

## UNIT IV DESIGN OF COLUMNS

### **Types of columns –Axially Loaded columns – Design of short Rectangular Square and circular columns –Design of Slender columns- Design for Uniaxial and Biaxial bending using Column Curves**

Compression members are structural elements primarily subjected to axial compressive forces and hence, their design is guided by considerations of strength and buckling. Examples of compression member pedestal, column, wall and strut.

**Effective length:** The vertical distance between the points of inflection of the compression member in the buckled configuration in a plane is termed as effective length  $l_e$  of that compression member in that plane. The effective length is different from the unsupported length  $l$  of the member, though it depends on the unsupported length and the type of end restraints. The relation between the effective and unsupported lengths of any compression member is

$$l_e = k l$$

Where  $k$  is the ratio of effective to the unsupported lengths.

**Pedestal:** Pedestal is a vertical compression member whose effective length  $l_e$  does not exceed three times of its least horizontal dimension  $b$ . The other horizontal dimension  $D$  shall not exceed four times of  $b$ .

**Column:** Column is a vertical compression member whose unsupported length  $l$  shall not exceed sixty times of  $b$  (least lateral dimension), if restrained at the two ends. Further, its unsupported length of a cantilever column shall not exceed  $100b^2/D$ , where  $D$  is the larger lateral dimension which is also restricted up to four times of  $b$

**Wall:** Wall is a vertical compression member whose effective height  $H_{we}$  to thickness  $t$  (least lateral dimension) shall not exceed 30. The larger horizontal dimension i.e., the length of the wall  $L$  is more than  $4t$ .

## Types of columns

A column may be classified based on different criteria such as:

### 1. Based on shape

- Rectangle
- Square
- Circular

### 2. Based on slenderness ratio or height

Short column and Long column or Short and Slender Compression Members

A compression member may be considered as short when both the slenderness ratios namely  $l_{ex}/D$  and  $l_{ey}/b$  are less than 12:

Where

$l_{ex}$  = effective length in respect of the major axis,  $D$  = depth in respect of the major axis,  $l_{ey}$  = effective length in respect of the minor axis, and  $b$  = width of the member.

It shall otherwise be considered as a slender or long compression member.

The great majority of concrete columns are sufficiently stocky (short) that slenderness can be ignored. Such columns are referred to as short columns. Short column generally fails by crushing of concrete due to axial force. If the moments induced by slenderness effects weaken a column appreciably, it is referred to as a slender column or a long column. Long columns generally fail by bending effect than due to axial effect. Long column carry less load compared to long column.

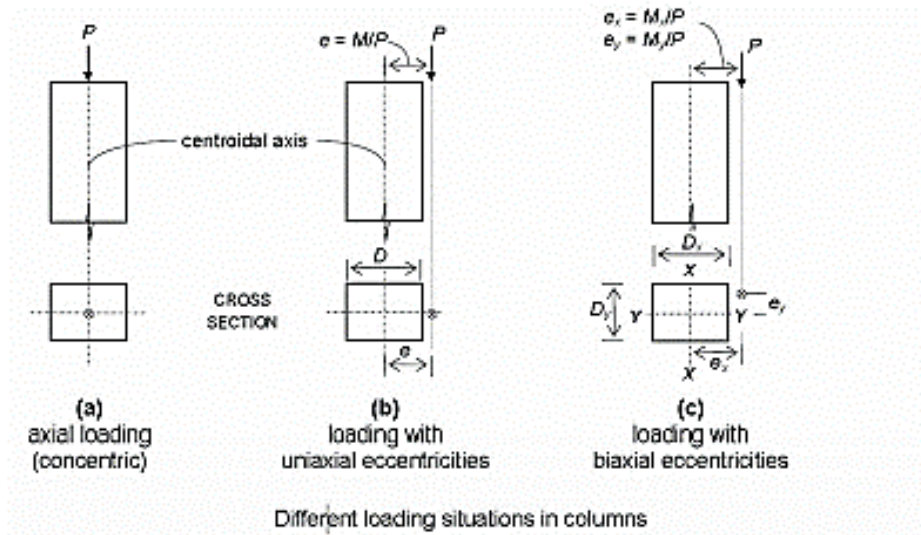
### 3. Based on pattern of lateral reinforcement

- Tied columns with ties as laterals
- Columns with Spiral steel as laterals or spiral columns

Majority of columns in any buildings are tied columns. In a tied column the longitudinal bars are tied together with smaller bars at intervals up the column. Tied columns may be square, rectangular, L-shaped, circular, or any other required shape. Occasionally, when high strength and/or high ductility are required, the bars are placed in a circle, and the ties are replaced by a bar bent into a helix or spiral. Such a column, called a spiral column. Spiral columns are generally circular, although square or polygonal shapes are sometimes used. The spiral acts to restrain the lateral expansion of the column core under high axial loads and, in doing so, delays the failure of the core, making the column more ductile. Spiral columns are used more extensively in seismic regions. If properly designed, spiral column carries 5% extra load at failure compared to similar tied column.

#### 4. Based on type of loading

- Axially loaded column or centrally or concentrically loaded column ( $P_u$ )
- A column subjected to axial load and uniaxial bending ( $P_u + M_{ux}$ ) or ( $P + M_{uy}$ )
- A column subjected to axial load and biaxial bending ( $P_u + M_{ux} + M_{uy}$ )



#### 5. Based on materials

- Timber
- Stone
- Masonry
- RCC
- PSC
- Steel
- Aluminium ,
- Composite column



## Minimum Eccentricity

In practical construction, columns are rarely truly concentric. Even a theoretical column loaded axially will have accidental eccentricity due to inaccuracy in construction or variation of materials etc. Accordingly, all axially loaded columns should be designed considering the minimum eccentricity

$$e_{x \min} \geq \text{greater of } (l/500 + D/30) \text{ or } 20 \text{ mm}$$

$$e_{y \min} \geq \text{greater of } (l/500 + b/30) \text{ or } 20 \text{ mm}$$

where  $l$ ,  $D$  and  $b$  are the unsupported length, larger lateral dimension and least lateral dimension, respectively.

## Longitudinal Reinforcement

The longitudinal reinforcing bars carry the compressive loads along with the concrete. stipulates the guidelines regarding the minimum and maximum amount, number of bars, minimum diameter of bars, spacing of bars etc. The following are the salient points:

- The minimum amount of steel should be at least 0.8 per cent of the gross cross-sectional area of the column required if for any reason the provided area is more than the required area.
- The maximum amount of steel should be 4 per cent of the gross cross-sectional area of the column so that it does not exceed 6 per cent when bars from column below have to be lapped with those in the column under consideration.
- Four and six are the minimum number of longitudinal bars in rectangular and circular columns, respectively.
- The diameter of the longitudinal bars should be at least 12 mm.
- Columns having helical reinforcement shall have at least six longitudinal bars within and in contact with the helical reinforcement. The bars shall be placed equidistant around its inner circumference.
- The bars shall be spaced not exceeding 300 mm along the periphery of the column.
- The amount of reinforcement for pedestal shall be at least 0.15 per cent of the cross-sectional area provided.

## Pitch and Diameter of Lateral Ties

Pitch: The maximum pitch of transverse reinforcement shall be the least of the following:

- (i) the least lateral dimension of the compression members;
  - (ii) sixteen times the smallest diameter of the longitudinal reinforcement bar to be tied; and
  - (iii) 300 mm.
- (b) Diameter: The diameter of the polygonal links or lateral ties shall be not less than one-fourth of the diameter of the largest longitudinal bar, and in no case less than 6 mm.

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### **Axially Loaded columns**

#### **Assumptions**

- The maximum compressive strain in concrete in axial compression is taken as 0.002.
- The maximum compressive strain at the highly compressed extreme fibre in concrete subjected to axial compression and bending and when there is no tension on the section shall be 0.0035 minus 0.75 times the strain at the least compressed extreme fibre.
- Plane sections normal to the axis remain plane after bending.
- The maximum strain in concrete at the outermost compression fibre is taken as 0.0035 in bending.
- The relationship between the compressive stress distribution in concrete and the strain in concrete may be assumed to be rectangle, trapezoid, parabola or any other shape which results in prediction of strength in substantial agreement with the results of test.
- An acceptable stress strain curve is given in IS:456-200. For design purposes, the compressive strength of concrete in the structure shall be assumed to be 0.67 times the characteristic strength. The partial safety factor  $\gamma$  of 1.5 shall be applied in addition to this. The tensile strength of the concrete is ignored.

#### **Short Axially Loaded Members in Compression**

The member shall be designed by considering the assumptions given and the minimum eccentricity. When the minimum eccentricity as per 25.4 does not exceed 0.05 times the lateral dimension, the members may be designed by the following equation:

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$P_u$  = axial load on the member,

$f_{ck}$  = characteristic compressive strength of the concrete,

$A_c$  = area of concrete,

$f_y$  = characteristic strength of the compression reinforcement, and

$A_s$  = area of longitudinal reinforcement for columns.

**P.1.Design the reinforcement in a column of size 400 mm x 600 mm subjected to an axial load of 2000 kN under service dead load and live load. The column has an unsupported length of 4.0 m and effectively held in position and restrained against rotation in both ends. Use M 25 concrete and Fe 415 steel.**

## Solution:

### Step 1: To check if the column is short or slender

Given  $l = 4000$  mm,  $b = 400$  mm and  $D = 600$  mm. Table 28 of IS 456 =  $l_{ex} = l_{ey} = 0.65(l) = 2600$  mm. So, we have

$$l_{ex}/D = 2600/600 = 4.33 < 12$$

$$l_{ey}/b = 2600/400 = 6.5 < 12$$

Hence, it is a short column.

### Step 2: Minimum eccentricity

$$e_{x \min} = \text{Greater of } (l_{ex}/500 + D/30) \text{ and } 20 \text{ mm} = 25.2 \text{ mm}$$

$$e_{y \min} = \text{Greater of } (l_{ey}/500 + b/30) \text{ and } 20 \text{ mm} = 20 \text{ mm}$$

$$0.05 D = 0.05(600) = 30 \text{ mm} > 25.2 \text{ mm } (= e_{x \min})$$

$$0.05 b = 0.05(400) = 20 \text{ mm} = 20 \text{ mm } (= e_{y \min})$$

Hence, the equation given in cl.39.3 of IS 456 (Eq.(1)) is applicable for the design here.

### Step 3: Area of steel

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

$$3000(103) = 0.4(25)\{(400)(600) - A_{sc}\} + 0.67(415) A_{sc} \text{ which gives,}$$

$$A_{sc} = 2238.39 \text{ mm}^2$$

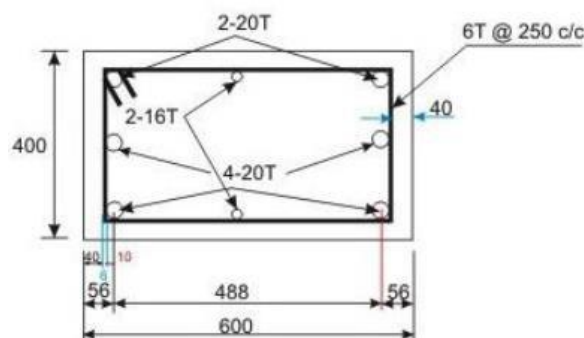
Provide 6-20 mm diameter and 2-16 mm diameter rods giving  $2287 \text{ mm}^2 (> 2238.39 \text{ mm}^2)$  and  $p = 0.953$  per cent, which is more than minimum percentage of 0.8 and less than maximum percentage of 4.0. Hence, o.k.

### Step 4: Lateral ties

The diameter of transverse reinforcement (lateral ties) is determined from cl.26.5.3.2 C-2 of IS 456 as not less than (i)  $\theta/4$  and (ii) 6 mm. Here,  $\theta$  = largest bar diameter used as longitudinal reinforcement = 20 mm. So, the diameter of bars used as lateral ties = 6 mm.

The pitch of lateral ties should be not more than the least of

- (i) the least lateral dimension of the column = 400 mm
- (ii) sixteen times the smallest diameter of longitudinal reinforcement bar to be tied =  $16(16) = 256$  mm
- (iii) 300 mm



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### Design of short Rectangular columns

Design the reinforcement in a column of size 450 mm × 600 mm, subject to an axial load of 2000 kN under service dead and live loads. The column has an unsupported length of 3.0m and its ends are held in position but not in direction. Use M<sub>20</sub> concrete and Fe<sub>415</sub> steel.

**Given:**

$$l_{eff} = 3000 \text{ mm}$$

$$b = 450 \text{ mm}$$

$$D = 600 \text{ mm}$$

$$P = 2000 \text{ kN}$$

$$M_{20}, Fe_{415}$$

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Ends are fixed.  $l_{ex} = l_{ey} = l = 3000 \text{ mm}$

$$\frac{l_{ex}}{D} = \frac{3000}{600} < 12$$

$$\frac{l_{ex}}{b} = \frac{3000}{450} < 12$$

Short Column

### Minimum eccentricity

$$e_{\min} = \frac{l_{eff}}{500} + \frac{D}{30}$$

In the longer direction

$$e_{\min} = \frac{3000}{500} + \frac{600}{30} = 26 \text{ mm or } 20 \text{ mm}$$

$$e_{\min} < 0.05D = 0.05 \times 600 = 30\text{mm}$$

In the Shorter direction

$$e_{\min} = \frac{3000}{500} + \frac{450}{30} = 21\text{mm or } 20\text{mm}$$

$$e_{\min} < 0.05D = 0.05 \times 450 = 22.5\text{mm}$$

Minimum eccentricities are within the limits and hence code formula for axially loaded short columns can be used.

$$\begin{aligned} P &= \text{service load} \times \text{partial load factor} \\ &= 2000 \times 1.5 = 3000 \text{ kN} \end{aligned}$$

### Design of Longitudinal Reinforcement (39.3-71)

$$P_u = 0.4 f_{ck} A_c + 0.67 f_y A_{sc}$$

or

$$\begin{aligned} P_u &= 0.4 f_{ck} A_c + (0.67 f_y - 0.4 f_{ck}) A_{sc} \\ 3000 \times 10^3 &= 0.4 \times 20 \times (450 \times 600) + (0.67 \times 415 - 0.4 \times 20) A_{sc} \\ A_{sc} &= 3111 \text{mm}^2 \end{aligned}$$

In view of the column dimensions (450 mm, 600 mm), it is necessary to place intermediate bars, in addition to the 4 corner bars:

Provide 4–25φ at corners

$$4 \times 491 = 1964 \text{ mm}^2 \text{ and } 4\text{--}20\phi$$

$$\text{additional ie, } 4 \times 314 = 1256 \text{ mm}^2$$

$$A_{sc} = 3220 \text{ mm}^2 > 3111 \text{ mm}^2$$

$$p = (100 \times 3220) / (450 \times 600)$$

$$= 1.192 > 0.8 \text{ (minimum steel), OK.}$$



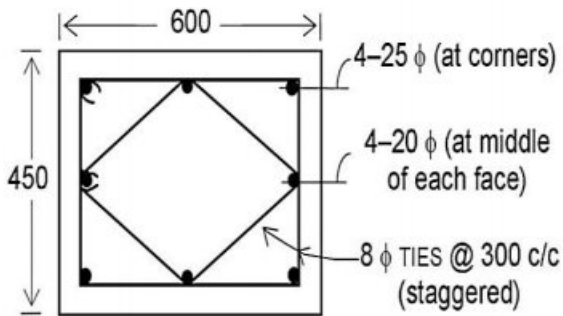
### Design of transverse steel

Diameter of tie =  $\frac{1}{4}$  diameter of main steel  
=  $\frac{25}{4} = 6.25$  mm or 6 mm

Spacing:

- i) 300 mm
- ii)  $16 \times 20 = 320$  mm
- iii) LLD = 450mm.

Provide ties 8mm @ 300 mm c/c



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### **Design for Uniaxial using Column Curves**

**Determine the reinforcement to be provided in a square column subjected to uniaxial bending with the following data**

Size of column 450 x 450 mm

Factored load 2500 kN

Factored moment 200 kN.m

Use M<sub>20</sub> concrete and Fe<sub>415</sub> steel.

Arrangement of reinforcement

(a) On two sides

(b) On four sides

$$d^1 = \text{Assume } 50\text{mm}$$

$$\frac{d^1}{D} = \frac{50}{450} = 0.11$$

$$\text{Chart for } \frac{d^1}{D} = 0.15$$

**Non Dimensional Parameters:**

$$\frac{P_u}{f_{ck} b D} = \frac{2500 \times 10^3}{25 \times 450 \times 450} = 0.494$$

$$\frac{M_u}{f_{ck} b D^2} = \frac{200 \times 10^6}{25 \times 450 \times 450^2} = 0.088$$

a) *Reinforcement on two sides*

*Referring to Chart 33*

$$\frac{P}{f_{ck}} = 0.09$$

*Percentage of reinforcement*

$$P = 0.09 \times 25 = 2.25 \%$$

$$A_s = \frac{PbD}{100} \\ = \frac{2.25 \times 450 \times 450}{100} = 4556 \text{ mm}^2$$

b) *Reinforcement on four sides*

*Referring Chart 45*

$$\frac{P}{f_{ck}} = 0.10$$

$$P = 0.10 \times 25 = 2.5 \%$$

$$A_s = \frac{PbD}{100} \\ = \frac{2.5 \times 450 \times 450}{100} = 5063 \text{ mm}^2$$

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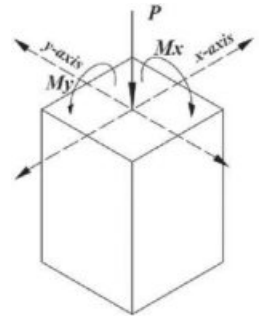
### Design for Biaxial bending using Column Curves

$$b = 400 \text{ mm}$$

$$D = 400 \text{ mm}$$

$$P_u = 1500 \text{ kN}$$

$$M_{ux} = M_{uy} = 50 \text{ kN}\cdot\text{m}$$



$$f_{ck} = 20 \text{ N/mm}^2$$

$$f_y = 415 \text{ N/mm}^2$$

*Equivalent moment*

The reinforcement in section is designed for the axial compressive load  $P_u$  and the equivalent moment

$$M_u = 1.15\sqrt{M_{ux}^2 + M_{uy}^2}$$

$$= 1.15\sqrt{50^2 + 50^2}$$

$$= 81.3 \text{ kN}\cdot\text{m}$$

### *Nondimensional parameters*

$$\left(\frac{P_u}{f_{ck} b D}\right) = \left(\frac{1500 \times 10^3}{20 \times 400 \times 400}\right) = 0.468$$

Assume  $d' = 40 \text{ mm}$   
 $(d'/D) = 0.10$

$$\left(\frac{M_u}{f_{ck} b D^2}\right) = \left(\frac{81.3 \times 10^6}{20 \times 400 \times 400^2}\right) = 0.063$$

Refer to Chart 44, SP:16

$$(p/f_{ck}) = 0.06$$

$$p = (20 \times 0.06) = 1.2$$

$$A_{st} = \left(\frac{pbD}{100}\right)$$

$$= \left(\frac{1.2 \times 400 \times 400}{100}\right)$$

$$= 1920 \text{ mm}^2$$

Provide 4 bars of 20 mm diameter and 4 bars of 16 mm diameter ( $A_{sc} = 2060 \text{ mm}^2$ ) distributed equally on all faces with 3 bars on each face.

$$p = (100 \times 2000)/(400 \times 400) = 1.28$$

$$(p/f_{ck}) = (1.28/20) = 0.064$$

Refer to Chart 44, SP:16 and readout ( $M_{ux1}/f_{ck}bD^2$ ) corresponding to the values of  $(P_u/f_{ck}bD) = 0.468$  and  $(p/f_{ck}) = 0.064$ .

$$\left(\frac{M_{ux1}}{f_{ck}bD^2}\right) = 0.068$$

$$\begin{aligned} M_{ux1} &= (0.068 \times 20 \times 400 \times 400^2) 10^{-6} \\ &= 87 \text{ kN}\cdot\text{m} \end{aligned}$$

Due to symmetry  $M_{ux1} = M_{uy1} = 87 \text{ kN}\cdot\text{m}$

$$\begin{aligned} P_{uz} &= [0.45f_{ck} A_c + 0.75f_y A_s] \\ &= (0.45 \times 20) [(400 \times 400) - 2060] + 0.75 \times 415 \times 2060 \end{aligned}$$

$$= 2062 \times 10^3 \text{ N}$$

$$= 2062 \text{ kN}$$

$$\left(\frac{P_u}{P_{uz}}\right) = \left(\frac{1500}{2062}\right) = 0.72$$

$$\alpha_n = 1.8$$

*Check for safety under biaxial bending*

$$\left[ \left(\frac{M_{ux}}{M_{ux1}}\right)^{\alpha_n} + \left(\frac{M_{uy}}{M_{uy1}}\right)^{\alpha_n} \right] \leq 1$$

$$\left[ \left(\frac{50}{87}\right)^{1.8} + \left(\frac{50}{87}\right)^{1.8} \right] = 0.736 < 1$$

**Hence the section is safe against bending**

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Determine the reinforcement to be provided in a circular column with the following data:

Diameter of column 500 mm  
Grade of concrete M20  
Factored moment 125 kN.m  
Characteristic strength 250 N/mm<sup>2</sup>  
Factored load 1600 kN

Lateral reinforcement:

- (a) Hoop reinforcement
- (b) Helical reinforcement

(Assume moment due to minimum eccentricity to be less than the actual moment).  
Assuming 25 mm bars with 40 mm cover,

$$d^1 = 40 + 12.5 = 52.5 \text{ mm}$$

$$d^1/D = 52.5/50 = 0.105$$

Charts for  $d^1/D = 0.10$  will be used.

Let  $b=D$

**(a) Column with hoop reinforcement**

$$\frac{P_u}{f_{ck} D^2} = \frac{1600 \times 10^3}{250 \times 500^2} = 0.32$$

$$\frac{M_u}{f_{ck} D^3} = \frac{125 \times 10^6}{250 \times 500^3} = 0.05$$

Referring to *Chart 52*, for  $f_y = 250 \text{ N/mm}^2$

$$\frac{P}{f_{ck}} = 0.87$$

Percentage of reinforcement,  $p = 0.87 \times 20 = 1.74 \%$

$$A_s = \frac{1.74}{100} \times \frac{\pi \times 500^2}{4} = 3416 \text{ mm}^2$$

### (b) Column with Helical Reinforcement

According to 38.4 of the Code, the strength of a compression member with helical reinforcement is 1.05 times the strength of a similar member with lateral ties. Therefore, the, given load and moment should be divided by 1.05 before referring to the chart.

$$\frac{P}{f_{ck} D^2} = \frac{1600 \times 10^3}{1.05 \times 20 \times 500^2} = 0.31$$

$$\frac{M_u}{f_{ck} D^3} = \frac{125 \times 10^6}{1.05 \times 20 \times 500^3} = 0.048$$

Hence, From Chart 52, for  $f_y = 250 \text{ N/mm}^2$ ,

$$\frac{P}{f_{ck}} = 0.078$$

$$p = 0.078 \times 20 = 1.56 \%$$

$$A_s = \frac{1.56}{100} \times \frac{\pi \times 500^2}{4} = 3063 \text{ mm}^2$$

According to 38.4.1 of the Code the ratio of the volume of helical reinforcement to the volume of the core shall not be less than

$$0.36 \left( \frac{A_g}{A_c} - 1 \right) \times \frac{f_{ck}}{f_y}$$

where  $A_g$  is the gross area of the section and  $A_c$  is the area of the core measured to the outside diameter of the helix. Assuming 8 mm dia bars for the helix

$$\text{Core diameter} = 500 - 2(40 - 8) = 436 \text{ mm}$$

$$\frac{A_g}{A_c} = \frac{500}{436} = 1.315$$

$$0.36 \left( \frac{A_g}{A_c} - 1 \right) \times \frac{f_{ck}}{f_y} = 0.36 \left( \frac{500}{436} - 1 \right) \times \frac{20}{250} = 0.0091$$



Volume of helical reinforcement / Volume of core

$$\frac{A_{sh} \pi \times 428 / (\pi / 4 \times 436^2) s_h}{S_h} \Rightarrow 0.9 \frac{A_{sh}}{S_h}$$

where,  $A_{sh}$  is the area of the bar forming the helix and  $s_h$  is the pitch of the helix. In order to satisfy the code requirement,

$$0.09 A_{sh} / s_h = 0.0091$$

For 8 mm dia bar,

$$s_h = 0.09 \times 50 / 0.0091 = 49.7 \text{ mm.}$$

Thus provide 48 mm pitch

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Problem:

A 4 m high column is effectively held in position at both ends and restrained against rotation at one end. Its diameter is restricted to 40 cm. Calculate the reinforcement if it is required to carry a factored axial load of 1500 kN Use M20 mix and Fe 415 grade steel.

**Solution:**

Effective height =  $0.80 L = 0.80 \times 400 = 320$  cm

Slenderness ratio =  $l/D = 320/40 = 8 < 12$

Hence, it is a short column.  $e_{min} = l/500 + D/30 = 320/500 + 40/30$   
 $= 1.97$  cm  $< 20$  mm

Also,  $0.05 D = 0.05 \times 400 = 20$  mm =  $e_{min}$

The strength of the column section is given by,

$$P_u = 0.4\sigma_{ck} A_c + 0.67\sigma_y A_{sc} \quad 1500 \times 1000$$

$$= 0.4 \times 20 \left( \frac{\pi}{4} \times 400^2 - A_{sc} \right) + 0.67 \times 415 A_{sc}$$

$$A_{sc} = 1830 \text{ mm}^2,$$

This steel should be more than minimum steel i.e., 0.8%

$$1500 \times 1000 = 0.4 \times 20 \left( \frac{\pi}{4} \times 400^2 - A_{sc} \right) + 0.67 \times 415 A_{sc}$$

Use 6 – 20 mm bars (Area 'A' =  $6 \times \pi \times 20^2 = 1884 \text{ mm}^2 > 1830 \text{ mm}^2$ ) (OK)  
Adopt 6 mm ties,

Pitch  $p \leq 400 \text{ mm}$

$\leq 16 \phi L$  ( $16 \times 20 = 320 \text{ mm}$ )

$\leq 300 \text{ mm}$

Adopt pitch of lateral ties as 300 mm c/c.

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### Design of Slender columns

#### **Slender Columns**

Columns having both  $l_{ex}/D$  and  $l_{ey}/b$  less than twelve are designated as short and otherwise, they are slender, where  $l_{ex}$  and  $l_{ey}$  are the effective lengths with respect to major and minor axes, respectively; and  $D$  and  $b$  are the depth and width of rectangular columns, respectively.

#### **Design of Slender Columns**

The design of slender compression members shall be based on the forces and the moments determined from an analysis of the structure, including the effect of deflections on moments and forces. When the effect of deflections are not taken into account in the analysis, additional moment given in cl no 39.7.1 of IS 456:2000 shall be taken into account in the appropriate direction.

The additional moments  $M_x$ , and  $M_y$ , shall be calculated by the following formulae:

$$M_{ax} = (P_u D/2000) (l_{ex}/D)^2$$

$$M_{ay} = (P_u b/2000) (l_{ey}/b)^2$$

Where  $P_u$  = axial load on the member,  
 $l_{ex}$  = effective length in respect of the major axis,  
 $l_{ey}$  = effective length in respect of the minor axis,  
 $D$  = depth of the cross-section at right angles to the major axis, and  
 $b$  = width of the member.

**Problem:**

Determine the reinforcement required for a braced column against sideway with the following data: size of the column = 350 x 450 mm concrete and steel grades = M 30 and Fe 415, respectively; effective lengths  $l_{ex}$  and  $l_{ey}$  = 7.0 and 6.0 m, respectively; unsupported length  $l$  = 8 m; factored load  $P_u$  = 1700 kN; factored moments in the direction of larger dimension = 70 kNm at top and 30 kNm at bottom; factored moments in the direction of shorter dimension = 60 kNm at top and 30 kNm at bottom. The column is bent in double curvature. Reinforcement will be distributed equally on four sides.

**Solution 1:****Step 1: Checking of slenderness ratios**

$$l_{ex}/D = 7000/450 = 15.56 > 12,$$

$$l_{ey}/b = 6000/350 = 17.14 > 12.$$

Hence, the column is slender with respect to both the axes.

**Step 2: Minimum eccentricities and moments due to minimum eccentricities**

$$e_{x \min} = l/500 + D/30 = 8000/500 + 450/30 = 31.0 > 20 \text{ mm}$$

$$e_{y \min} = l/500 + b/30 = 8000/500 + 350/30 = 27.67 > 20 \text{ mm}$$

$$M_{ox} (\text{Min. ecc.}) = P_u(e_{x \min}) = (1700)(31)(10^{-3}) = 52.7 \text{ kNm}$$

$$M_{oy} (\text{Min. ecc.}) = P_u(e_{y \min}) = (1700)(27.67)(10^{-3}) = 47.04 \text{ kNm}$$

**Step 3: Additional eccentricities and additional moments****Table I of SP-16**

For  $l_{ex}/D = 15.56$ , Table I of SP-16 gives:

$$e_{ax}/D = 0.1214, \text{ which gives } e_{ax} = (0.1214)(450) = 54.63 \text{ mm}$$

For  $l_{ey}/D = 17.14$ , Table I of SP-16 gives:

$$e_{ay}/b = 0.14738, \text{ which gives } e_{ay} = (0.14738)(350) = 51.583 \text{ mm}$$

**Step 4: Primary moments and primary eccentricities**

$$M_{ox} = 0.6M_2 - 0.4M_1 = 0.6(70) - 0.4(30) = 30 \text{ kNm, which should be } \geq 0.4 M_2 (= 28 \text{ kNm}). \text{ Hence, o.k.}$$

$$M_{oy} = 0.6M_2 - 0.4M_1 = 0.6(60) - 0.4(30) = 24 \text{ kNm, which should be } \geq 0.4 M_2 (= 24 \text{ kNm}). \text{ Hence, o.k.}$$

Primary eccentricities:

$$e_x = M_{ox}/P_u = (30/1700)(10^3) = 17.65 \text{ mm}$$

$$e_y = M_{oy}/P_u = (24/1700)(10^3) = 14.12 \text{ mm}$$

Since, both primary eccentricities are less than the respective minimum eccentricities (see Step 2), the primary moments are revised to those of Step 2. So,  $M_{ox} = 52.7$  kNm and  $M_{oy} = 47.04$  kNm.

### Step 5: Modification factors

To determine the actual modification factors, the percentage of longitudinal reinforcement should be known. So, either the percentage of longitudinal reinforcement may be assumed or the modification factor may be assumed which should be verified subsequently. So, we assume the modification factors of 0.55 in both directions.

### Step 6: Total factored moments

$$\begin{aligned} M_{ux} &= M_{ox} + (\text{Modification factor}) (M_{ax}) = 52.7 + (0.55) (92.548) \\ &= 52.7 + 50.9 = 103.6 \text{ kNm} \end{aligned}$$

$$\begin{aligned} M_{uy} &= M_{oy} + (\text{Modification factor}) (M_{ay}) = 47.04 + (0.55) (87.43) \\ &= 47.04 + 48.09 = 95.13 \text{ kNm} \end{aligned}$$

### Step 7: Trial section

The trial section is determined from the design of uniaxial bending with  $P_u = 1700$  kN and  $M_u = 1.15(M_{ux}^2 + M_{uy}^2)$

$$\text{So, we have } = (1.15)\{(103.6)^2 + (95.13)^2\}^{1/2} = 161.75 \text{ kNm.}$$

With these values of  $P_u (= 1700 \text{ kN})$  and  $M_u (= 161.75 \text{ kNm})$ ,

we use chart of SP-16 for the  $d' / D = 0.134$ . We assume the diameters of longitudinal bar as 25 mm, diameter of lateral tie = 8 mm and cover = 40 mm, to get  $= 40 + 8 + 12.5 = 60.5$  mm.

Accordingly,  $d' / D = 60.5/450 = 0.134$  and  $d' / b = 60.5/350 = 0.173$ .

$$P_u / f_{ck} bD = 1700(10^3) / (30)(350)(450) = 0.3598$$

$$M_u / f_{ck} bD^2 = 161.75(10^6) / (30)(350)(450)(450) = 0.076$$

We have to interpolate the values of  $p/f_{ck}$  for  $d'/D = 0.134$  obtained from Charts 44 (for  $d'/D = 0.1$ ) and 45 ( $d'/D = 0.15$ ). The values of  $p/f_{ck}$  are 0.05 and 0.06 from Charts 44 and 45, respectively. The corresponding values of  $p$  are 1.5 and 1.8 per cent, respectively. The interpolated value of  $p$  for  $d'/D = 0.134$  is 1.704 per cent, which gives  $A_{sc} = (1.704)(350)(450)/100 = 2683.8 \text{ mm}^2$ . We use  $4-25 + 4-20$  ( $1963 + 1256 = 3219 \text{ mm}^2$ ), to have  $p$  provided = 2.044 per cent giving  $p/f_{ck} = 0.068$ .

### Step 8: Calculation of balanced loads $P_b$

The values of  $P_{bx}$  and  $P_{by}$  are determined using Table 60 of SP-16. For this purpose, two parameters  $k_1$  and  $k_2$  are to be determined first from the table. We have

$p/f_{ck} = 0.068$ ,  $d'/D = 0.134$  and  $d'/b = 0.173$ . From Table 60,  $k_1 = 0.19952$  and  $k_2 = 0.243$  (interpolated for  $d'/D = 0.134$ ) for  $P_{bx}$

So, we have:  $P_{bx}/f_{ck}bD = k_1 + k_2(p/f_{ck}) = 0.19952 + 0.243(0.068) = 0.21044$ ,

which gives  $P_{bx} = 0.21044(30)(350)(450)(10^{-3}) = 1020.81 \text{ kN}$ .

Similarly, for  $P_{by}$ :  $d'/D = 0.173$ ,  $p/f_{ck} = 0.068$ . From Table 60 of SP-16,  $k_1 = 0.19048$  and  $k_2$

$= 0.1225$  (interpolated for  $d'/b = 0.173$ ). This gives  $P_{by}/f_{ck}bD = 0.19048 + 0.1225(0.068) = 0.19881$ ,

which gives  $P_{by} = (0.19881)(30)(350)(450)(10^{-3}) = 939.38 \text{ kN}$ .

Since, the values of  $P_{bx}$  and  $P_{by}$  are less than  $P_u$ , the modification factors are to be used.

### Step 9: Determination of $P_{uz}$

$$P_{uz} = 0.45 f_{ck} A_g + (0.75 f_y - 0.45 f_{ck}) A_{sc}$$

$$= 0.45(30)(350)(450) + \{0.75(415) - 0.45(30)\}(3219) = 3084.71 \text{ kN}$$

### Step 10: Determination of modification factors

$$k_{ax} = (P_{uz} - P_u)/(P_{uz} - P_{ubx})$$

or

$$k_{ax} = (3084.71 - 1700)/(3084.71 - 1020.81) = 0.671 \text{ and}$$

$$k_{ay} = (P_{uz} - P_u)/(P_{uz} - P_{uby})$$

or

$$k_{ay} = (3084.71 - 1700)/(3084.71 - 939.39) = 0.645$$

The values of the two modification factors are different from the assumed value of 0.55 in Step 5. However, the moments are changed and the section is checked for safety.

**Step 11: Total moments incorporating modification factors**

$$M_{ux} = M_{ox} \text{ (from Step 4)} + (k_{ax}) M_{ax} \text{ (from Step 3)}$$

$$= 52.7 + 0.671(92.548) = 114.8 \text{ kNm}$$

$$M_{uy} = M_{oy} \text{ (from Step 4)} + k_{ay} (M_{ay}) \text{ (from Step 3)}$$

$$= 47.04 + (0.645)(87.43) = 103.43 \text{ kNm.}$$

**Step 12: Uniaxial moment capacities**

The two uniaxial moment capacities  $M_{ux1}$  and  $M_{uy1}$  are determined as stated: (i) For  $M_{ux1}$ , by interpolating the values obtained from Charts 44 and 45, knowing the values of  $P_u/f_c k b D = 0.3598$  (see Step 7),

$p/f_c k = 0.068$  (see Step 7),  $d'/D = 0.134$  (see Step 7), (ii) for  $M_{uy1}$ , by interpolating the values obtained from Charts 45 and 46, knowing the same values of  $P_u/f_c k b D$  and  $p/f_c k$  as those of (i) and  $d'/D = 0.173$  (see Step 7). The results are given below:

(i)  $M_{ux1}/f_c k b D^2 = 0.0882$  (interpolated between 0.095 and 0.085)

(ii)  $M_{uy1}/f_c k b b^2 = 0.0827$  (interpolated between 0.085 and 0.08)

So, we have,  $M_{ux1} = 187.54 \text{ kNm}$  and  $M_{uy1} = 136.76 \text{ kNm}$ .

**Step 13: Value of  $\alpha_n$**

We have  $P_u/P_{uz}$

$$= 1700/3084.71 = 0.5511.$$

We have  $n \alpha = 0.67 + 1.67 (P_u/P_{uz}) = 1.59$ .

**Step 14: Checking of column for safety**

$$(M_{ux} / M_{ux1})^{\alpha_n} + (M_{uy} / M_{uy1})^{\alpha_n} \leq 1$$

Here, putting the values of  $M_{ux}$ ,  $M_{ux1}$ ,  $M_{uy}$ ,  $M_{uy1}$  and  $n \alpha$ , we get:  $(114.8/187.54)^{1.5452} + (103.43/136.76)^{1.5852} = 0.4593 + 0.6422 = 1.1015$ . Hence, the section or the reinforcement has to be revised.