3.1 APPLICATION OF THE MOMENTUM EQUATION FOR RVF

The hydraulic jump is an important feature in open channel flow and is an example of rapidly varied flow. A hydraulic jump occurs when a super-critical flow and a sub-critical flow meet. The jump is the mechanism for the to surface to join. They join in an extremely turbulent manner which causes large energy losses.

Because of the large energy losses the energy or specific energy equation cannot be use in analysis, the momentum equation is used instead.

Resultant force in x- direction = F1 - F2

Momentum change = M2 – M1

$$F_1 - F_2 = M_1 - M_2$$

Or for a constant discharge

$$F_1 + M_1 = F_2 + M_2 = \text{constant}$$

For a rectangular channel this may be evaluated using

$$F_1 = \rho g \frac{y_1}{2} y_1 b \qquad F_2 = \rho g \frac{y_2}{2} y_2 b$$

$$F_1 = \rho g v_1 \qquad F_2 = \rho g v_2$$

$$= \rho g \frac{Q}{y_1 b} \qquad = \rho g \frac{Q}{y_1 b}$$

Substituting for these and rearranging gives

$$y_2 = \frac{y_1}{2}\left(\sqrt{1 + 8 f_{r1}^{2}} - 1\right)$$

$$y_1 = \frac{y_2}{2}\left(\sqrt{1 + 8 f_{r2}^{2}} - 1\right)$$

So knowing the discharge and either one of the depths on the upstream or downstream side of the jump the other – or conjugate depth – may be easily computed.

More manipulation with Equation and the specific energy give the energy loss in the jump as

$$\Delta E = \frac{(y_2 - y_1)^3}{4\ y_1\, y_2}$$

These are useful results and which can be used in gradually varied flow calculations to determine water surface profiles.

In summary, a hydraulic jump will only occur if the upstream flow is super-critical. The higher the upstream Froude number the higher the jump and the greater the loss of energy in the jump.

3.2 HYDRAULIC JUMPS

Hydraulic jump defined as, the rise of water level, which takes place due to the transformation of unstable shooting flow (super-critical) to the stable streaming flow (sub-critical flow).It frequently occurs in a canal below a regulating sluice, at the toe of a spillway, at downstream of narrow channel or at the place where a steep channel slope suddenly turns flat.

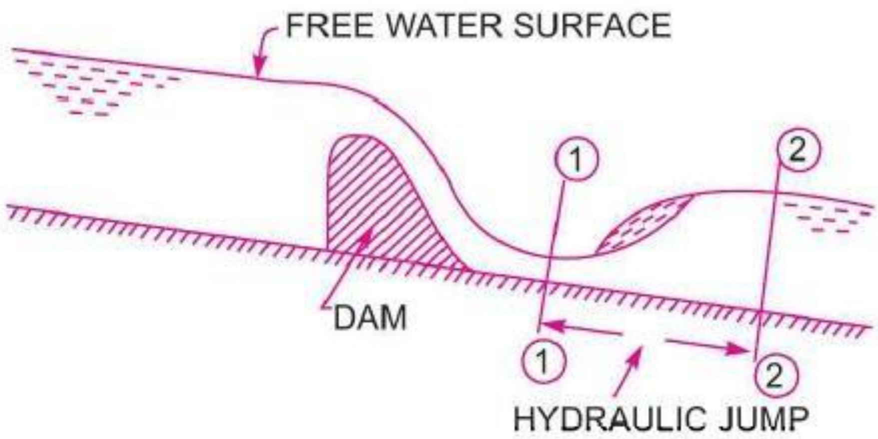

Figure 3.1 Hydraulic Jumps

[Source:Fluid Mechanics and Hydraulic Machines by Dr.R.K.Bansal,page 783]

Consider the flow of water over a dam as shown in Fig.3.1 The depth of water at the section 1-1 is small, but it increases towards downstream rapidly over a short length of the channel. This is because at the section 1-1, the flow is a shooting flow as the depth of water at section 1-1 is less than the critical depth. Shooting flow is an unstable type of flow and does not continue on the downstream side. Then this shooting will convert itself into a streaming or tranquil flow and hence depth of water will increase. This sudden increased of depth of water is called a hydraulic jump or a standing wave. When hydraulic jump take place, a loss of energy due to eddy formation and turbulence occurs.

Expression for Hydraulic Jump

Following assumption are made before deriving the expression for the depth of hydraulic jump,

1. Flow is uniform and pressure distribution is due to hydrostatic before and after the jump.

2. Losses due to friction on the surface of the bed of the channel are negligible
3. The slope of the bed of the channel is small, so that the component of the weight of the fluid in the direction of flow is negligibly small.

Consider a hydraulic jump formed in a channel of horizontal bed as shown in figure 12.2.

Consider there are two sections 1-1 and 2-2 before and after hydraulic jump.

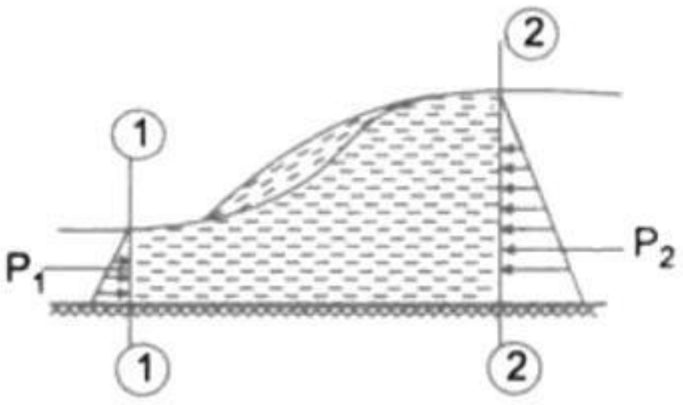

Figure 3.2 Hydraulic Jumps

[Source:Fluid Mechanics and Hydraulic Machines by Dr.R.K.Bansal,page 784]

Let

d_1 is the depth of flow at section 1-1 i.e. initial depth;

d_2 is the depth of flow at section 2-2 i.e. depth of flow after the hydraulic jump, also known as sequent depth or theoretical tail water depth. The depth pair at section 1-1 and 2-2 are called conjugate depth.

v_2 is thevelocity of flow at section 1-1;

v_2 is the velocity of flow at section 2-2;

z_1 is the depth of centroid of area at section 1-1 below free surface;

z_2 is the depth of centroid of area at section 2-2 below free surface;

A_1 is the cross sectional area at section 1-1;

A_2 is the cross sectional area at section 2-2.

Consider unit width of the channel.

The force acting on the mass of water between sections 1-1 and 2-2 are:

(i) Pressure force P_1 and P_2 on section 1-1 and 2-2 respectively.

(ii) Frictional force on the floor of the channel, which assumed to be negligible.

Let q = discharge per unit width.

$$v_1 d_1 = v_2 d_2$$

Now, pressure force on section 1-1

$$P_1 = \rho g A_1 \bar{Z}_1 = \rho g (d_1 \times 1) \times \frac{d_1}{2}$$

(Since we are considering unit width hence, $A_1 = d_1 \times 1$)

$$= \frac{\rho g d_1^2}{2}$$

Similarly pressure force on section 2-2

$$P_2 = \frac{\rho g d_2^2}{2}$$

Net force acting on the mass of water between section 1-1 and 2-2

$$= P_2 - P_1$$

$$= \frac{\rho g}{2}[d_2^2 - d_1^2] \qquad (12.2)$$

But from the momentum principle, the net force acting on a mass of fluid must be equal to the rate of change of momentum in the same section.

Rate of change of momentum in the direction of force

∴Rate of change of momentum in the direction of force

= mass of water per second × change of velocity in direction of force

Now,mass of water per second

= ρ × discharge per unit width × width

$$=\rho \times q \times 1 = \rho q m^3/s$$

Change of velocity in the direction of force $v_1 - v_2$

Hence, rate of change of momentum in the direction of force

$$= \rho q(v_1 - v_2)$$

Hence, according to the momentum principle, the expression given by eq. (12.2) is equal to the expression given by eq. (12.3)

or $\frac{\rho g}{2}[d_2^2 - d_1^2] = \rho q(v_1 - v_2)$

But from equation (1),

$$v_1 = q/d_1 \text{ and } \quad v_2 = q/d_2$$

Substituting the value of and in eq. (12.4) and by solving we get,

$$d_2 + d_1 = \frac{2q^2}{gd_1d_2}$$

Multiplying both sides by d_2 , we get

$$d_2^2 + d_1 d_2 = \frac{2q^2}{gd_1}$$

$$d_2^2 + d_1 d_2 - \frac{2q^2}{gd_1} = 0$$

By solving eq. (12.6) we get

$$d_2 = \frac{-d_1 \pm \sqrt{d_1^2 + \frac{2q^2}{gd_1}}}{2}$$

Neglecting the negative sign, we get

$$d_2 = -\frac{d_1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2q^2}{gd_1}}$$

$$= -\frac{d1}{2} + \sqrt{\frac{d_1^2}{4} + \frac{2(v_1 d_1)^2}{gd_1}}$$

$$= -\frac{d1}{2} + \frac{d1}{2}\sqrt{1 + \frac{8v1^2}{gd_1}}$$

Now, depth of hydraulic jump = $(d_2 - d_1)$

Froude number on the upstream side is given by

$$F_{r1} = \frac{v_1}{\sqrt{g d_1}}$$

Substituting the value in equation (12.7) we get,

$$d_2 = \frac{d_1}{2}\left(-1 + \sqrt{1 + 8F_{r1}^2}\right)$$

ENERGY DISSIPATION DUE TO HYDRAULIC JUMP

When hydraulic jump take place, a loss of energy due to eddies formation and turbulence occurs. This loss of energy is equal to the difference of specific energies at sec. 1-1 and 2-2.

Let E_1, and E_2 are the energy at section 1-1 and 2-2 respectively. Loss of energy due to hydraulic jump

$$\Delta E = E_1 - E_2$$

$$= \left(d_1 + \frac{v_1^2}{2g}\right) - \left(d_2 + \frac{v_2^2}{2g}\right)$$

$$= \left(\frac{v_1^2}{2g} - \frac{v_2^2}{2g}\right) - (d_2 - d_1)$$

Since $v_1 = q/d_1$ and $v_2 = q/d_2$

$$= \left(\frac{q^2}{2gd_1^2} - \frac{q^2}{2gd_2^2}\right) - (d_2 - d_1)$$

$$= \frac{q^2}{2g}\left(\frac{d_2^2 - d_1^2}{d_1^2 d_2^2}\right) - (d_2 - d_1)$$

$$q^2 = g d_1 d_2 \left(\frac{d_2 + d_1}{2}\right)$$

solving the expression, we get

$$\Delta E = \frac{(d_2 - d_1)^3}{4 d_1 d_2}$$

3.3 HYDRAULIC JUMPS – TYPES

Based on Froude number (F), hydraulic jump can be classified into 5 types.

a. **Undulation jump**: The Froude number F ranges from 1 to1.7 and the liquid surface does not rise shortly but having undulations of radically decreasing size.

b. **Weak jump**: The Froude number F ranges from1.7 to 2.5 and the liquid surface remains smooth.

c. **Oscillating jump**: The Froude number F ranges from 2.5 to 4.5 and there is an oscillating jet which enters the jump bottom and oscillating to the surface.

d. **Steady jump**: The Froude number F ranges from 4.5 to9 and energy loss due to steady jump in between 45 and 70%.

e. **Strong jump**: The Froude number greater than 9 and the downstream water surface is rough. Energy loss due to strong jump may be up to 85%.

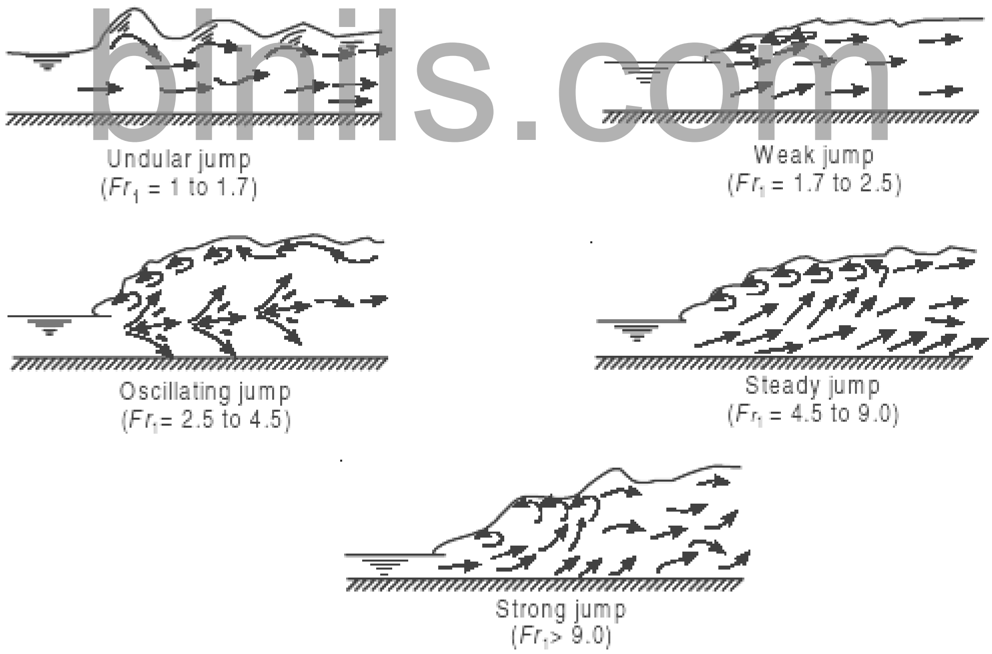

Figure 3.3 Types of hydraulic jump

[*Source: Hydraulics And Fluid Mechanics Including Hydraulic Machines By Dr. P.N.Modi , page 804*]

APPLICATION OF HYDRAULIC JUMP.

1. Generally, the use of hydraulic jump reverses the flow of water. The hydraulic jump can be used to mix chemicals for pure water.
2. It usually maintains a high water level on the downstream side. It is used for high-level water for irrigation purposes.
3. It can be used to remove the sewage lines to prevent air locking and air from a water supply.
4. hydraulic jump prevents the scouring action dam structure on the downstream side.

Problem 1

The depth of flow of water certain section of a rectangular channel of 4m wide in 0.5m.The discharge through the channel in 16m^3/s .If a hydraulic jump takes place on a down stream side.find the depth of flow after the jump

Given data:

b= 4m

Y_1 = 0.5 m

Q= 16m^3/s

To find :

Depth of flow after the jump ?

Solution :

$$Y_2 = -\frac{y1}{2} + \sqrt{\frac{y_1{}^2}{4} + \frac{2q^2}{gy_1}}$$

V = q/y q = vy

q= Q/b = 16/4 = 4 m^2/s

$$Y_2 = -\frac{0.5}{2} + \sqrt{\frac{0.5^2}{4} + \frac{2\times4^2}{9.81\times0.5}}$$

$=2.3163$ m

Result :

$y_2 = 2.316$ m

Problem 2

The depth of flow of water at a certain section of a rectangular channel of wide in 0.3m.The discharge through the channel in 1.5 m^3/s ,determine whether a hydraulic jump will occur and if so,find its height and loss of energy per kg of water

Given data :

b = 2m

$Y_1 = 0.3$ m

$Q = 1.5 m^3/s$

To find :

i) Hydraulic jump will occur =?

ii)Height = ?

iii) loss =?

solution :

$y_c = (q^2/g)^{1/3}$

whereq = Q/b = 1.5/2 = 0.75 m^2/s

$y_c=(0.75^2/9.81)^{1/3}= 0.385$ m

hence $y < yc$

hydraulic jump will occur

Height of hydraulic jump $= y_2 - y_1$

$$Y_2 = -\frac{y1}{2} + \sqrt{\frac{y_1{}^2}{4} + \frac{2q^2}{gy_1}}$$

$$Y_2 = -\frac{0.3}{2} + \sqrt{\frac{0.3^2}{4} + \frac{2*0.75^2}{9.81*0.3}}$$

$Y_2 = 0.4862$ m

Height of hydraulic jump

= 0.4862 - 0.3

= 0.1862 m

Loss of energy

$$h_L = \frac{(y_2 - y_1)^3}{4y_1y_2}$$

$$= \frac{(0.4862 - 0.3)^3}{4\times0.4862\ \times 0.3}$$

$h_L = 0.011$ m

Problem 3

A sluice gate discharge water into the horizontal rectangular channel with a velocity of $10m^3/s$ and the depth of flow of 1m.Determine the depth of flow after the jump and consequent loss in total.

Given data :

V = 10 m/s

$y_1 = 1m$

To find :

y2 = ?

$h_L = ?$

Solution :

$$Y_2 = -\frac{y1}{2} + \sqrt{\frac{y_1{}^2}{4} + \frac{2y_1v^2}{g}}$$

$$Y_2 = -\frac{1}{2} + \sqrt{\frac{1^2}{4} + \frac{2\times1\times10^2}{9.81}}$$

$= 4.043$ m

Loss of energy

$$h_L = \frac{(y_2-y_1)^3}{4y_1y_2}$$

$$= \frac{(4.043-1)^3}{4\times4.043\times1}$$

$\mathbf{h_L = 1.742m}$

Problem 4

A hydraulic jump form at the down stream end of spillway carrying 17.73m^3/s discharge.if the depth before jump is 0.80m.determine the depth after the jump and energy loss

Given data :

$Q = 17.93\ m^3/s$

$Y_1 = 0.8$

To find :

$y_2 = ?$

$h_L = ?$

Solution :

q = Q/b

Take b = 1m

$$Y_2 = -\frac{y1}{2} + \sqrt{\frac{y_1{}^2}{4} + \frac{2q^2}{gy_1}}$$

$$Y_2 = -\frac{0.8}{2} + \sqrt{\frac{0.8^2}{4} + \frac{2\times17.73^2}{9.81*0.8}}$$

$= - 0.4 + 8.5912$

$y_2 = 8.66$ m

$$h_L = \frac{(y_2-y_1)^3}{4y_1y_2}$$

$$= \frac{(8.1912-0.8)^3}{4\times0.8\times8.1912}$$

$\mathbf{h_L = 17.52}$ **m**

Problem 5

A control sluice in spanning the entry of 4m wide rectangular channel having a mild slope at mid 16m^3/s at a velocity of 3m/s .find whether a hydraulic jump is expected in the channel downstream from the sluice

Given data :

b = 4m

Q = 16 m^3/s

V = 3 m/s

To find

Hydraulic jump will occur or not

Solution

$Y_c = (q^2/g)^{1/3}$

$q = Q/b$

$=16/4$

$= 4$ m^2/s

$yc =(4^2/9.81)^{1/3}$

= 1.177 m

V = Q/A

= Q /b× y = q/y

3= 4/y

y = 4/3 = 1.33 m

y > yc

Hydraulic jump is not expected

Problem 6

Water flows at the rate of 1×10^6 cm^3/s along a channel of rectangular section 1.75 m in width.calculate the critical depth if a hydraulic jump formed at a point where the upstream depth is 25cm.what would be the raise in water level and power lost in jump.

Given data :

Q = 1×10^6cm^3/s

= 1×10^6 × $(10^{-2})^3 m^3/s$

= 1× 10^6 × $10^{-6} m^3/s$ = 1 m^3/s

b = 1.75 m

y_1 = 25 cm = 0.25 m

To find

critical depth = ?

Raise in water level y_2 = ?

Power lost = ?

Solution

$Y_c=(q^2/g)^{1/3}$

q= 1/1.75 = 0.5714 m^2/s

$Y_c=(0.5714^2/9.81)^{1/3}$

$Y_c= 0.3217$ m

Here Y < Yc

Hydraulic jump will occur

$$Y_2 = -\frac{y_1}{2} + \sqrt{\frac{y_1^2}{4} + \frac{2q^2}{gy_1}}$$

$$Y_2 = -\frac{0.25}{2} + \sqrt{\frac{0.25^2}{4} + \frac{2\times0.5714^2}{9.81\times0.25}}$$

=0.40589 m

Height of hydraulic jump

$=y_2 - y_1$

=0.40589 - 0.25

= 0.15589m

Loss of energy

$$h_L=\frac{(y_2-y_1)^3}{4y_1y_2}$$

$$= \frac{(0.405-0.25)^3}{4\times0.25\times0.405}$$

$= 9.33\times10^{-3}$

Power lost

$P = \rho g Q h_L$

$= 1000\times 9.81 \times 9.33\times10^{-3}$

= 91.523 w

3.4 ENERGY DISSIPATION

The flow velocity at the toe of a high-head spillway is usually high and may cause serious scour and erosion of the downstream channel if proper precautions are not taken. For this purpose, energy dissipators are provided to dissipate sufficient amount of energy before water enters the downstream channel.

Three types of energy dissipators have been commonly used:

- stilling basins,
- flip buckets, and
- roller buckets

Each dissipator has certain advantages and disadvantages and may be selected for a particular project depending upon the site characteristics. A brief description of these dissipators is given in the following paragraphs

Stilling Basins

The hydraulic jump is used for energy dissipation in a stilling basin. Typically, this basin may be used for heads less than 50 m. At higher heads, cavitation becomes a problem. A concrete apron is provided for the length of the jump and the invert level of the apron is set such that the downstream water level provides the necessary sequent depth for the flow depth and the Froude number at the entrance to the jump. Long apron lengths and low apron levels are needed for such a stilling basin. Low apron levels require large amount of excavation and concrete. For economic reasons as well as to make the stilling basin operate efficiently over a wide range of flows, other devices and accessories may be provided to stabilize the jump, to reduce the length of the jump,
and to permit the apron at a higher elevation. These devices include chute blocks, baffle blocks, and end sills

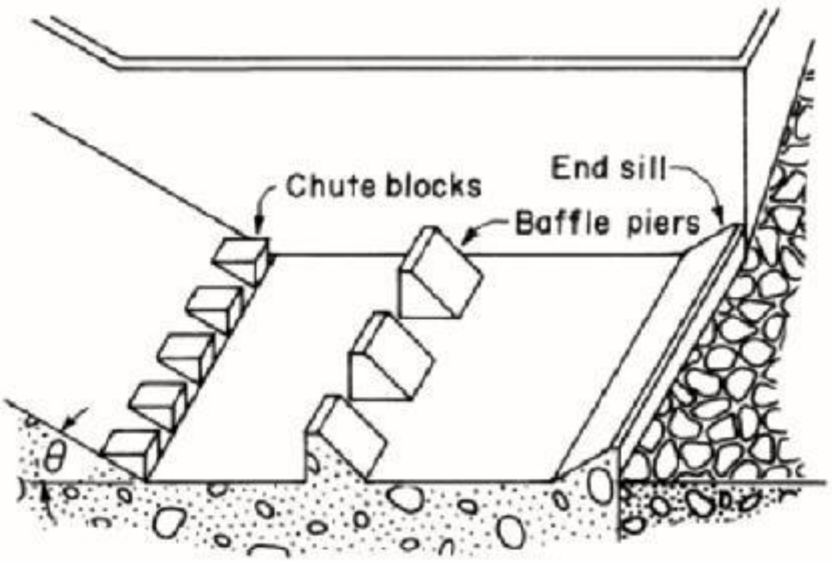

Figure 3.4 Stilling Basins

source

The chute blocks serrate the flow entering the basin and lift up part of the jet. This produces more eddies increasing energy dissipation, the jump length is decreased, and the tendency of the jump to sweep out of the basin is reduced. The baffle blocks stabilize the jump and dissipate energy due to impact. The sill mainly stabilizes the jump and inhibits the tendency of the jump to sweep out.

Flip Buckets

The flip bucket energy disspator is suitable for sites where the tailwater depth is low (which would require a large amount of excavation if a hydraulic jump dissipator were used) and the rock in the downstream area is good and resistant to erosion. The flip bucket, also called ski-jump dissipator, throws the jet at a sufficient distance away from the spillway where a large scour hole may be produced. Initially, the jet impact causes the channel bottom to scour and erode. The scour hole is then enlarged by a ball-mill motion of the eroded rock pieces in the scour hole. A plunge pool may be excavated prior to the first spill for controlled erosion and to keep the plunge pool in a desired location. A small amount of the energy of the jet is dissipated by the internal turbulence and the shearing action of the surrounding air as it travels in the air. However, most of the energy of the jet is dissipated in the plunge pool. During the operation of a flip bucket, a large amount of spray is produced, which may be undesirable for roads, bridges, and electrical equipment, such as transmission lines or grid stations. In addition, large water-level fluctuations may be produced in the tailrace area by the plunging jet and the associated return currents and eddies. These water level oscillations near the draft tube exits may impair the operation of Francis turbines, since these oscillations may result in load swings.

Roller Buckets

A roller bucket may be used for energy dissipation if the downstream depth is significantly greater than that required for the formation of a hydraulic jump. In this dissipator, the dissipation is caused mainly by two rollers: counterclockwise roller near the water surface above the bucket and a roller on the channel bottom downstream of the bucket. The movement of these rollers along with the intermixing of the incoming flows results in the dissipation of energy. Since then two types of roller buckets – solid and slotted – have been developed through hydraulic model studies and used successfully on several projects. In a solid bucket, the ground roller may bring material towards the bucket and deposit it in the bucket during periods of unsymmetrical operation. In a slotted bucket, part of the flow passes through the slots, spreads laterally, and is distributed over a greater area. Therefore, the flow concentration is less than that in a solid bucket. In addition, any material that might get into the bucket is washed out. The sweepout in a slotted bucket occurs at a slightly higher tailwater level than that in a solid bucket

3.5 CELERITY

The celerity is defined as the wave velocity with respect to the velocity of the medium in which the wave is traveling.

To derive an expression for the wave celerity, let us consider a small wave in a horizontal, frictionless channel, The wave is considered to be small if $|\delta y| << y$. Let us assume that this wave is traveling in the downstream direction with absolute wave velocity, Vw and that, as a result of the motion, the flow velocity is changed from V to V + δV . By superimposing a constant velocity Vw in the upstream direction, the component of the weight of water in the control volume in the downstream direction is zero. Similarly, there is no shear force acting on the channel boundary, since the channel is assumed to be frictionless.

$$Vw = V \pm \sqrt{gy}$$

celerity, c, is thewave velocity relative to the medium in which the wave is traveling – i.e., Vw = V ± c.

$$c = \sqrt{gy}$$

We proved in a previous section that the Froude number Fr = 1 when the flow is critical. By substititing the expression for Fr and using subscript c to denote various quantities for critical flow, we obtain

$$Vc \sqrt{gy} = 1$$

$$Vc = \sqrt{gy}$$

$$Vc = c$$

Thus, the celerity of a small wave is equal to the flow velocity when the flow is critical. Since, V < Vc in subcritical flows, it follows that V < c in these flows. Similarly, we may prove that V >c in the supercritical flows. Three different flow situations for the propagation of a disturbance are possible depending upon the relative magnitudes of V and c, i.e., whether the flow is subcritical, critical, or supercritical

3.6 RAPIDLY VARIED UNSTEADY FLOWS (POSITIVE AND NEGATIVE SURGES)

A surge or surge wave is a moving wave front which brings about an abrupt change in depth of flow. A surge is also often referred to as moving hydraulic jump and is caused by sudden increase or decrease in flow, such as that caused by sudden opening or closing of a gate fixed in the channel. Surges are usually classified as positive surges and negative surges. A positive surge is one which results in an increase in the depth of flow and a negative surge causes a decrease in the depth of flow.

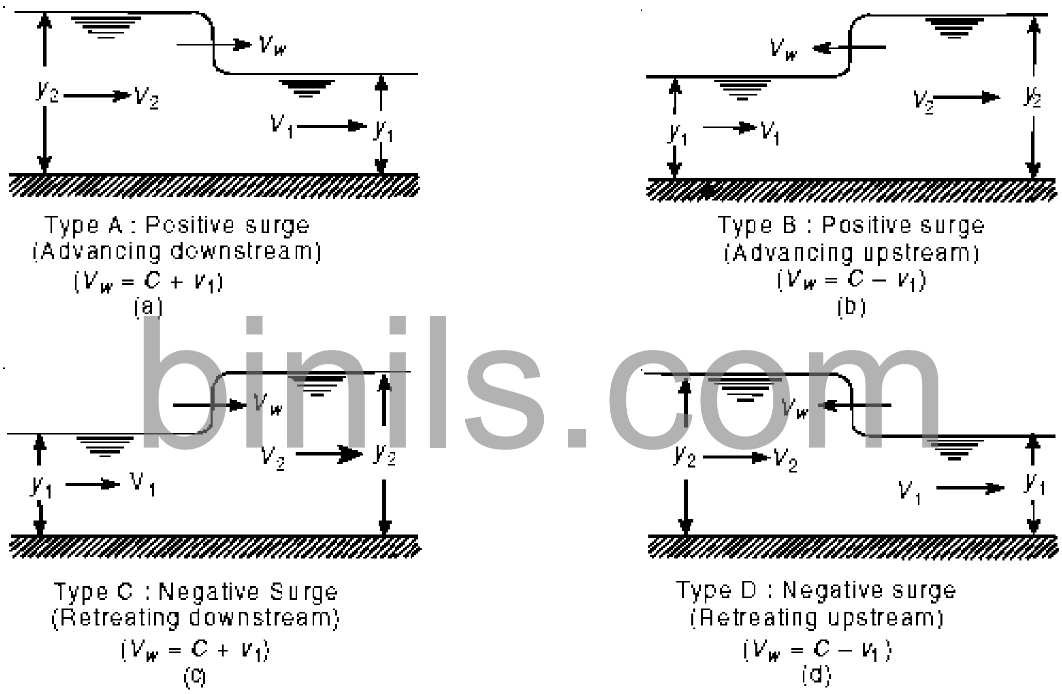

Figure 3.3 Types of hydraulic jump

[*Source: Hydraulics And Fluid Mechanics Including Hydraulic Machines By Dr. P.N.Modi , page 808*]

Figures (a) and (b) shows two types of positive surges and Figs. (c) and (d) shows two types of negative surges. Type A (Fig a) is a positive surge having an advancing wave front moving downstream. Type *B* (Fig. b) is also a positive surge having an advancing wave front moving upstream.Type *C* (Fig. c) is a negative surge having a retreating wave front moving downstream. Type *D* (Fig. d) is also a negative surge having a retreating wave front moving upstream. The positive surge of type *A* may occur when a gate provided at the head of a channel is suddenly opened. The positive surge of the type *B* may occur when a gate provided at the tail end of a channel is suddenly closed. The negative surge of type *C* may occur when a gate provided at the head of a channel is suddenly closed. The negative surge of type *D* may occur when a gate provided at the tail end of a channel is suddenly opened. Although the occurrence of a surge is an usteady flow phenomenon, but when the surge is moving at a constant velocity, it can be converted into a case of a steady flow by applying a velocity of the same magnitude but in opposite direction to the flowing stream as well as to the surge.

Positive Surges

Case (a) Surge due to sudden increase of flow

Consider a positive surge of type A created in a rectangular channel by suddenly opening a gate. Let Vw be the absolute velocity of the surge moving towards right and let V1 and V2 be the velocities and y1 and y2 be the corresponding depths at sections 1 and 2 respectively. In order to make it a case of steady flow, apply velocity Vw in opposite directions to the velocities V1 and V2 and the surge. Thus the velocities at sections 1 and 2 becomes (V1 – Vw) and (V2 – Vw) respectively.

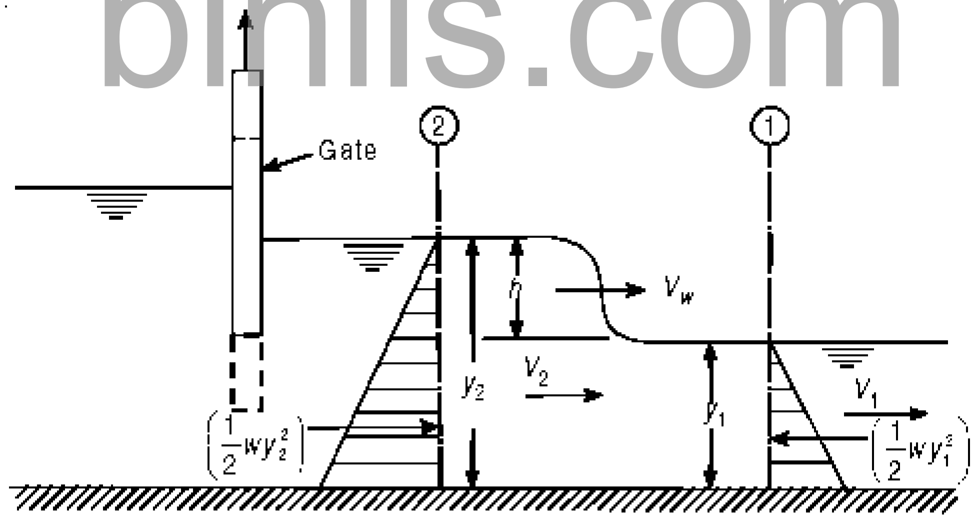

Figure 3.4 positive surges

[*Source: Hydraulics And Fluid Mechanics Including Hydraulic Machines By Dr. P.N.Modi , page 809*]

Case (b) Surge due to sudden reduction of flow

Consider a positive surge of type *B* created in a rectangular channel by sudden reduction of flow due to partial or complete closing of a gate. If *Vw* is the absolute velocity of the surge moving towards left then in order to make it a case of steady flow apply a velocity V_w in the direction opposite to that of surge. The velocities at sections 1 and 2 will then be $(V_1 + V_w)$ and $(V_2 + V_w)$ respectively. The continuity equation in this case is

$$y_1(V_1 + V_w) = y_2(V_2 + V_w)$$

Negative Surges

Negative surges are not stable in form because as indicated below the upper portions of the wave travel faster than the lower portions. if y_2 is the depth below the top a_1 b_1 of the wave and y_1 is the depth below the bottom or trough c_1 d_1 of the wave, then since celerity $C = gy$, its value at the top a_1 b_1 of the wave is $C_2 = gy_2$ and at the bottom c_1 d_1 of the wave is $C_1 = gy_1$; and C_2 is greater than C_1. Thus if the initial profile of the surge is assumed to have a steep front, it will soon flatten out as the surge moves through the channel. If the height of the surge is moderate or small compared with the depth of flow, the equations derived for a positive surge can be applied to determine approximately the propagation of the negative surge. However, if the height of the surge is relatively large, a more elaborate analysis as indicated below may be adopted.