\frac{W}{4} - \frac{R}{2}\right]\left[\frac{-L^3}{6}\right]$$

$$\frac{\partial U_b}{\partial R} = \frac{\partial U_{DG}}{\partial R} + \frac{\partial U_{GE}}{\partial R} + \frac{\partial U_{EF}}{\partial R}$$

$$= \frac{1}{EI}\left[\frac{3W}{4} - \frac{R}{2}\right]\left[\frac{-L^3}{48}\right] + \frac{1}{EI}\left[\left(\frac{3W}{4} - \frac{R}{2}\right)\left(\frac{-7L^3}{48}\right) + W\left(\frac{5L^3}{96}\right)\right] + \frac{1}{EI}\left[\frac{W}{4} - \frac{R}{2}\right]\left[\frac{-L^3}{6}\right]$$

$$= \frac{1}{EI}\left[-\frac{11WL^3}{96} + \frac{RL^3}{6}\right]$$

Total strain energy in the system

Hence;

$$U = U_s + U_b$$

$$\frac{\partial U}{\partial R} = \frac{\partial U_S}{\partial R} + \frac{\partial U_b}{\partial R}$$

$$= -\frac{W}{8}(3f_1 + f_3) + \frac{R}{4}(f_1 + 4f_2 + f_3) + \frac{1}{EI}\left[-\frac{11WL^3}{96} + \frac{RL^3}{6}\right]$$

Since;

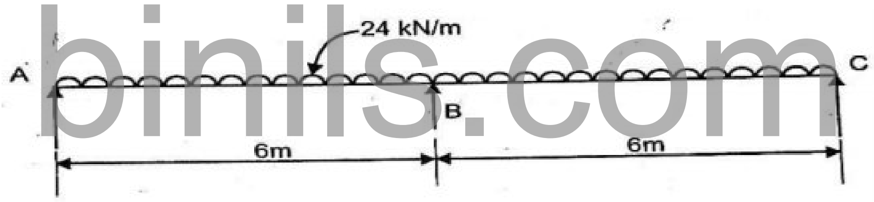
$$\frac{\partial U}{\partial R} = 0 \Rightarrow -\frac{W}{8}(3f_1 + f_3) + \frac{R}{4}(f_1 + 4f_2 + f_3) + \frac{1}{EI}\left[-\frac{11WL^3}{96} + \frac{RL^3}{6}\right] = 0$$

$$R\left(\frac{L^3}{6EI} + \frac{f_1}{4} + f_2 + \frac{f_3}{4}\right) = W\left(\frac{11}{96}\frac{L^3}{EI} + \frac{3}{8}f_1 + \frac{f_3}{8}\right)$$

$$\therefore \quad R = \frac{W\left(\frac{11}{96}\frac{L^3}{EI} + \frac{3}{8}f_1 + \frac{f_3}{8}\right)}{\left(\frac{L^3}{6EI} + \frac{f_1}{4} + f_2 + \frac{f_3}{4}\right)}$$

Problem No:03

Analyse the continuous beam loaded as shown in fig., by the strain energy method.EI is constant.



Solution:

For beams with vertical loads only,

Statically indeterminacy,I = number of vertical reactions + number of end moments – 2

= 3 + 0 – 2 = 1

Hence the beam is statically indeterminate to the first degree.Let us treat RB as the redundant.Since the beam and loading are symmetrical,

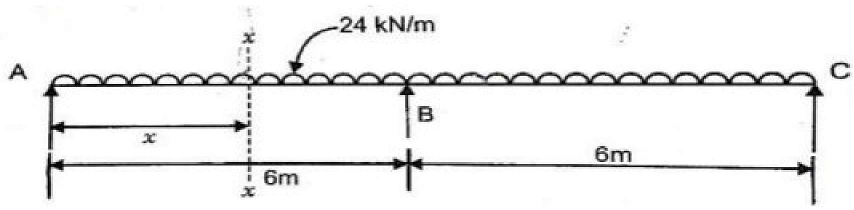
RA = RC = Total load – RB/2

RA = RC = 24 x 12 – RB/2

= 144 – 0.5 RB

The partial derivative of the total strain energy U in the beam AC with respect to RB is zero,since ΔB = 0

∂U/∂RB = 0



Considering a section xx at a distance x from A in AB,

Mx = RA.x – w.x.x/2

= (144 – 0.5 RB)x – 24(x^2/2)

Mx = 144x – 0.5 RB.x – 12x^2

∂Mx/∂RB = - 0.5x

The intergration limits are 0 and 6m.

$$\frac{\partial U}{\partial R_B} = \left\{\frac{1}{EI}\int_0^6 [144x - 0.5R_B\,x - 12x^2]\,[-0.5x]\,dx\right\} \times 2 = 0$$

$$\left\{\frac{1}{EI}\int_0^6 [-72x^2 + 0.25R_B\,x^2 + 6x^3]\,dx\right\} \times 2 = 0$$

- 72 x 6^3/3 + 0.25 x RB.6^3/3 + 6 x 6^4/4 = 0

RB = 5184 – 1944/18;

RB = 180 KN = RA

Substituting this in equ(1);

Mx = 144.x – 0.5 x 180x – 12x^2

When x = 0;

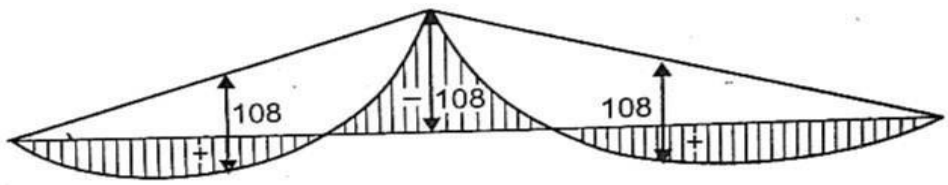
MA = MC = 0

When x = 6m;

MB = 144 x 6 – 0.5 x 180 x 6 – 12 x 6^2 = - 108 kNm

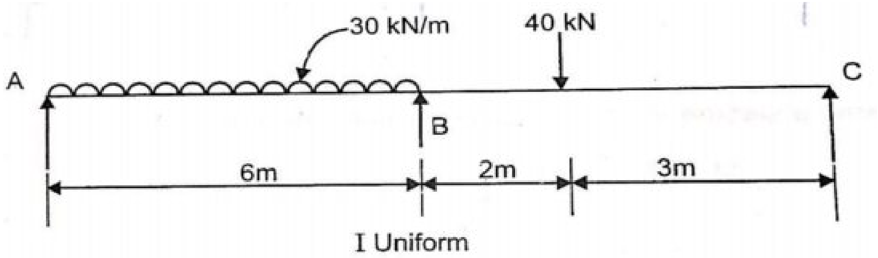
Free Bending Moments:

Span AB = BC =wl^2/8 = 24 x 6^2/8 = 108 kNm



Problem No:04

Analyse the continuous beam loaded as shown in fig., by the strain energy method.EI is constant.



Solution:

For beams with vertical loads only,

Statically indeterminacy,I = number of vertical reactions + number of end moments – 2

= 3 + 0 – 2 = 1

Treating RB as redundant and taking of moments about C,

RA x 11 + RB x 5 – 30 x 6 x (5 + 6/2) – 40 x 3 = 0

11RA + 5RB = 1440 +120 = 1560

RA = 1560 – 5RB/11

RA = (141.82 – 0.45RB)

RC = Total load – RA – RB

= 30 x 6 + 40 – (141.82 – 0.45RB) – RB

= (78.18 – 0.545 RB)

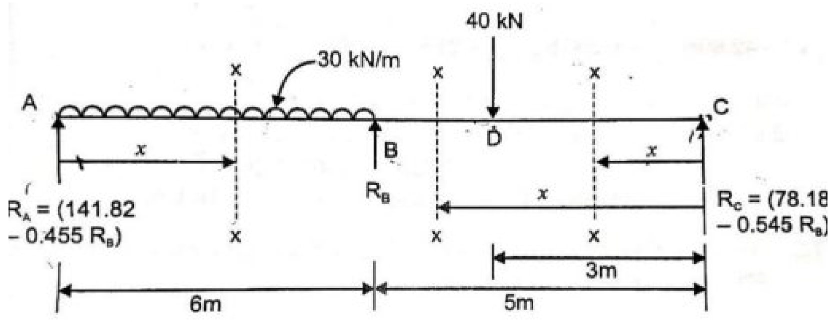


Fig. 7.7

∂U/∂RB = 0 (Since δB = 0)

EI is constant and can be removed

The integration will be done separately for the 3 zones, AB,CD and DB

Portion	Origin	Limits (m)	Mx (or) M	∂U/∂H
AB	A	0 to 6	- H.x	-0.455x
CD	C	0 to 3	30x – H x 5	-0.545x
DB	C	3 to 5	15x – H x 5	-0.545x

Substituting the values from the tables in equations;

$(- 64.53x^3/3 + 0.207RB.x^3/3 + 6.825.x^4/4) = 0$

- 4646.16 + 14.904 RB + 2211.3 – 383.47 +2.673RB – 1391.86 + 9.702RB + 712.13 – 523.2 = 0

Hence;

RB = 147.41KN; RA = 74.75 KN; RC = - 2.16 KN

1.4 ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES BY STRAIN ENERGY METHOD

(PLANE FRAMES)

Let a statically indeterminate structure has degree of indeterminacy as n . On the selected basic determinate structure apply the unknown forces , R_1.. R_2 and . R_n Using the Eq. (4.16) the displacement Δ_j in the direction of R_j is expressed by

$$\Delta_j = \frac{\partial U}{\partial R_j}$$

$$(j = 1, 2, \ldots \ldots n) \quad (5.1)$$

The equations (5.1) will provide the n linear simultaneous equations with n unknowns R_1 , R_2 ..... and R_n. Since the Δ_j is known, therefore, the solution of simultaneous equations will provide the desired R_j (j =1, 2,…., n).

For structures with members subjected to the axial forces only (i.e. pin-jointed structures), the equation (5.1) is re-written as

$$\Delta_j = \frac{\partial}{\partial R_j}\sum\left(\frac{P^2 L}{2AE}\right) = \sum\left(\frac{P\frac{\partial P}{\partial R_j}L}{AE}\right) \quad (5.2)$$

Where;

P is the force in the member due to applied loading and unknown R_j (j =1, 2,…., n); and L and AE are length and axial rigidity of the member, respectively.

For structures with members subjected to the bending moments (i.e. beams and rigid-jointed frames), the equation (5.1) is re-written as

$$\Delta_j = \frac{\partial}{\partial R_j}\left(\int \frac{M^2 dx}{2EI}\right) = \int \frac{M\frac{\partial M}{\partial R_j}dx}{EI} \quad (5.3)$$

where;

M is the bending moment due to applied loading and unknown R_j(j =1, 2,…., n) at a small element of length dx ; and EI is the flexural rigidity.

Problem No:1

Determine the force in various members of the pin-jointed frame shown in Figure 5.20(a).Length and AE is constant for all members.

Solution:

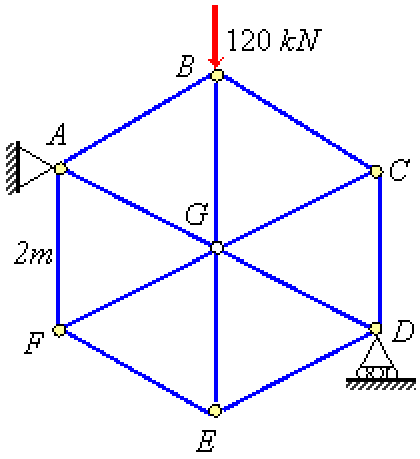


Figure 5.20(a)

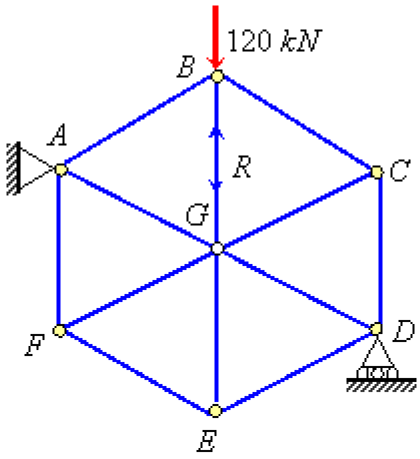


Figure 5.20(b)

The static indeterminacy of the pin-jointed frame is =12+3–7×2 = 1. Let the force in the member BG be R as shown in Figure 5.20(b). According to the Castigliano's theorem

$$\frac{\partial U}{\partial R} = 0$$

The computation of $\partial U / \partial R$ is made in Table 5.6.

$$\frac{\partial U}{\partial R} = \frac{1}{AE}\left(P\frac{\partial P}{\partial R}L\right) = 0$$

$$\frac{1}{AE}(24R - 1920) = 0$$

The final force in various members of the frame is shown in Table 5.6

Member	Length, $L(m)$	P	$\frac{\partial P}{\partial R}$	$P\frac{\partial P}{\partial R}L$	Final force (kN)
AB	2	-120 +*R*	1	2 *R*-240	-40
AG	2	120 -*R*	-1	2 *R*-240	40
AF	2	-60 +*R*	1	2 *R*-120	20
BC	2	-120 +*R*	1	2 *R*-240	-40
BG	2	-*R*	-1	2 *R*	-80
CD	2	-120 +*R*	1	2 *R*-240	-40
CG	2	120 -*R*	-1	2 *R*-240	40
DE	2	-60 +*R*	-1	2 *R*-120	20
DG	2	60 -*R*	-1	2 *R*-120	-20
EF	2	-60 +*R*	1	2 *R*-120	20
EG	2	60 -*R*	-1	2 *R*-120	-20
FG	2	60 -*R*	-1	2 *R*-120	-20

$$\sum \quad 24R - 1920$$

$$\frac{\partial U}{\partial R} = \frac{1}{AE}\left(P\frac{\partial P}{\partial R}L\right) = 0$$

or

$$\frac{1}{AE}(24R - 1920) = 0$$

$$\therefore \qquad R = 80 \text{ kN}$$

The final force in various members of the frame is shown in Table 5.6.

Problem No:2

Determine the force in various members of the pin-jointed frame as shown in Figure 5.21(a), if the member *BC* is short by an amount of Δ. All members of the frame have same axial rigidity as AE.

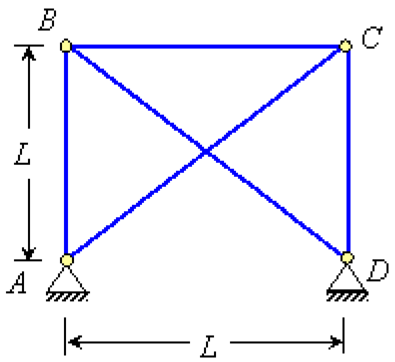


Figure 5.21(a)

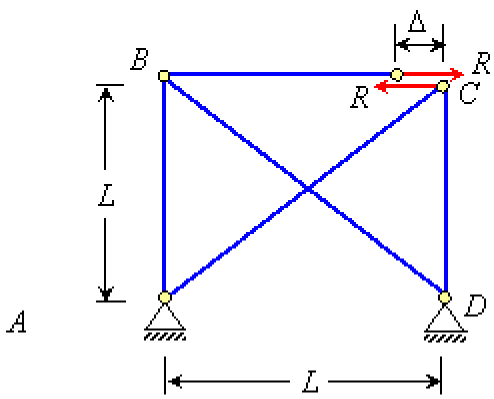


Figure 5.21(b)

Solution:

The static indeterminacy of the pin-jointed frame is =5 + 4 - 2×4=1. Since the member BC is short by an amount of Δ, therefore, apply a force *R* in the member BC such that displacement in the direction of *R* is Δ. Thus, according to the Castigliano's theorem.

$$\frac{\partial U}{\partial R} = \Delta$$

The computation of $\partial U / \partial R$ is made in Table 5.7.

Table 5.7

Member	Length,*L*(*m*)	*F*	$\frac{\partial F}{\partial R}$	$F \frac{\partial F}{\partial R} L$	Final force
AB	*L*	*R*	1	*RL*	1
AC	$\sqrt{2}L$	$-\sqrt{2}R$	$-\sqrt{2}$	$2\sqrt{2}RL$	$-\sqrt{2}$
BC	*L*	*R*	1	*RL*	1
BD	*L*	$-\sqrt{2}R$	$-\sqrt{2}$	$2\sqrt{2}RL$	$-\sqrt{2}$
CD	$\sqrt{2}L$	*R*	1	*RL*	1

$$\Sigma\left(\frac{AE\Delta}{(3+4\sqrt{2})L}\right)$$

$$(3+4\sqrt{2})RL$$

or
$$\frac{\partial U}{\partial R}=\frac{1}{AE}\left(F\frac{\partial F}{\partial R}L\right)=\Delta$$

$$\therefore \quad \frac{1}{AE}\left(3+4\sqrt{2}\right)RL=\Delta$$

$$R=\frac{AE\Delta}{(3+4\sqrt{2})L}$$

The final force in various members of the frame is shown in Table 5.7.

Problem No:3

Determine the horizontal reaction of the portal frame shown in Figure 5.22(a) by energy method. Also, calculate the horizontal reaction when the member BC is subjected to distributed load, *w* over entire length.

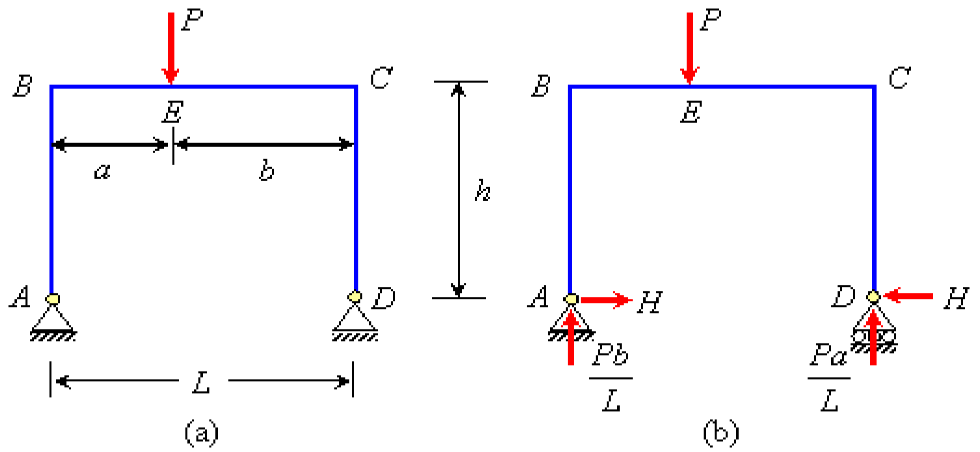


Figure 5.22(a)-(b)

Solution:

Static indeterminacy of the frame = 1.

Let the horizontal reaction, *H* at D be the redundant. The reaction at A and D are

$$H_A=H; \qquad H_D=H \qquad R_A=Pb/L \quad \text{and} \quad R_D=Pa/L$$

For the span AB (x measured from A),

$$M_x = Hx \quad \text{and} \quad \frac{\partial M_x}{\partial H} = x$$

$$\frac{\partial U_{AB}}{\partial H} = \frac{Hh^3}{3EI}$$

For the span BE (x measured from B),

$$M_x = Hh - \frac{Pb}{L}x \quad \text{and} \quad \frac{\partial M_x}{\partial H} = h$$

$$\frac{\partial U_{BE}}{\partial H} = \frac{1}{EI}\int_0^a \left[Hh - \frac{Pb}{L}x\right] h\,dx = \frac{Hh^2 a}{EI} - \frac{Pa^2 bh}{2EIL}$$

For the span CD (X Measured from D), $M_x = Hx \quad \text{and} \quad \frac{\partial M_x}{\partial H} = x$

$$\frac{\partial U_{CD}}{\partial H} = \frac{Hh^3}{3EI}$$

For the span CE (x measured from C),

$$M_x = Hh - \frac{Pa}{L}x \quad \text{and} \quad \frac{\partial M_x}{\partial H} = h$$

$$\frac{\partial U_{CE}}{\partial H} = \frac{Hh^2 b}{EI} - \frac{Pab^2 h}{2EIL}$$

Since $\frac{\partial U}{\partial H} = 0 \Rightarrow \frac{Hh^3}{3EI} + \frac{Hh^2 a}{EI} - \frac{Pa^2 bh}{2EIL} + \frac{Hh^2 b}{EI} - \frac{Pab^2 h}{2EIL} + \frac{Hh^3}{3EI}$

$$\frac{2Hh^2}{3} + HhL - \frac{Pab}{2} = 0$$

$$H = \frac{3Pab}{2h(2h + 3L)}$$

Horizontal reaction due to udl, w over BC :

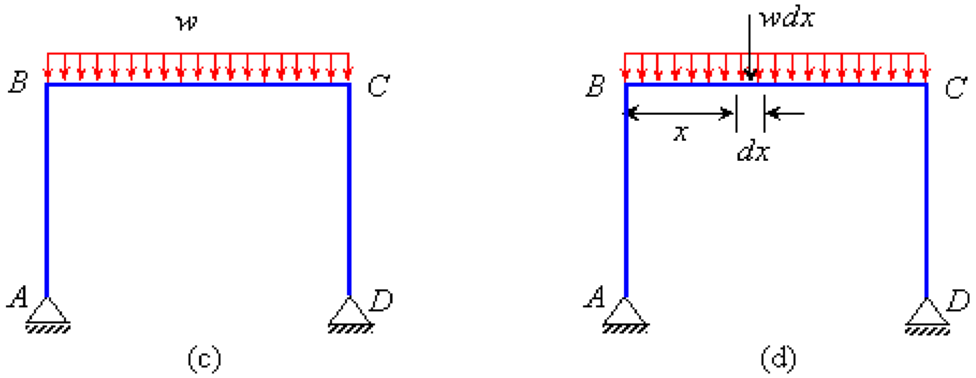


Figure 5.22(c)-(d)

The horizontal reaction due to small incremental load *wdx* is given by

$$dH = \frac{3w\,dx.x(L-x)}{2h(2h+3L)}$$

(using the expression derived earlier for concentrated force and putting

$P = wdx$, $a = x$ and $b = L - x$).

The horizontal reaction due to entire distributed load,

$$H = \int dH$$

$$= \int_0^L \frac{3w\,dx.x(L-x)}{2h(2h+3L)}$$

$$= \frac{3w}{2h(2h+3L)} \int_0^L x(L-x)dx$$

$$= \frac{wL^3}{4h(2h+3L)}$$

Problem No:4

Analyze the portal frame shown in Figure 5.23 by strain energy method.

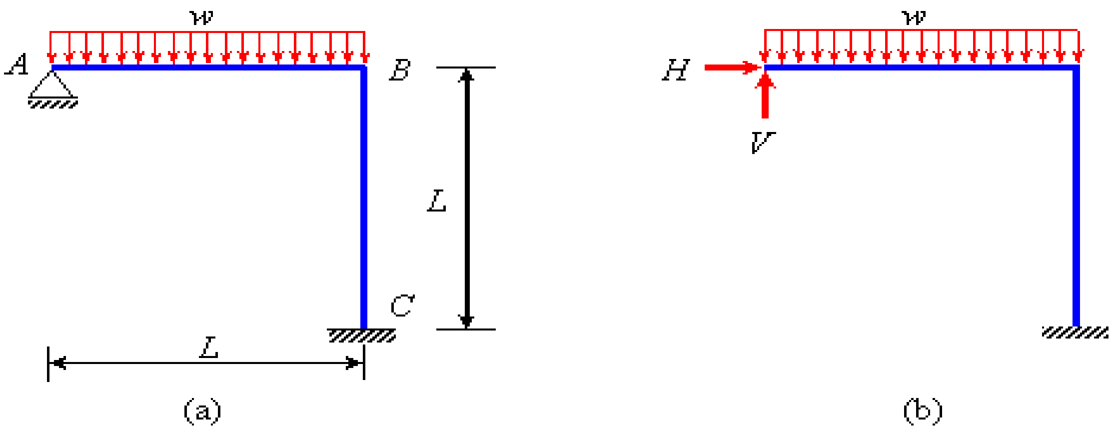


Figure 5.23

Solution:

Static indeterminacy of the frame = 2. Horizontal and vertical reactions at A are taken as redundant.

For the span *AB* (*x* measured from *A*),

$$M_x = Vx - wx^2/2$$

$$\frac{\partial M_x}{\partial H} = 0 \qquad \text{and} \qquad \frac{\partial M_x}{\partial V} = x$$

$$\frac{\partial U_{AB}}{\partial H} = 0$$

$$\frac{\partial U_{AB}}{\partial V} = \frac{1}{EI}\int_0^L \left[Vx - wx^2/2\right][x]\,dx = \frac{VL^3}{3EI} - \frac{wL^4}{8EI}$$

$$M_x = Hx + VL - wL^2/2$$

$$\frac{\partial M_x}{\partial H} = x \qquad \text{and} \qquad \frac{\partial M_x}{\partial V} = L$$

$$\frac{\partial U_{BC}}{\partial H} = \frac{1}{EI}\int_0^L \left[Hx + VL - wL^2/2\right][x]\,dx = \frac{VL^3}{2EI} + \frac{HL^3}{3EI} - \frac{wL^4}{4EI}$$

$$\frac{\partial U_{BC}}{\partial V} = \frac{1}{EI}\int_0^L \left[Hx + VL - wL^2/2\right][L]\,dx = \frac{VL^3}{EI} + \frac{HL^3}{2EI} - \frac{wL^4}{2EI}$$

Since;

$$\frac{\partial U}{\partial H} = 0 \quad \Rightarrow \frac{VL^3}{2EI} + \frac{HL^3}{3EI} - \frac{wL^4}{4EI} = 0 \qquad \Rightarrow \frac{VL^3}{2EI} + \frac{HL^3}{3EI} - \frac{wL^4}{4EI} = 0$$

or $\qquad 6\,V + 4\,H = 3\,wL$ ------ (i)

and

$$\frac{\partial U}{\partial V} = 0 \quad \Rightarrow \frac{VL^3}{3EI} - \frac{wL^4}{8EI} + \frac{VL^3}{EI} + \frac{HL^3}{2EI} - \frac{wL^4}{2EI} = 0$$

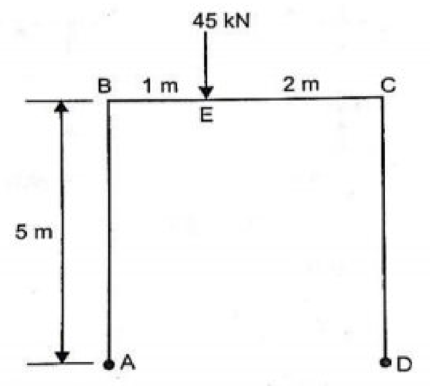
or $\qquad 32\,V + 12\,H = 15\,Wl$ ------ (ii)

Solving Eqs (i) and (ii) for *H* and *V*,

$$H = \frac{3wL}{28} \qquad \text{and} \quad V = \frac{3wL}{7}$$

Problem No:5

The simple portal frame shown in fig., is asymmetrically loaded.EI is constant.Analyse the frame by the strain energy method.Sketch the bending moment diagram.



Solution:

- **Finding the Redundant Force:**
  Degree of static indeterminacy = 1 x 3 – 2 = 1
  Let us treat the horizontal reaction at D as redundant.Since there is no other horizontal force,

  HA = - HD = H

  Since D is hinged,Δd = 0

  ∂U/∂H = 0

  $$\frac{1}{EI}\int \frac{M\,\partial M}{\partial H}\,dx \quad \text{--- (1)}$$

  VA x 3 – 45 x 2 = 0

  VA = 15 KN

Portion	Origin	Limits (m)	Mx (or) M	∂U/∂H
AB	A	0 to 5	- H.x	-x
BE	B	0 to 1	30x – H x 5	-5
CE	C	0 to 2	15x – H x 5	-5
DC	D	0 to 5	- H.x	-x

Substituting the values in equation (1)

$$\frac{1}{EI}\left\{\int_0^5 (-Hx)(-x)\,dx + \int_0^1 (30x - 5H)(-5)\,dx + \int_0^2 (15x - 5H)(-5)\,dx + \int_0^5 (-Hx)(-x)\,dx\right\} = 0$$

83.33 H – 75 + 25 H – 150 + 50 H = 0

158.33 H = 225

H = 1.421 KN

- **Determining the Bending Moments:**

  Span AB,

  x = 0,x = 5m,

  Mx = -Hx = -1.421x

  MA = - 7.11 kNm

  Span BE,

  x = 0,x = 1m,

  Mx = 30x - 5H

  MB = - 7.11 kNm

  ME = 22.89 kNm

  Span CE,

  x = 0,x = 2m,

  Mx = 15x - 5H

  MC = - 7.11 kNm

  ME = 22.89 kNm

  Span DC,

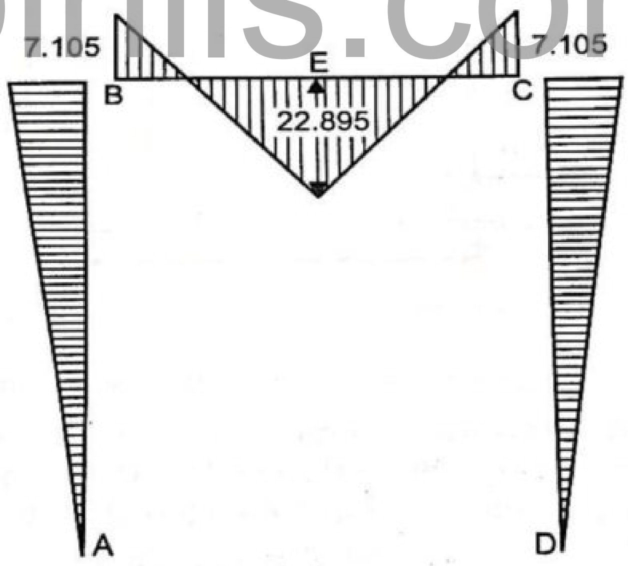
  x = 0,x = 5m,

  Mx = -Hx = -1.421x

  MD = 0 kNm

  ME = - 7.11 kNm

- **Bending Moments Diagram:**



1.5 ANALYSIS OF STATICALLY INDETERMINATE STRUCTURES BY STRAIN ENERGY METHOD

(PLANE TRUSSES)

1.5.1 STATIC INDETERMINACY OF STRUCTURES

If the number of independent static equilibrium equations (refer to Section 1.2) is not sufficient for solving for all the external and internal forces (support reactions and member forces, respectively) in a system, then the system is said to be statically indeterminate.

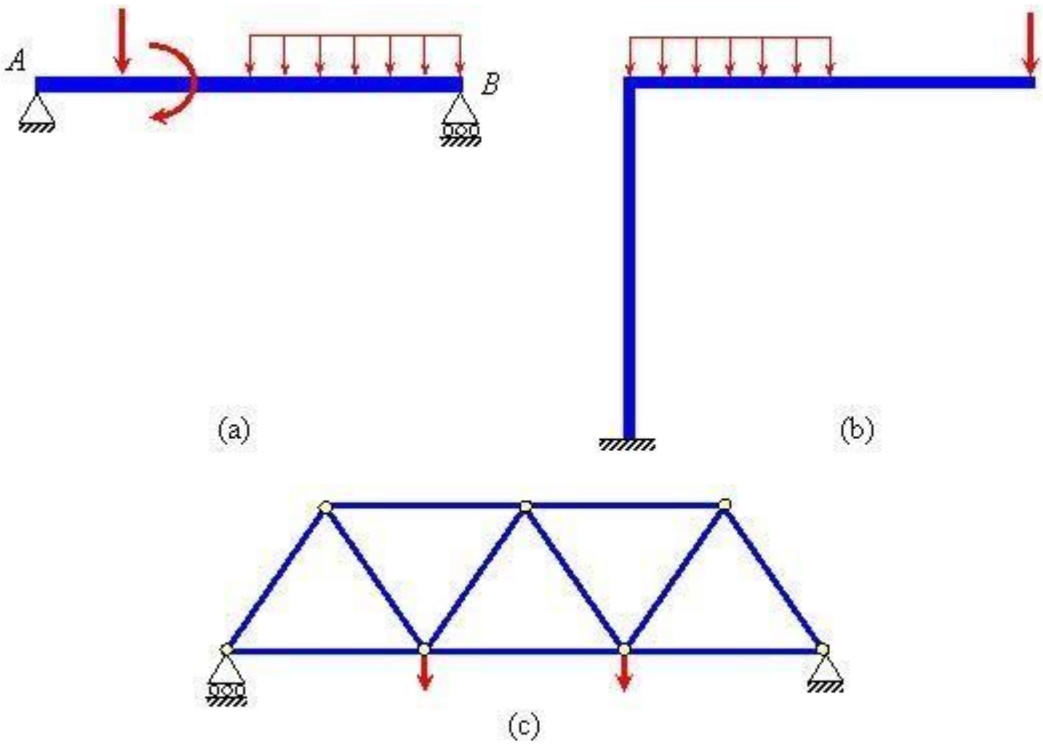
A statically determinate system, as against an indeterminate one, is that for which one can obtain all the support reactions and internal member forces using only the static equilibrium equations.

For example, for the system in Figure 1.10, idealized as one-dimensional, the number of independent static equilibrium equations is just 1 ($R_A \& R_B$), while the total number of unknown support reactions are 2 ($\sum F_x = 0$), that is more than the number of equilibrium equations available.

Therefore, the system is considered statically indeterminate. The following figures illustrate some example of statically determinate
(Figures 1.11a-c) and indeterminate structures (Figures 1.12a-c).

In Section 1.2, the equilibrium equations are described as the necessary and sufficient conditions to maintain the equilibrium of a body. However, these equations are not always able to provide all the information needed to obtain the unknown support reactions and internal forces.

The number of external supports and internal members in a system may be more than the number that is required to maintain its equilibrium configuration. Such systems are known as indeterminate systems and one has to use compatibility conditions and constitutive relations in addition to equations of equilibrium to solve for



the unknown forces in that system.

Figure 1.11 Statically determinate structures

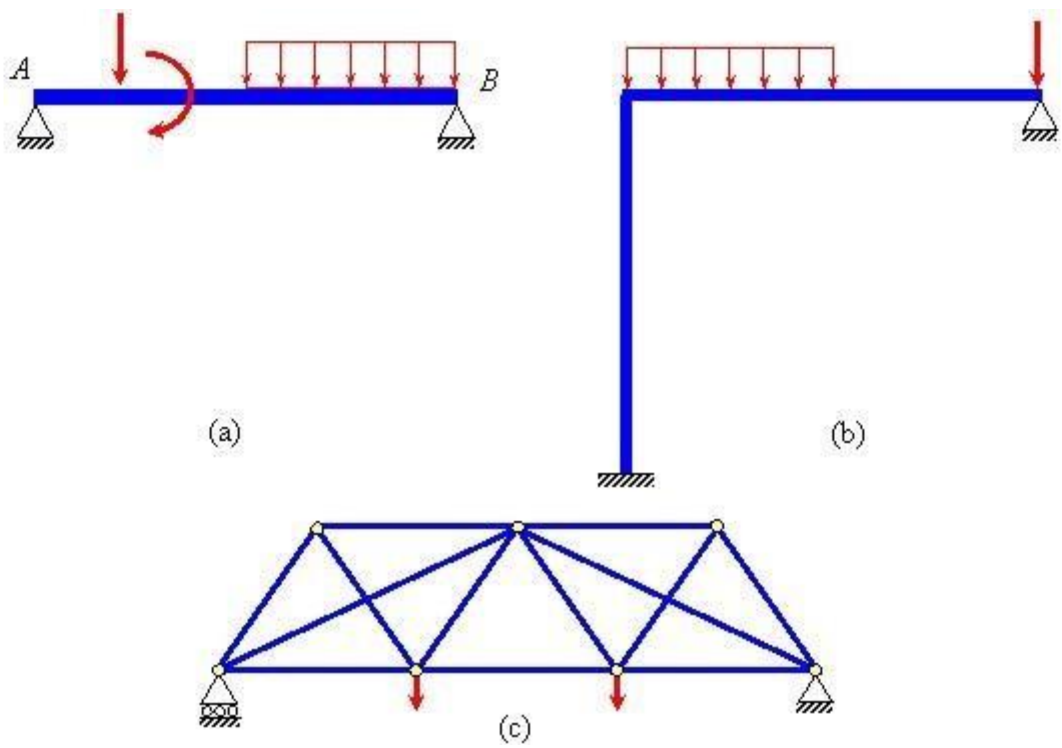


Figure 1.12 Statically indeterminate structures

For an indeterminate system, some support(s) or internal member(s) can be removed without disturbing its equilibrium. These additional supports and members are known as redundants . A determinate system has the exact number of supports and internal members that it needs to maintain the equilibrium and no redundants. If a system has less than required number of supports and internal members to maintain equilibrium, then it is considered unstable.

For example, the two-dimensional propped cantilever system in (Figure 1.13a) is an indeterminate system because it possesses one support more than that are necessary to maintain its equilibrium. If we remove the roller support at end B (Figure 1.13b), it still maintains equilibrium. One should note that here it has the same number of unknown support reactions as the number of independent static equilibrium equations. The unknown

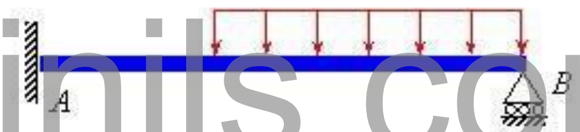


Figure 1.13a Propped cantilever *AB*

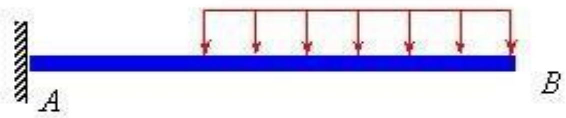


Figure 1.13b Cantilever *AB* with no roller support at *B*

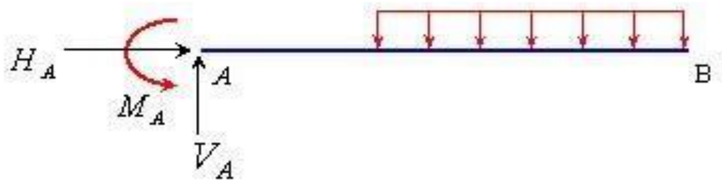


Figure 1.13c Free body diagram of cantilever *AB* (with no roller support at *B*)

The reactions are H_A, V_A, & M_A (Fig.1.13.c) and the equilibrium equations are;

$$\sum F_x = 0 \quad \text{-----} \quad (1.18)$$

$$\sum F_y = 0 \quad \text{-----} \quad (1.19)$$

$$\sum M_z(\text{about any point}) = 0 \quad \text{----} \quad (1.20)$$

An indeterminate system is often described with the number of redundant, it posses and this number is known as its degree of static indeterminacy.

Thus,mathematically:

Degree of Static Indeterminacy = Total number of unknowns (External & Internal)

- number of independent equations of equilibrium

---- (1.21)

It is very important to know exactly the number of unknown forces and the number of independent equilibrium equations. Let us investigate the determinacy/indeterminacy of a few two-dimensional pin-jointed truss systems.

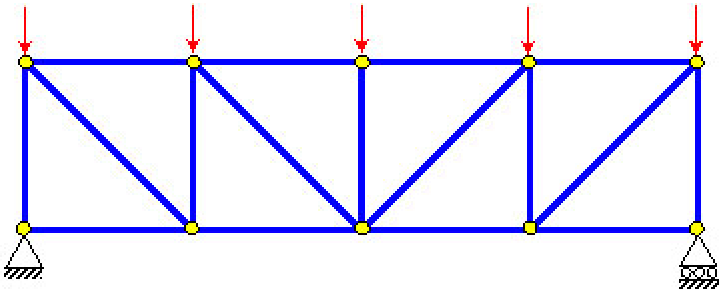
Let *m* be the number of members in the truss system and *n* be the number of pin (hinge) joints connecting these members. Therefore, there will be *m* number of unknown internal forces (each is a two-force member) and 2 *n* numbers of independent joint equilibrium equations ($\sum F_x = 0$ and $\sum F_y = 0$ for each joint, based on its free body diagram). If the support reactions involve *r* unknowns, then:

Total number of unknown forces = $m + r$

Total number of independent equilibrium equations = $2n$

So, degree of static indeterminacy = $(m + r) - 2n$

For the trusses in Figures 1.14a, b & c, we have:



Figure; 1.14a Determinate truss

1.14a: *m* = 17, *n* = 10, and *r* = 3. So, degree of static indeterminacy = 0, that means it is a statically determinate system.

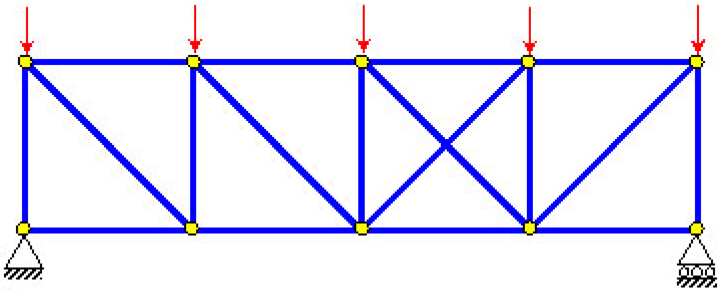


Figure 1.14b (Internally) indeterminate truss

1.14b: *m* = 18, *n* = 10, and *r* = 3. So, degree of static indeterminacy = 1.

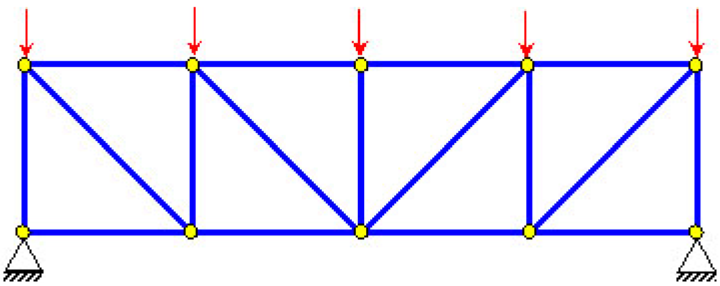


Figure 1.14c (Externally) indeterminate truss

1.14c: *m* = 17, *n* = 10, and *r* = 4. So, degree of static indeterminacy = 1.

It should be noted that in case of 1.14b, we have one member more than what is needed for a determinate system (i.e., 1.14a), where as 1.14c has one unknown reaction component more than what is needed for a determinate system. Sometimes, these two different types of redundancy are treated differently; as internal indeterminacy and external indeterminacy . Note that a structure can be indeterminate either externally or internally or both externally and internally.

We can group external and internal forces (and equations) separately, which will help us understand easily the cases of external and internal indeterminacy. There are *r* numbers of external unknown forces, which are the support reactions components. We can treat 3 system equilibrium equations as external equations. This will lead us to:

Degree of external static indeterminacy = r - 3.

The number of internal unknown forces is m and we are left with (2 n -3) equilibrium equations. The 3 system equilibrium equations used earlier were not independent of joint equilibrium equations, so we are left with (2 n - 3) equations instead of 2 n numbers of equations.So:

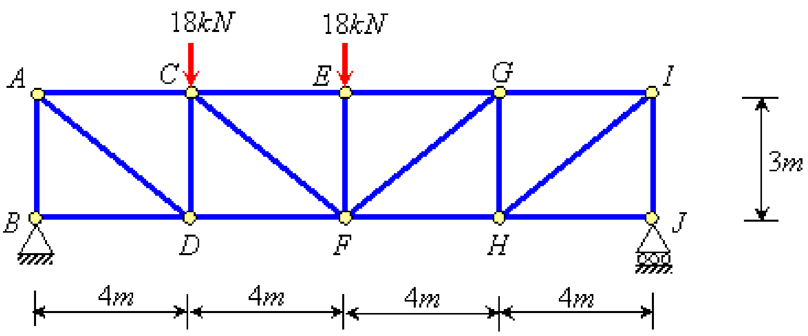
Degree of internal static indeterminacy = m - (2 n - 3).

Please note that the above equations are valid only for two-dimensional pin-jointed truss systems. For example, for three-dimensional ("space") pin-jointed truss systems, the degree of static indeterminacy is given by ($m + r - 3n$). Similarly, the expression will be different for systems with rigid (fixed) joints, frame members, etc.

Examble problems on Trusses;

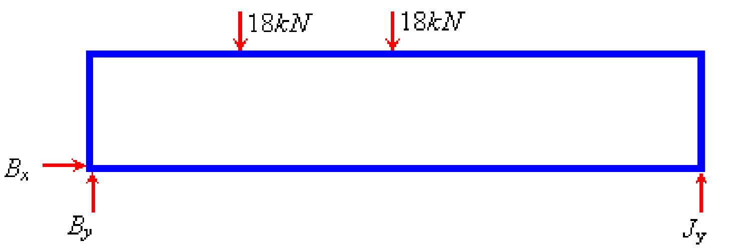
Problemn no:01

Find the forces in AB , AD and AC in the following Figure E2.1.



Solution:

FBD of the whole system

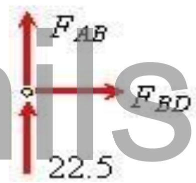


$$\sum F_x = B_x = 0$$

$$\sum M(about\,B) = J_y(16) - 18(8) - 18(4) = 0 \Rightarrow J_y = 13.5\,kN$$

$$\sum F_y = B_y + J_y - 18 - 18 = 0 \Rightarrow B_y = 22.5\,kN$$

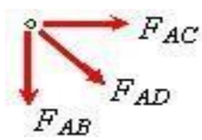
FBD of joint B;



$$\sum F_x = F_{BD} = 0$$

$$\sum F_y = 22.5 + F_{AB} = 0 \Rightarrow F_{AB} = -22.5\,kN$$

FDB of the joint A;



$$\sum F_y = -F_{AB} - \frac{3}{5}F_{AD} = 0 \qquad \Rightarrow F_{AD} = 37.5\,kN$$

$$\sum F_x = -F_{AC} + \frac{4}{5}F_{AD} = 0 \qquad \Rightarrow F_{AC} = -30\,kN$$

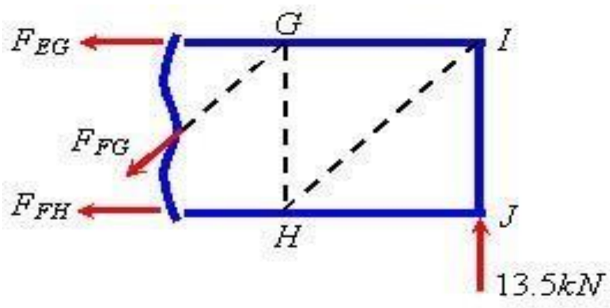
Force in AB = 22.5 kN (Compression).

Force in AD = 37.5 kN (Tension)

Force in AC = 30.0 kN (Compression).

Problemn no:02

Find the forces in EG , FG and FH in the following Figure E2.1.



$$\sum F_y = 13.5 - \frac{3}{5}F_{FG} = 0 \Rightarrow F_{FG} = 22.5\ kN$$

$$\sum M(about\ G) = 13.5(4) - F_{FH}(3) = 0 \Rightarrow F_{FH} = 18\ kN$$

$$\sum F_x = -F_{EG} - \frac{4}{5}F_{FG} - F_{FH} = 0 \Rightarrow F_{EG} = -36\ kN$$

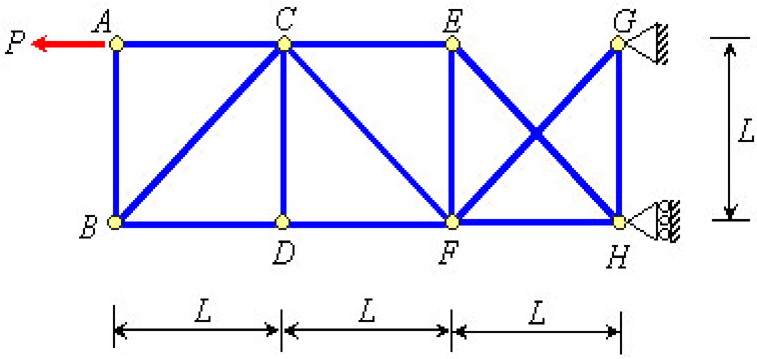
Force in EG = 36.0 kN (Compression).

Force in FG = 22.5 kN (Tension).

Force in FH = 18.0 kN (Compression).

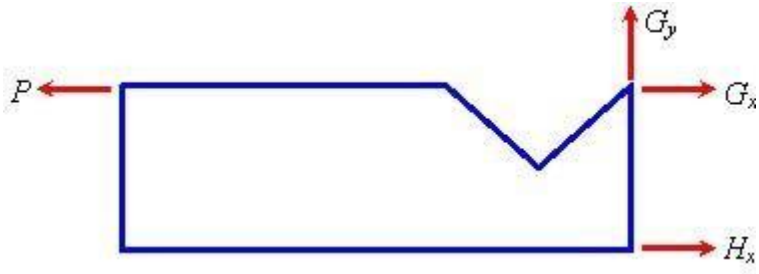
Problemn no:03

Find the forces in all members in the Figure E2.2.



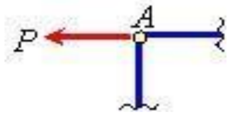
Solution:

From equilibrium of the whole body;



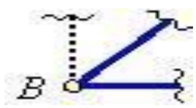
$$G_x = P,\ G_y = 0,\ H_x = 0$$

Looking at joint A :



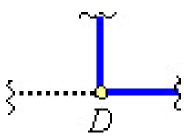
AB is a zero-force member and FAC = P

Looking at joint B :



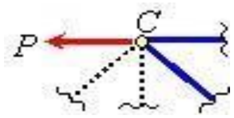
Both BC and BD are zero-force member.

Looking at joint D :



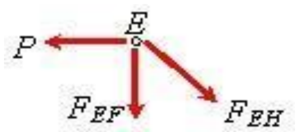
Both CD and DF are zero-force member.

Looking at joint C :



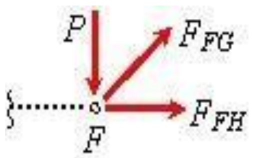
CF is a zero-force member and FCE= P

Equilibrium of joint B :



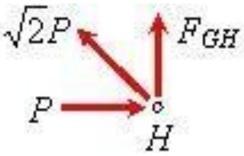
$F_{EF} = -P$ and $F_{EH} = \sqrt{2}P$

Equilibrium of joint F :



$F_{EF} = -P$ and $F_{EH} = \sqrt{2}P$

Equilibrium of joint H :

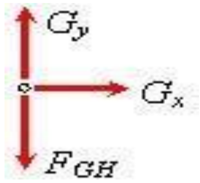


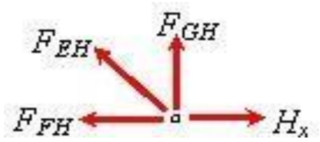
$F_{GH} = -P$

Sign convention: Tension +ve, compression –ve.

Note:

That we have not obtained the support reactions before finding the member forces. It was not necessary for this specific problem. Find out these reactions at supports *G* and *H* and check if joint equilibrium is satisfied at these two joints with the member forces that we have found already.


G_y
G_x
F_{GH}


F_{EH}
F_{GH}
F_{FH}
H_x