Catalog

UNIT V

THIN CYLINDERS AND SPHERES

5.1. INTRODUCTION

The vessels such as boilers, compressed air receivers etc., are of cylindrical and spherical forms. These vessels are generally used for storing fluids (liquid or gas) under pressure. The walls of such vessels are thin as compared to their diameters. If the thickness of the wall of the cylindrical vessels is less than 1/15 to 1/20 of its internal diameter, the cylindrical vessel is known as thin cylinder. In case of thin cylinders, the stress distribution is assumed uniform over the thickness of the wall.

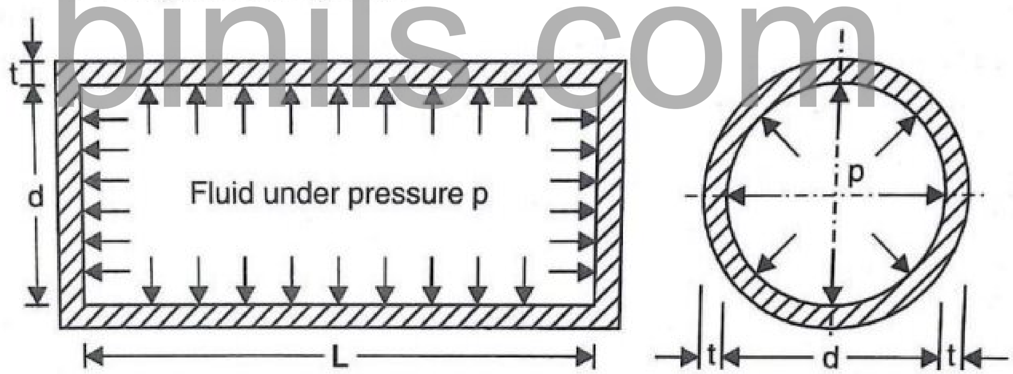
5.2. THIN CYLINDRICAL VESSEL SUBJECTED TO INTERNAL PRESSURE

Let d = Internal diameter of the thin cylinder

t = Thickness of the wall of the cylinder

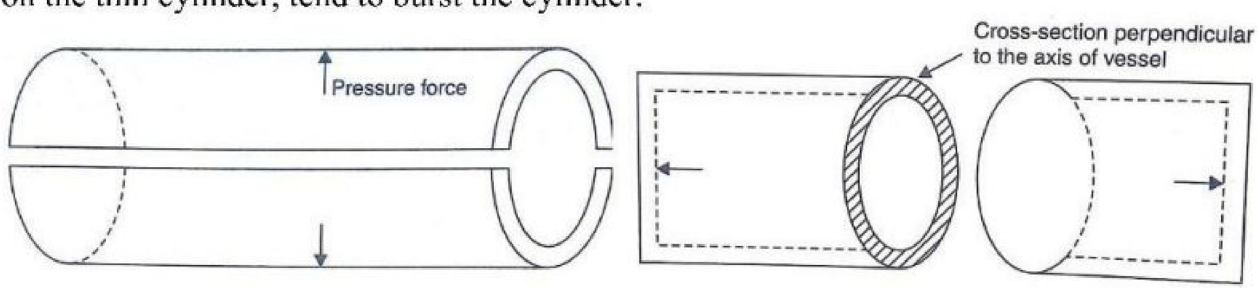
p = Internal pressure of the fluid

L = Length of the cylinder



One of the internal pressure p, the cylindrical vessel may fail by splitting up in any one of the two ways.

The forces, due to pressure of the fluid acting vertically upwards and downwards on the thin cylinder, tend to burst the cylinder.



The forces, due to pressure of the fluid, acting at the thin cylinder, tend to burst the thin cylinder.

5.3. STRESSES IN A THIN CYLINDRICAL VESSEL SUBJECTED TO INTERNAL PRESSURE

When a thin cylindrical vessel is subjected to internal fluid pressure, the stresses in the wall of the cylinder on the cross section along the axis and on the cross section perpendicular to the axis are set up. These stresses are tensile and are known as:

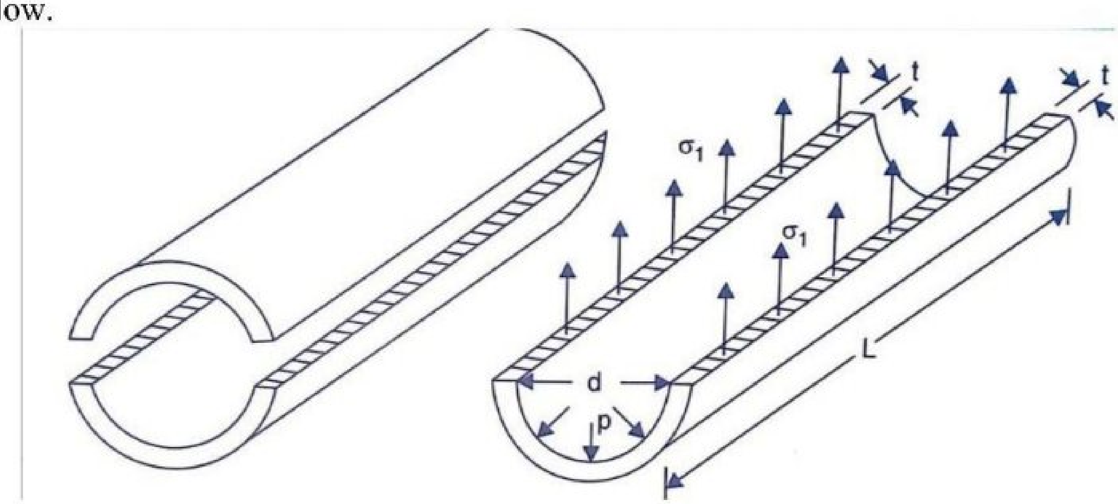
1. Circumferential stress (or hoop stress) and
2. Longitudinal stress

The name of the stress is given according to the direction in which the stress is acting. The stress acting along the circumference of the cylinder is called circumferential stress whereas the stress acting along the length of the cylinder (i.e., in the longitudinal direction) is known as longitudinal stress. The circumferential stress is also known as hoop stress. The stress set up in is circumferential stress whereas the stress set up in is longitudinal stress.

5.4. EXPRESSION FOR CIRCUMFERENTIAL STRESS (OR) HOOP STRESS

Consider a thin cylinder vessel subjected to an internal fluid pressure. The circumferential stress will be set up in the material of the cylinder, if the bursting of the cylinder takes place.

The expression for hoop stress or circumferential stress is obtained as given below.



Let p =Internal pressure of the fluid

d =Internal diameter of the cylinder

t =Thickness of the wall of the cylinder

σ_1 =Circumferential or hoop stress in the material

The bursting will take place if the force due to fluid pressure is more than the resisting force due to circumferential stress set up in the material. In the limiting case, the two forces should be equal.

Forces due to fluid pressure = p ×Area on which p is acting

$= p \times (d \times L)$……………(i)

Forces due to circumferential stress

$= \sigma_1 \times$ Area on which σ_1 is acting

$= \sigma_1 \times (L \times t + L \times t)$

$= \sigma_1 \times 2Lt = 2\sigma_1 \times L \times t$……………..(ii)

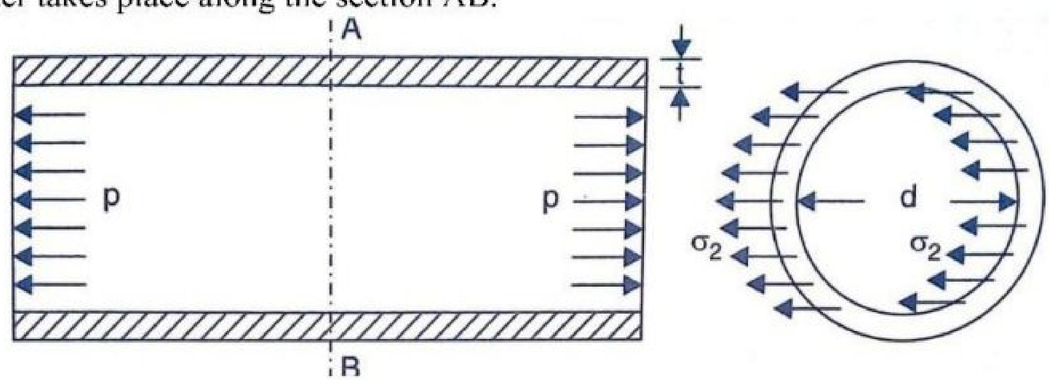
Equating (i) and (ii) we get

$$P \times d \times L = 2\sigma_1 \times L \times t$$

$$\sigma_1 = \frac{Pd}{2t}$$ This stresses is tensile.

5.5. EXPRESSION FOR LONGITUDINAL STRESS

Consider a thin cylindrical vessel subjected to internal fluid pressure. The longitudinal stress will be set up in the material of the cylinder, if the bursting of the cylinder takes place along the section AB.



The longitudinal stress (σ_2) developed in the material is obtained as:

Let p =Internal pressure of fluid stored in thin cylinder

d =Internal diameter of cylinder

t =Thickness of the cylinder

σ_2 =Longitudinal stress in the material

Thus bursting will take place if the force due to fluid pressure acting on the ends of the cylinder is more than the resisting force due to longitudinal stress developed in the material. In the limiting case, both the forces should be equal.

Forces due to fluid pressure $= p \times$ Area on which p is acting

$= p \times \frac{\pi}{4} d^2$

Resisting force $= \sigma_2 \times$ area on which σ_2 is acting

$= \sigma_2 \times \pi d \times t$

Hence in the limiting case

Force due to fluid pressure = resisting force

$$p \times \frac{\pi}{4} d^2 = \sigma_2 \times \pi d \times t$$

$$\sigma_2 = \frac{pd}{4t}$$

The stress σ_2 is also tensile equation can be written as

$$\sigma_2 = \frac{pd}{2 \times 2t}$$

$$\sigma_2 = \frac{1}{2} \times \sigma_1 \qquad (\because \quad \sigma_1 = \frac{pd}{2t})$$

or Longitudinal stress=Half of circumferential stress

This also means that circumferential stress is two times the longitudinal stress Hence in the material of the cylinder the permissible stress should be less than the circumferential stress should not be greater than the permissible stress.

Maximum shear stress At any point in the material of the cylindrical shell, there are two principle stresses, namely a circumferential stress of magnitude $\sigma_1 = pd/2t$ acting circumferentially and a longitudinal stress of magnitude $\sigma_2 = pd/4t$ acting parallel to the axis of the shell. These two stresses are tensile and perpendicular to each other.

Maximum shear stress $\tau_{max} = \sigma_1 - \frac{\sigma_2}{2}$

$$= \frac{pd}{4t} - \frac{\frac{pd}{4t}}{2}$$

$$\tau_{max} = \frac{pd}{8t}$$

Problem 5.1:A cylindrical pipe of diameter 1.5m and the thickness1.5 cm is subjected to an internal fluid pressure of 1.2N/mm^2 Determine (i) Longitudinal stress developed in the pipe, and (II)circumferential stress developed in the pipe.

Given data:

Diameter of pipe $d=1.5$ m $= 1.5\times10^3$ mm

Thickness $t=1.5$cm $= 15$ mm

Internal fluid pressure $p=1.2$ N/mm^2

To find:

Longitudinal stress $\sigma_2 = ?$

Circumferential stress $\sigma_1 = ?$

Solution:

As the ratio $\frac{t}{d} = \frac{1.5\times10-2}{1.5} = \frac{1}{100}$, which is less than $\frac{1}{20}$ hence this is a case of thin cylinder.

Here unit of pressure (p) in N/mm^2 Hence the unit of σ_1 and σ_2 will also be in N/mm^2

(i)The longitudinal stress(σ_2) is given by equation

$$\sigma_2 = \frac{pd}{4t} = \frac{1.2\times1.5\times10^3}{4\times15} = 30 \text{ N/mm}^2$$

(ii)The circumferential stress (σ_1) is given by equation

$$\sigma 1 = \frac{pd}{2t} = \frac{1.2\times1.5\times10^3}{2\times15} = 60 \text{ N/mm}^2$$

Result:

Longitudinal stress $\sigma_2 =$ **30 N/mm^2**

Circumferential stress $\sigma_1 =$ **60 N/mm^2**

Problem5.2: A cylinder of internal diameter 2.5m and of thickness 5 cm contains a gas. If the tensile stress in the material is not to exceed 80 N/mm^2 determine the internal pressure of the gas.

Given data:

Internal diameter of cylinder $d = 2.5$m $= 2.5\times10^3$ mm

Thickness of the cylinder $t = 5$cm$=50$ mm

Maximum permissible stress $=80$ N/mm^2

To find:

Internal pressure of the gas $p = ?$

Solution:

Maximum permissible stress is available in the circumferential stress (σ_1)

$$\therefore \quad \text{circumferential stress } (\sigma_1) = \frac{pd}{2t}$$

$$80 = \frac{p \times 2.5\times10^3}{2\times50}$$

$$\gg \quad p = 3.2 N/mm^2$$

Result:

Internal pressure of the gas $p =$ **$3.2 N/mm^2$**

Problem 5.3: A cylinder of internal diameter 0.50 m contains air at a pressure of $7 N/mm^2$(gauge). If the maximum permissible stress induced in the material is $80 N/mm^2$, find the thickness of the cylinder.

Given data:

Internal dia of cylinder $d = 0.50 m = 500$ mm

Internal pressure of air, $p = 7 N/mm^2$

Circumferential stress, $\sigma_1 = 80 N/mm^2$ ($\because$ maximum permissible stress)

To find:

Thickness of cylinder $t = ?$

Solution:

$$\text{Wkt} \quad \text{Circumferential stress } (\sigma_1) = \frac{pd}{2t}$$

$$80 = \frac{7\times500}{2\times t}$$

$$\gg \quad t = 21.88 mm$$

If the value t is taken more than 21.875 mm(sat t=21.88 mm), the stress induced will be less than $80 N/mm^2$.

Hence take t=21.88 mm or say 22 mm

Result:

Thickness of cylinder $t =$ **22 mm**

Problem 5.4: A thin cylinder of internal diameter 1.25m contains a fluid at an internal pressure of 2N/mm^2. Determine the maximum thickness of the cylinder if (i) The longitudinal stress is not to exceed 30N/mm^2 and (ii)The circumferential stress is not to exceed 45N/mm^2

Given data:

Internal dia of cylinder, $d = 1.25\text{ m} = 1.25\times10^3$ mm

Internal pressure of fluid, $p = 2\text{N/mm}^2$

Longitudinal stress $\sigma_2 = 30\text{n/mm}^2$

Circumferential stress, $\sigma_1 = 45\text{N/mm}^2$

To find:

Thickness of cylinder $t = ?$

Solution:

Wkt Circumferential stress $(\sigma_1) = \frac{pd}{2t}$

$$45 = \frac{2\times1.25\times10^3}{2\times t}$$

$$\gg \quad t = 27.7\text{ mm}$$

Wkt, longitudinal stress $\sigma_2 = \frac{pd}{4t}$

$$30 = \frac{2\times1.25\times10^3}{4\times t}$$

$$\gg \quad t = 28.0\text{ mm}$$

from the above two thickness value it is clear that t should not be less than 27.7mm. Hence take t=28. mm.

Result:

Thickness of cylinder $t = 28$ mm

Problem 5.5: A water main 80 cm diameter contains water at a pressure head of 100m. If the weight density of water is 9810N/m^3, find the thickness of the metal required for the water main given the permission stress as 20N/mm^2.

Given data:

Diameter of main, d=80 cm = 800 mm

Pressure head of water, $h = 100\ m = 100 \times 10^3\ mm$

Weight density of water $\omega = \rho \times g = 1000 \times 9.81 = 9810\ N/m^3$

Permissible stress $= 20 N/mm^2$

To find:

Thickness of the metal $t = ?$

Solution:

Permissible stress is equal to circumferential stress(σ1)

Pressure of water inside the water main,

$$p = \rho \times g \times h = \omega.h = 9810 \times 100\ N/m^2$$

Here σ_l is in N/mm^2 hence pressure (p) should be N/mm^2. The value of p in N/mm^2 is given as

$$P = 9810 \times 100/1000^2$$

$$= 0.981 N/mm^2$$

Wkt Circumferential stress $\sigma_1 = \dfrac{pd}{2t}$

$$20 = \frac{0.981 \times 800}{2 \times t}$$

» $t = 20\ mm$

Result:

Thickness of the metal $t = 20\ mm$

5.6. EFFICIENCY OF A JOINT

The cylindrical shells such as boilers are having two types of joints namely longitudinal joint and circumferential joint. In case of a joint, holes are made in the material of the shell for the rivets. Due to the holes, the area offering resistance decreases. Due to the decreases in area, the stress developed in the material of the shell will be more.

Hence in case of riveted shell the circumferential and longitudinal stresses are greater than what are given by eqn. If the efficiency of a longitudinal joint and circumferential joint are given then the circumferential and longitudinal stresses are obtained as:

Let η_l = Efficiency of a longitudinal joint, and

η_c = Efficiency of the circumferential joint.

Then the circumferential stress (σ_1) is given as

$$\sigma_1 = \frac{pd}{2t \times \eta_l}$$

and the longitudinal stress (σ_2) is given as

$$\sigma_2 = \frac{pd}{4t \times \eta_c}$$

Problem 5.6: A boiler is subjected to an internal steam pressure of 2N/mm^2. The thickness of boiler plate is 2.0cm and permissible tensile stress is 120N/mm^2. Find out the maximum diameter when efficiency of longitudinal joint is 90% and that of circumferential joint is 40%

Given data:

Internal steam pressure p = 2N/mm^2

Thickness of boiler plate t = 2.0cm = 20mm

Permissible tensile stress =120N/mm^2

Efficiency of Longitudinal joint, η_l=90%=0.90

Efficiency of circumferential joint, η_c=40%=0.40

To find:

Find the maximum diameter = ?

Solution:

For Circumferential stress σ_1= 120 N/ mm^2

Wkt Circumferential stress $\sigma_1 = \frac{pd}{2t \times \eta_l}$

$$120 = \frac{2 \times d}{2 \times 20 \times 0.90}$$

» d = 2160 mm ……………………..(i)

For longitudinal stress σ_2 = 120 N/ mm^2

Wkt, longitudinal stress $\sigma_2 = \frac{pd}{4t \times \eta_c}$

$$120 = \frac{2 \times d}{4 \times 20 \times 0.40}$$

» d = 1920 mm ………………..(ii)

hence suitable maximum diameter d=1920 mm.

Note: If d is taken as equal to216cm the longitudinal stress will be more than the given permissible value as shown below.

$$\sigma_2 = \frac{pd}{4t \times \eta_c}$$

$$\sigma_2 = \frac{2 \times 216}{4 \times 20 \times 0.40} = 135 \text{ N/ mm}^2$$

Problem 5.7: A boiler shell is to be mad of 15mm thick plate having a limiting tensile stress of 120 N/mm^2. If the efficiencies of the longitudinal and circumferential joints are 70% and 30% respectively determine;

(i) The max permissible diameter of the shell for an internal pressure of 2N/mm^2

(ii) permissible intensity of internal pressure when the shell diameter is 1.5m

Given data:

Thickness of boiler shell, t =15mm

Limiting tensile stress =120N/mm^2

Efficiency of longitudinal joint η_l=70%=0.70

Efficiency of circumferential joint η_c=30%=0.30

To find:

Maximum permissible diameter d =?

Internal pressure p =?

Solution:

i) Maximum permissible diameter for an internal pressure p = 2 N/mm^2

The boiler shell should be designed for the limiting tensile stress of 120N/mm^2. First consider the limiting tensile stress as circumferential stress and then as longitudinal stress. The minimum diameter of the two case will satisfy the condition.

(a)Taking limiting tensile stress =circumferential stress σ_1=120N/mm^2

Wkt the circumferential stress σ_1

$$\sigma_1 = \frac{pd}{2t \times \eta_l}$$

$$120 = \frac{2 \times d}{2 \times 15 \times 0.70}$$

$$\gg \quad d = 2160 \text{ mm} \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

(b)Taking limiting tensile stress = longitudinal stress σ_2=120N/mm^2

Wkt the longitudinal stress σ_2

$$\sigma_2 = \frac{pd}{4t \times \eta_c}$$

$$120 = \frac{2 \times d}{4 \times 15 \times 0.30}$$

$$\gg \quad d = 1080 \text{ mm}$$

(ii) Permissible intensity of internal pressure when the shell diameter is 1.5m

d =1.5m=1500mm

(a)Taking limiting tensile stress=circumferential stress(σ_1) =120 N/mm^2

Wkt the circumferential stress σ_1

$$\sigma_1 = \frac{pd}{2t \times \eta_l}$$

$$120 = \frac{p \times 1500}{2 \times 15 \times 0.70}$$

$$\gg \quad p = 1.68 \text{ N/mm}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

(b)Taking limiting tensile stress = longitudinal stress σ_2=120 N/mm^2

Wkt the longitudinal stress σ_2

$$\sigma_2 = \frac{pd}{4t \times \eta_c}$$

$$120 = \frac{p \times 1500}{4 \times 15 \times 0.30}$$

$$\gg \quad p = 1.44 \text{ N/mm}^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\text{(ii)}$$

value of pressure given by (i) & (ii)

Max permissible internal pressure is taken as the minimum value of (i) & (ii)

p =1.44N/mm^2

$$\sigma_2 = \frac{pd}{4t \times \eta_c}$$

$$= \frac{1.44 \times 1500}{4 \times 15 \times 0.30} = 140\text{N/mm}^2.$$

Problem 5.8: A cylinder of thickness 1.5cm, has to withstand maximum internal pressure of 1.5N/mm2. If the ultimate tensile stress in the material of the cylinder is

300N/mm2 factor of safety 3.0 and joint efficiency 80% determine the diameter of the cylinder.

Given Data:

Thickness of cylinder $t = 1.5cm = 15$ mm

internal pressure $p = 1.5N/mm^2$

ultimate tensile stress $= 300N/mm^2$

FOS $=3.0$

joint efficiency $= 80\%$

To find:

Diameter of the cylinder $d = ?$

Solution:

Working stress, σ_1 = Ultimate tensile stress/FOS

$= 300/3$

$=100N/mm^2$

Joint efficiency, $\eta = 80\% = 0.80$

Joint efficiency means the efficiency of longitudinal joint η_l

The stress corresponding to longitudinal joint is given by equation

$$\sigma_1 = \frac{pd}{2t \times \eta_l}$$

$$100 = \frac{1.5 \times d}{2 \times 15 \times 0.80}$$

$\gg$ $d = 1600$ mm$=1.6$m

5.7. EFFECT OF INTERNAL PRESSURE ON THE DIMENSIONS OF A THIN CYLINDRICAL SHELL

When a fluid having internal pressure (p) is stored in a thin cylindrical shell, due to internal pressure of the fluid the stresses set up at any point of the material of the shell are:

(i)Hoop circumferential stress(σ_1),acting on longitudinal section.

(ii)Longitudinal stress(σ_2) acting on the circumferetial section.

These stresses are principal stresses, as they are acting on principal planes. The stress in the third principal plane is zero as the thickness (t) of the cylinder is very small.

Actually the stress in the third principal plane is radial stress which is very small for thin cylinders and can be neglected.

Let p = Internal pressure of fluid

L= Length of cylindrical shell

d = Diameter of the cylindrical shell

t = Thickness of the cylindrical shell

E = Modulus of Elasticity for the material of the shell

σ_1 = Hoop stress in the material

σ_2 = Longitudinal stress in the material

δd = change in diameter due to stresses set up in the material

δL = change in length

δv = change in volume

μ =poison ratio

The value of σ_1 and σ_2 are given by eqn and as

$$\sigma_1 = \frac{pd}{2t}$$

$$\sigma_2 = \frac{pd}{4t}$$

Let e_1 = circumferential strain,

e_2 = Longitudinal strain,

Then circumferential strain,

$$e_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E}$$

$$= \frac{pd}{2tE} - \mu\,\frac{pd}{4tE}$$

$$= \frac{pd}{2tE}\left(1 - \frac{\mu}{2}\right)$$

and longitudinal strain,

$$e_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$$

$$= \frac{pd}{4tE} - \mu\,\frac{pd}{2tE}$$

$$= \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right)$$

But circumferential strain is also given as,

$$e_1 = \frac{\text{Change in circumferential due to pressure}}{\text{original circumference}}$$

$$= \frac{\text{Final circumference} - \text{original circumference}}{\text{original circumference}}$$

$$= \frac{\pi(d+\delta d) - \pi d}{\pi d}$$

$$= \frac{\pi d + \pi \delta d - \pi d}{\pi d}$$

$$= \frac{\delta d}{d}$$

Equating the two values of e_1 given by equations and we get

$$\frac{\delta d}{d} = \frac{pd}{2tE}\left(1 - \frac{\mu}{2}\right)$$

Change in diameter

$$\delta d = \frac{pd^2}{2tE}\left(1 - \frac{\mu}{2}\right)$$

similarly longitudinal strain is also given as,

e_2=change in length due to pressure/original length

$= \delta L / L$

Equating the two values of e_2 given by equation

$$\delta L / L = \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right)$$

Change in length

$$\delta L = \frac{pdL}{2tE}\left(\frac{1}{2} - \mu\right)$$

Volumetric strain.

It is defined as change in volume divided by original volume

Volumetric strain $= \frac{\delta V}{V}$

But change in volume (δV) = Final volume - Original volume

Original volume (V) = Area of cylindrical shell × Length

$$= \frac{\pi}{4} \times d^2 \times L$$

Final volume = (Final area of cross section)×Final length

$$= \frac{\pi}{4} (d + \delta d)^2 \times (L + \delta L)$$

$$= \frac{\pi}{4} \left[d^2 + (\delta d)^2 + 2d\delta d\right] \times [L + \delta L]$$

$$= \frac{\pi}{4} [d^2 L + (\delta d)^2 L + 2dL\delta d + d^2 \delta L + (\delta d)^2 \delta L + 2d\delta d \delta L]$$

Neglecting the smaller quantities such as $(\delta d)^2 L$, $(\delta d)^2 \delta L$ and $2d\delta d\delta L$, we get

Final volume $= \frac{\pi}{4} \left[d^2 L + 2dL\delta d + d^2 \delta L\right]$

Change in volume (δV)

$$= \frac{\pi}{4} \left[d^2 L + 2dL\delta d + d^2 \delta L\right] - \frac{\pi}{4} \times d^2 \times L$$

$$= \frac{\pi}{4} \left[2dL\delta d + d^2 \delta L\right]$$

Then volumetric strain = $\delta V/V$

$$= \frac{\frac{\pi}{4}[2dL\delta d + d^2 \delta L]}{\frac{\pi}{4} \times d^2 \times L}$$

$$= 2\frac{\delta d}{d} + \frac{\delta L}{L} = 2e_1 + e_2 \qquad \left[\because \frac{\delta d}{d} = e_1, \frac{\delta L}{L} = e_2\right]$$

$$= 2 \times \frac{pd}{2tE}\left(1 - \frac{\mu}{2}\right) + \frac{pd}{2tE}\left(\frac{1}{2} - \mu\right)$$

Substitutes the value of e_1 and e_2

$$= \frac{pd}{2tE}\left(2 - \frac{2\mu}{2} + \frac{1}{2} - \mu\right)$$

$$= \frac{pd}{2tE}\left(2 + \frac{1}{2} - \mu - \mu\right)$$

$$= \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)$$

Also change in volume (δV)=V($2e_1 + e_2$)

Problem 5.9 Calculate (i) the change in diameter, (ii) change in length (iii)change in volume of a thin cylindrical shell 100 cm diameter, 1cm thick and 5m long when subjected to internal pressure of 3N/mm2. Take the value of E=2×105N/mm^2 and poisons ratio $\mu = 0.3$

Given data:

Diameter of shell d =100cm = 1000mm

Thickness of shell $t = 1\text{cm} = 10$ mm

Length of shell $L = 5\text{m} = 5\times10^3$ mm

Internal pressure $p = 3\text{N/mm}^2$

Young's modulus $E = 2\times10^5$

Poisson's ratio $\mu = 0.30$

To find:

(i) change in diameter $\delta d = ?$

(ii) change in length $\delta L = ?$

(iii) change in volume $\delta V = ?$

Solution:

(i)Change in diameter (δd) is given by equation

$$\delta d = \frac{pd^2}{2tE}\left(1 - \frac{\mu}{2}\right)$$

$$= \frac{3\times 1000^2}{2\times10\times2\times10^5}\left(1 - \frac{0.30}{2}\right)$$

$$= 0.6375 \text{ mm}$$

(ii) Change in length (δL) is given by equation

$$\delta L = \frac{pdL}{2tE}\left(\frac{1}{2} - \mu\right)$$

$$= \frac{3\times 1000\times5\times10^3}{2\times10\times2\times10^5}\left(\frac{1}{2} - 0.30\right)$$

$$= 0.75 \text{ mm}$$

(iii)change in volume (δV) is given by equation

$$\delta V = V[2e_1 + e_2]$$

$$= V[2\times\frac{\delta d}{d} + \frac{\delta L}{L}]$$

substituting the values of δd, δL, d and L, we get

$$\delta V = V[2\times\frac{0.06375}{1000} + \frac{0.075}{5\times10^3}]$$

Where V=original volume = $\frac{\pi}{4}\times d^2\times L = \frac{\pi}{4}\times1000^2\times 5\times10^3 = 3.92\times10^9$ mm^3

$$\delta V = 3.92\times10^9\ [2\times\frac{0.06375}{1000} + \frac{0.075}{5\times10^3}]$$

$$= 5.595\times10^6 \text{ mm}^3$$

Problem 5.10: A cylindrical shell 90cm long20cm internal diameter having thickness of metal as 8mm is filled with fluid at atmospheric pressure. If an additional 20cm^3 of fluid is pumped into the cylinder find (i)the pressure exerted by the fluid on the cylinder and (ii) the hoop stress induced. Take E=2×10^5 N/mm^2 and μ=0.3

Given data:

Length of cylinder	L=90cm = 900mm
Diameter od cylinder	d=20cm = 200 mm
Thickness of cylinder	t=8mm
Increase in volume	δV=Volume of additional fluid =20×10^3 mm^3
	E = 2×10^{5}N/mm^2
	μ = 0.3

To find:

(i) pressure exerted by the fluid

(ii) hoop stress induced

Solution:

volume of cylinder $V=\frac{\pi}{4}\times d^2\times L = \frac{\pi}{4}\times 200^2\times 900 = 2.827\times 10^7$ mm^3

(i)Let p=pressure of exerted by fluid on the cylinder

Now using eqn volumetric strain is given as

$$\frac{\delta V}{V} = 2e_1 + e_2$$

$$\frac{20\times 10^3}{2.827\times 10^7} = 2e_1+e_2 \qquad \text{..............................(i)}$$

But e_1 and e_2 are circumferential and longitudinal strains and given by equation and respectively as

$$e_1 = \frac{pd}{2tE}\left(1-\frac{\mu}{2}\right)$$

$$e2 = \frac{pd}{2tE}\left(\frac{1}{2}-\mu\right)$$

substitute these values in eqn (i) we get

$$\frac{20\times 10^3}{2.827\times 10^7} = 2\frac{pd}{2tE}\left(1-\frac{\mu}{2}\right)+\frac{pd}{2tE}\left(\frac{1}{2}-\mu\right)$$

$$\frac{20\times10^3}{2.827\times10^7} = 2\frac{p\times200}{2\times8\times2\times10^5}\left(1-\frac{0.3}{2}\right)+\frac{p\times200}{2\times8\times2\times10^5}\left(\frac{1}{2}-0.3\right)$$

$$p = 5.386\ N/mm^2$$

(ii) Hoop stress (σ_1) is given by equation

$$\sigma_1 = \frac{pd}{2t} = \frac{5.386\times200}{2\times8}$$

$$= 67.33 N/mm^2$$

Result:

(i) pressure exerted by the fluid (p) = 5.386 N/mm²

(ii) hoop stress induced (σ_1) = 67.33N/mm²

Problem 5.11: A cylindrical vessel whose ends are closed by means of rigid flanges plates, is made of steel plate 3mm thick. The length and the internal diameter of the vessel are 50cm and 25cm respectively. Determine the longitudinal and hoop stress in the cylindrical shell due to an internal fluid pressure of 3N/mm2. Also calculate the increase in length, diameter and volume of the vessel. Take E=2×10^7N/mm2 and μ=0.3

Given data:

Thickness	t = 3mm
Length of the cylindrical vessel	L= 50cm = 500 mm
Internal diameter	d = 25cm = 250 mm
Internal fluid pressure	p = 3N/mm²
Young's modulus	$E = 2\times10^5\ N/mm^2$
Poisson's ratio	μ = 0.3

To find:

Longitudinal stress and hoop stress =?

Increase in length, diameter and volume =?

Solution:

Using equation for hoop stress $\sigma_1 = \frac{pd}{2t} = \frac{3\times250}{2\times3}$

$= 125 N/mm^2$

Using equation for longitudinal stress $\sigma_2 = \frac{pd}{4t} = \frac{3\times250}{4\times3}$

$= 62.5\ N/mm^2$

Using equation for circumferential strain $e_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_2}{E} = = \frac{1}{E}\left(\sigma_1 - \frac{\sigma_2}{2}\right)$

$$= \frac{1}{2\times10^5}\left(125 - \frac{62.5}{2}\right)$$

$$= 53.125\times10^{-5}$$

And longitudinal strain, $e_2 = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$

But circumferential strain is also given by equation

$$c_1 = \delta d/d$$

Equating the two values of circumferential strain e_1 we get

$$\delta d/d = 53.125\times10^{-5}$$

$$\delta d = 53.125\times10^{-5}\times d$$

$$= 53.125\times10^{-5}\times250$$

$$= 0.133 \text{ mm}$$

Increase in diameter δd=0.133mm

Longitudinal strain is given by equation, $e_2 = \delta L/L$, But $e_2 = \frac{\sigma_2}{E} - m\frac{\sigma_1}{E}$

Then

$$\delta L/L = \frac{\sigma_2}{E} - \mu\frac{\sigma_1}{E}$$

$$\delta L/L = \frac{1}{E}(\sigma_2 - \mu\sigma_1)$$

$$= \frac{1}{2\times10^5}(62.5\sigma_2 - \mu125)$$

$$=12.5\times10^{-5}$$

Increase in length $\delta L = 12.5\times10^{-5}\times L$

$$= 12.5\times10^{-5}\times500$$

$$= 0.0625 \text{ mm.}$$

Volumetric strain is given as $\delta V/V = 2\times e_1 + e_2$

$$= 2\times53.125\times10^{-5} + 12.5\times10\text{-}5$$

$$= 118.75\times10^{-5}$$

Increase in volume $\delta V = 118.75\times10^{-5}\times V$

$= 118.75\times10^{-5}\times\frac{\pi}{4}\times250^2\times500$

$= 29.13\times10^3\ mm^3$

Result :

Hoop stress and Longitudinal stress σ_1 **= 125 N/mm²; σ_2= 62.5 N/mm²**

Increase in length, diameter and volume **δL = 0.0625 mm**

δd = 0.133 mm ;

δV = 29.13 ×10³ mm³

Problem 5.12: A cylindrical vessel is 1.5m diameter and 4m long is closed at ends by rigid plates. It is subjected to an internal pressure of 3N/mm². If the maximum principal stress is not to exceed 150N/mm², find the thickness of the shell. Assume E=2×10⁵N/mm² and poison's ratio=0.25 Find the changes in diameter, Length and volume of the shell.

Given Data:

Diameter	d = 1.5m=1500mm
Length	L = 4m=4000mm
Internal pressure	p = 3N/mm²
Max principal stress is as	σ_1= 150N/mm²
	E = 2×10⁵N/mm²
poison's ratio	μ = 0.25

To find:

Thickness of cylinder t =?

Change in length, diameter and volume =?

(i)Using hoop stress equations $\sigma_1 = \frac{pd}{2t}$

$$t = \frac{pd}{2\times\sigma_1} = \frac{3\times1500}{2\times150}$$

=15mm

(ii) Change in diameter $\delta d = \frac{pd^2}{2tE}\left(1-\frac{\mu}{2}\right)$

$$= \frac{3\times1500^2}{2\times15\times2\times10^5}\left(1-\frac{0.30}{2}\right)$$

$= 0.984mm$

(iii) Change in length $\delta L = \frac{pdL}{2tE}\left(\frac{1}{2} - \mu\right)$

$= \frac{3\times 1500\times4\times10^3}{2\times15\times2\times10^5}\left(\frac{1}{2} - 0.30\right)$

$= 0.75mm$

(iv) Change in volume $\delta V/V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)$

$\delta V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)V$

$= \frac{3\times 1500}{2\times15\times2\times10^5}\left(\frac{5}{2} - 2\times 0.30\right)\times[\frac{\pi}{4}\times250^2\times500]$

$=10602875mm^3$

Result:

Thickness of cylinder **t = 15 mm**

Change in length, diameter and volume **δL = 0.984 mm**

δd = 0.75 mm ;

δV =10602875 mm³

Problem 5.13: A cylindrical shell 3m long which is closed as the ends has an internal diameter of 1m and a wall thickness of 15mm. Calculate the circumferential and longitudinal stresses induced and also changes in the dimensions of the shell, if it is subjected to an internal pressure of 1.5N/mm². Take $E=2\times10^5$ N/mm² and μ=0.3

Given data:

Length of shell	L=3m=3000 mm
Internal diameter	d=1m=1000 mm
Wall thickness	t=15mm
Internal pressure	$p=1.5N/mm^2$
Young's modulus	$E= 2\times10^5$ N/mm²
Poison's ratio	μ =0.3

To find:

Longitudinal stress and hoop stress =?

Increase in length, diameter and volume =?

soln:

Using equation for hoop stress $\sigma_1 = \frac{pd}{2t} = \frac{1.5 \times 1000}{2 \times 15}$

$= 50 N/mm^2$

Using equation for longitudinal stress $\sigma_2 = \frac{pd}{4t} = \frac{1.5 \times 1000}{4 \times 15}$

$= 25 N/mm^2$

Change in dimensions

Using equation for the change in diameter (δd)

$$\delta d = \frac{pd^2}{2tE}\left[1 - \frac{1}{2} \times \mu\right]$$

$$= \frac{1.5 \times 1000^2}{2 \times 1.5 \times 2 \times 10^5}\left[1 - \frac{1}{2} \times 0.3\right]$$

$$= 0.2125 \times 10^{-2} \text{ mm}$$

Using equation for change in length we get

$$\delta L = \frac{pdL}{2tE}\left(\frac{1}{2} - \mu\right)$$

$$= \frac{1.5 \times 1000^2}{2 \times 1.5 \times 2 \times 10^5}\left(\frac{1}{2} - 0.3\right)$$

$$= 0.15 \text{ mm}$$

Using volumetric strain equation we get,

$$\delta V/V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)$$

$$\delta V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)V$$

$$= \frac{1.5 \times 1000}{2 \times 1.5 \times 2 \times 10^5}\left(\frac{5}{2} - 2 \times 0.30\right) \times [\frac{\pi}{4} \times 1000^2 \times 3000]$$

$$= 1119190.85 \text{ mm}^3$$

Result:

Hoop stress and Longitudinal stress **σ_1 = 50 N/mm²; σ_2= 25 N/mm²**

Change in length, diameter and volume **δL = 0. 0.002125mm**

δd = 0.15 mm ;

δV =1119190.85 mm³

Problem 5.14: A thin cylindrical shell with following dimensions is filled with a liquid at atmospheric pressure Length=1.2m external diameter =20cm, thickness of metal=8mm.

Find the value of the pressure exerted by the liquid on the walls of the cylinder and the hoop stress induced if an additional volume of 25cm³ of liquid is pumped into the cylinder. Take $E=2.1\times10^5 N/mm^2$ and poison ratio=0.33

Given data:

Length	L=1.2m=1200mm
External diameter	D = 20cm=200 mm
Thickness	t = 8mm
Internal diameter	d = D-(2×t) = 200-(2×8) = 184mm

Additional Volume $\delta V=25cm^3 = 25\times10^3 mm^3$

To find:

Pressure exerted by the liquid on the walls p =?

Hoop stress induced σ_1 =?

solution:

Volume of liquid or inside volume of cylinder

$$V = \frac{\pi}{4}\times d^2\times L = \frac{\pi}{4}\times 184^2\times 1200$$

$$= 31908528 \text{ mm}^3$$

Using volumetric strain equation we get,

$$\delta V/V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)$$

$$\frac{25000}{31908528} = \frac{p\times184}{2\times8\times2.1\times10^5}\left(\frac{5}{2} - 2\times0.33\right)$$

$$p = \frac{2500\times2\times8\times2.1\times10^5}{31908528\times184\times(2.5-0.66)} = 7.77 \text{ N/mm}^2$$

Using Circumferential stress equations

$$\sigma_1 = \frac{pd}{2t} = \frac{7.77\times184}{2\times8} = 89.42 \text{ N/mm}^2$$

Result:

Pressure exerted by the liquid on the walls p = **7.77 N/mm²**

Hoop stress induced σ_1 = **89.42 N/mm²**

Problem 5.15: A hallow cylindrical drum 600mm in diameter and 3m long, has a shell thickness of 10mm. If the drum is subjected to an internal air pressure of 3 N/mm^2, determine the increases in its volumes. Take $E=2\times10^5$ N/mm^2 and poisons ratio =0.3 for the material.

Given data:

External diameter	D = 600mm
Length of drum	L = 3m=3000mm
Thickness of drum	t = 10mm
Internal pressure	p = $3N/mm^2$
Young's modulus	E = 2×10^5 N/mm^2
Poison's ratio	μ = 0.3

To find:

Increases in volumes δV =?

Solution:

Internal diameter d = D – (2×t) = 600 – (2×10)

= 580mm

Using volumetric strain equation we get,

$$\delta V/V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)$$

$$\delta V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)V \qquad \left[\because V = \frac{\pi}{4}\times d^2 \times L\right]$$

$$= \frac{3\times580}{2\times10\times2\times10^5}\left(\frac{5}{2} - 2\times0.3\right)\times\left[\frac{\pi}{4}\times580^2\times3000\right]$$

$$= 792623000 \text{ mm}^3$$

Result:

Increases in volumes **δV = 792623000 mm^3**

5.8. A THIN CYLINDRICAL VESSEL SUBJECTED TO INTERNAL FLUID PRESSURE AND A TORQUE

When a thin cylinder vessel is subjected to internal fluid pressure (p) the stresses set up in the material of the vessel are circumferential stress and longitudinal stress. These two stresses are tensile and are acting perpendicular to each other. If the cylindrical vessel is subjected to a torque, shear stresses will also be set up in the material of the vessel.

Hence at any point in the material of the cylindrical vessel there will be two tensile stresses mutually perpendicular to each other accompanied by a shear stress. The major principal stress, the minor principle stress and maximum shear stress will be obtained is given in Art.

Let σ_1 = Circumferential stress

σ_2 = Longitudinal stress

τ = shear stress due to torque

The major principal stress $= \frac{\sigma_1+\sigma_2}{2} + \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$

Minor principal stress $= \frac{\sigma_1+\sigma_2}{2} - \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$

And maximum permissible stress $= \frac{1}{2}$[major principle stress-minor principle stress]

$$= \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$$

Problem 5.16: A thin cylindrical tube 80mm internal diameter and 5mm thick, is closed at the ends and is subjected to an internal pressure of 6 N/mm^2. A torque of 2009600 Nmm is also applied to the tube. Find the hoop stress, longitudinal stress, maximum and minimum principal stresses and the maximum shear stress.

Given data:

Internal diameter d = 80mm

Thickness of tube t = 5mm

Internal pressure p = 6N/mm^2

Torque applied T = 2009600Nmm

To find:

Hoop stress, longitudinal stress =?

Maximum and minimum principal stresses =?

Maximum shear stress =?

Solution:

Using equation for hoop stress

$$\sigma_1 = \frac{pd}{2t} = \frac{6\times80}{2\times5}$$

$$= 48\ N/mm^2$$

Using equation for longitudinal stress

$$\sigma_2 = \frac{pd}{4t} = \frac{7.77\times184}{4\times8}$$

$$= 24\ N/mm^2$$

Maximum and minimum principle stresses

Let τ=Shear stress in the wall of the tube

Wkt, shear force = shear stress × shear Area

$$= \tau\times(\pi d\times t)$$

$$= \tau\times\pi\times80\times5 = 400\pi\tau$$

But, torque, T = shear force $\times\frac{d}{2}$

⋙ $= 400\pi\times\tau\times\frac{80}{2} = 16000\pi\times\tau$ Nmm

But torque applied (T)=2009600 Nmm

Equating the two values of the torque, we get

⋙ $16000\pi \times \tau = 2009600$

$\tau = 2009600/16000\pi$ $=40\ N/mm^2$

Hence the material of the tube is subjected to two tensile stresses(σ_1=48 N/mm² and σ_2=24N/mm²)accompanied by a shear stress (τ=40 N/mm²)

Then Maximum permissible stress

$$= \frac{\sigma_1+\sigma_2}{2} + \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$$

$$= \frac{48+24}{2} + \sqrt{\left(\frac{48-24}{2}\right)^2 + 40^2}$$

$$= 77.7\ N/mm^2$$

Minimum principal stress $= \frac{\sigma_1+\sigma_2}{2} - \sqrt{\left(\frac{\sigma_1-\sigma_2}{2}\right)^2 + \tau^2}$

$$= \frac{48+24}{2} - \sqrt{\left(\frac{48-24}{2}\right)^2 + 40^2}$$

$= -5.76$ N/mm²

Maximum shear stress $= \frac{\text{max principle stress} - \text{Min principal stress}}{2}$

$$= \frac{77.76-(-5.76)}{2}$$

$= 41.76$ N/mm²

Result:

Hoop stress, longitudinal stress $\sigma_1 = 48$ **N/mm** $\sigma_2 = 24$ **N/mm²**

Maximum principal stress = **77.7 N/mm²**

Minimum principal stress = **-5.76 N/mm²**

Maximum shear stress = **41.76 N/mm²**

Problem 5.17: A copper cylinder 90 cm long, 40cm external diameter and wall thickness 6mm has its both ends closed by rigid blank flanges. It is initially full of oil at atmosphere pressure. Calculate the additional volume of oil which must be pumped into it in order to raise the oil pressure to 5 N/mm² above atmospheric pressure. For copper assume E=1.0×10⁵ N/mm² and poisons ratio =1/3. Take the bulk modulus of oil as 2.6×10³ N/mm³.

Given data:

Length of cylinder L = 90cm = 900 mm

External diameter D = 40cm = 400 mm

Wall thickness t = 6mm

Internal diameter, d = External diameter – (2×Wall thickness)

= 400 – (2×6) = 388 mm

Initial volume of oil, V= Internal volume of cylinder

$$V = \frac{\pi}{4} \times d^2 \times L$$

$$= \frac{\pi}{4} \times 388^2 \times 900$$

$= 1.06413 \times 10^8$ mm³

Increase in oil pressure p = 5 N/mm²

Young's modulus for copper $E=1.0\times10^5$ N/mm^2

poisons ratio $\mu=1/3 = 0.333$

Bulk modulus of oil $K= 2.6\times10^3$ N/mm^3.

To find:

Additional volume of oil pumped into the cylinder =?

Solution:

Due to internal pressure of fluid inside the cylinder, there will be a change in the dimensions of the cylinder. Due to this, there will be a increase in the volume of the cylinder. Let us first calculate the increase in volume of the cylinder.

Let

δV_1=Increase in volume of cylinder

Then volumetric strain $= \delta V_1/V$

But Volumetric strain due to fluid pressure is given by equation

$$\delta V_1/V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)$$

$$\delta V = \frac{pd}{2tE}\left(\frac{5}{2} - 2\mu\right)V$$

$$= \frac{5\times388}{2\times6\times1.0\times10^5}\left(\frac{5}{2} - 2\times\frac{1}{3}\right)\times[1.06413\times10^8]$$

$$= 3.15\times10^3 \text{ mm}^3$$

As bulk modulus of oil is gives, then due to increase of fluid pressure on the oil, the original volume of oil will decrease. Let us to find this decrease in volume of the oil.

Let

δV_2 = Decreases in volume of oil due to increase of pressure

Bulk modulus is given as

$$k = \frac{\text{Increase in pressure of oil}}{\left(\frac{\text{Increase in pressure of oil}}{\text{original volume of oil}}\right)}$$

$$= \frac{p}{\left(\frac{\delta V2}{V}\right)}$$

$$\frac{\delta V_2}{V} = \frac{P}{K}$$

$$\delta V_2 = \frac{P}{K} \times V$$

$$= \frac{5}{2.6\times10^3} \times 1.06413\times10^8$$

$$= 204.64 \times 10^3 \text{ mm}^3$$

Resultant additional space created in the cylinder

= Increase in volume of cylinder + Decrease in volume of oil

$= \delta V_1 + \delta V_2$

$= 314.98 \times 10^3 + 204.64\times10^3$

$= 519.62 \times 10^3 \text{mm}^3$

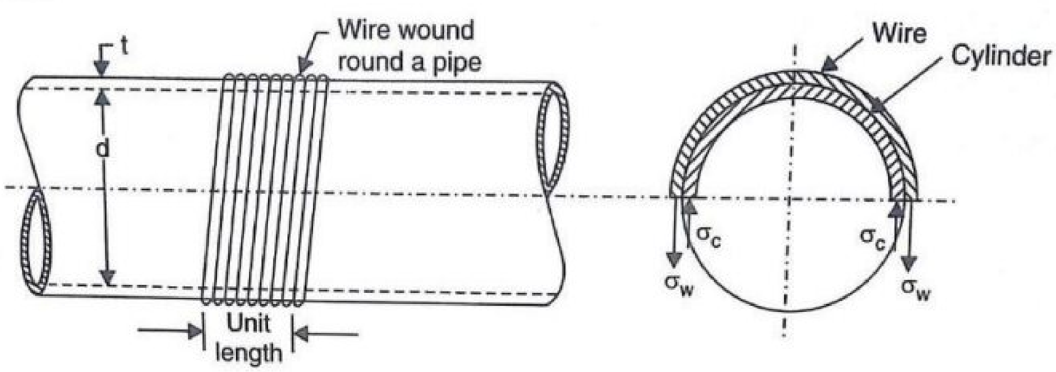
Additional quantity of oil which must be pumped in order to raise the oil pressure to 5 N/mm^2 = 519.62 $\times 10^3$mm^3

Result:

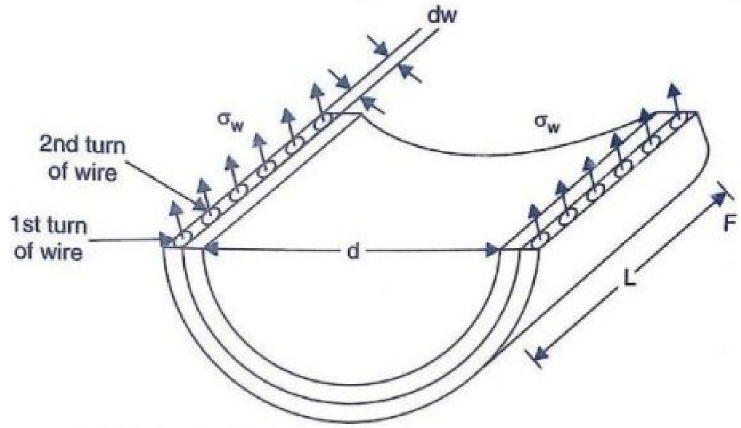
Additional volume of oil pumped into the cylinder = **519.62 $\times$10^{3}mm^3**

5.9. WIRE WINDING OF THIN CYLINDER

We have seen in previous articles that hoop stress is two times the longitudinal stress in a thin cylinder, when the cylinder is subjected to internal fluid pressure. Hence the failure of a thin cylinder will be due to hoop stress. Also the hoop stress which is tensile in nature is directly proportional to the fluid pressure inside the cylinder. Hence the maximum fluid pressure inside the cylinder is limited corresponding to the condition that the hoop stress reached the permissible value. In case of cylinders which have to carry high internal fluid pressures, some methods of reducing the hoop stresses have to be devised.



One method is to wind strong steel wire under tension on the walls of the cylinder will be subjected to hoop tensile stress. The net effect of the initial compressive stress due to wire winding and those due to internal fluid pressure is to make resultant stress less. The resultant stress in the material of the cylinder will be the hoop stress due to internal fluid pressure minus the initial compressive stress. Whereas the stress in the wire will be equal to the sum of the tensile stress due to internal pressure in the cylinder and initial tensile winding stress.



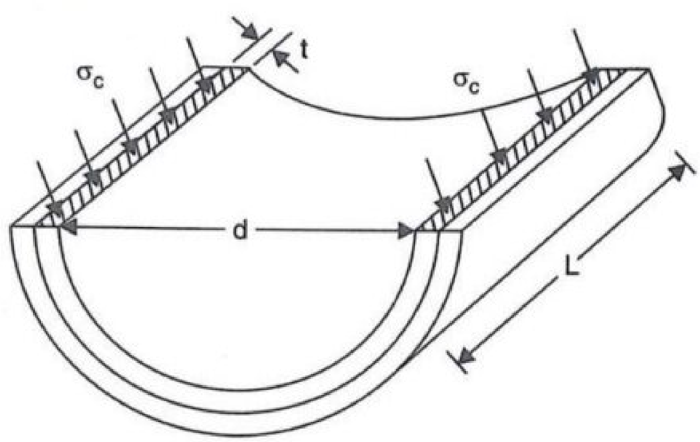
IF σ_w =Initial winding stress in wire

Initial tensile force in wire for length L = n $\times \left(2 \times \frac{\pi}{4} \times d_w^2\right) \times \sigma_w$

Where n = number of turns of wire in length L

d_w = Diameter of wire

Then n = $\frac{L}{d_w} \times \left(2 \times \frac{\pi}{4} \times d_w^2\right) \times \sigma_w$



σ_c = Compressive circumferential stress exerted by wire on cylinder

$= L \times \frac{\pi}{2} \times d_w \times \sigma_w$

Compressive force exerted by wire on cylinder for length L=2×L×t×σ_c

For equilibrium

Initial tensile force in wire=Compressive force on cylinder

$$L\times\frac{\pi}{2}\times d_w\times\sigma_w = 2\times L\times t\times\sigma_c$$

or
$$\sigma_c = \frac{\pi\times d_w}{4t}\times\sigma_w$$

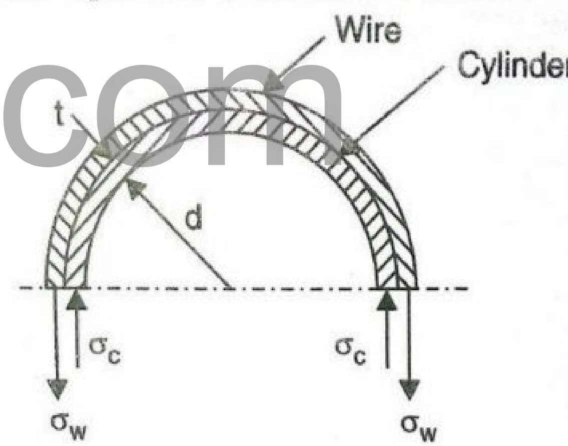
σ_c^* = Circumferential stress developed in the cylinder due to fluid pressure only

σ_w^* = Stress developed in the wire due to fluid pressure only

Then Resultant stress in the cylinder $= (\sigma_c^* - \sigma_c)$

The resultant stress in wire $= (\sigma_w + \sigma_w^*)$

Problem 5.18: A cast iron pipe of 200mm internal diameter and 12mm thick is wound closely with a single layer of circular steel wire of 5mm diameter under a tension of 60 N/mm^2. Find the initial compressive stress in the pipe section. Also find the stresses set up in the pipe and steel wire, when water under a pressure of 3.5N/mm^2. Poisons ratio=0.3



Given data:

Internal dia of pipe d = 200mm

pipe thickness t = 12mm

Diameter of wire d_w = 5mm

Tension in wire = 60N/mm^2

Water pressure p = 3.5N/mm^2

E for C.I., E_C = 1×105 N/mm^2

E for steel E_S = 2×105 N/mm^2

μ = 0.3

To find:

Initial compressive stress due to wire winding =?

Resultant stress =?

Solution:

(i) Before the fluid under pressure is admitted in the cylinder

σ_w =60 N/mm^2

Consider 1cm length of pipe Number of turns of the wire of 1cm pipe length

$$= \frac{\text{Lengthofpipe}}{\text{Diaofwire}} = \frac{1}{0.5} = 2$$

The compressive force exerted by one turn of the wire on the cylinder

$$= 2 \times \text{Area of cross section of wire} \times \sigma_w$$

$$= 2 \times \frac{\pi}{4} \times 5^2 \times 60 \text{ N}$$

Total compressive force exerted by the wire on the cylinder per cm length of the pipe

= No.of turns × Force exerted by one turn

$$= 2 \times (2 \times \frac{\pi}{4} \times 5^2 \times 60)$$

= 4712N

Sectional area of the cylinder which takes this compressive force

$$= 2 \times l \times t$$

$$= 2 \times 10 \times 12 \text{ mm}^2$$

Here l=1cm=10mm and t=12mm

Initial compressive stress in the material of the cylinder due to wire windings

σ_c = Total compressive force on the cylinder/sectional area of cylinder

$$= \frac{4712}{2} \times 10 \times 12$$

=19.63 N/mm²

(ii) Stresses due to fluid pressure alone

Let σc* = stresses in the pipe due to fluid pressure 3.5 N/mm²

σw*= Stresses in the wire due to pressure 3.5 N/mm²

The force of fluid which tends to burst the cylinder along longitudinal section

$$= p.d.l = 3.5 \times 200 \times 10$$

=7000N ....(i)

Resisting force of cylinder

= Stresses in the cylinder × Area of cylinder resisting

$$= \sigma c^* \times 2l \times t$$

$$= \sigma c^* \times 2 \times 10 \times 12$$

$$= 240\sigma c^*$$

Resisting force of wire = No of turns × $(2 \times \frac{\pi}{4} \times 5^2)$ × Stress in wire due to fluid pressure

$= 2 \times (2 \times \frac{\pi}{4} \times 5^2) \times \sigma w^*$

$= 78.54\sigma w^*$

Total resisting force

$=240\sigma c^* + 78.5\sigma w^*$ ………………..(ii)

Equating the resisting force (i) and (ii)

$=240\sigma c^*+78.54\sigma w^*=7000$ ………………..(iii)

But circumferential strain in cylinder

$= \frac{\text{Circumferential stress}}{E} - \frac{\text{longitudinal stress}}{E} \times \mu$

$= \frac{\sigma_c^*}{Ec} - \frac{(pd/4t)}{Ec} \times \mu$

$= \frac{1}{Ec}(\sigma c^* - \left(\frac{3.5\times200}{4\times12}\right) \times 0.3)$

$= \frac{1}{Ec}(\sigma c^* - 4.375)$ ………………..(iv)

strain in wire $= \frac{\sigma_w^*}{Es}$ ………………..(v)

Equating eqn (iv) and (v) we get

$\frac{1}{Ec}(\sigma c^* - 4.375) = \frac{\sigma_w^*}{Es}$

$\sigma w^* = \frac{Es}{Ec}(\sigma c^* - 4.375)$

$= \frac{2\times10^5}{1\times10^5}(\sigma c^* - 4.375)$

$= 2(\sigma c^* - 4.375)$ ……………..(vi)

substitute the above value in equation (iii) we get

$240\sigma c^*+78.54\times[2(\sigma c^*-4.375)]=7000$

$397.08\sigma c^*=7867.225$

$\sigma c^*=19.36 N/mm^2$

Substitute the value of σc* in equation (vi) we get

$\sigma w^* = 2(19.36-4.375)$

$= 29.97 N/mm^2$

(iii) Resultant stresses in pipe and wire

Resultant stress in pipe

= Initial stress in pipe + Stress due to fluid pressure alone

=19.63(compressive) + 19.36(tensile)

= **0.27 N/mm² (compressive)**

Resultant stress in wire

= Initial stress in wire + stresses due to fluid pressure alone

= 60(tensile) + 29.97 (tensile)

= **89.97N/mm² (tensile)**

Result:

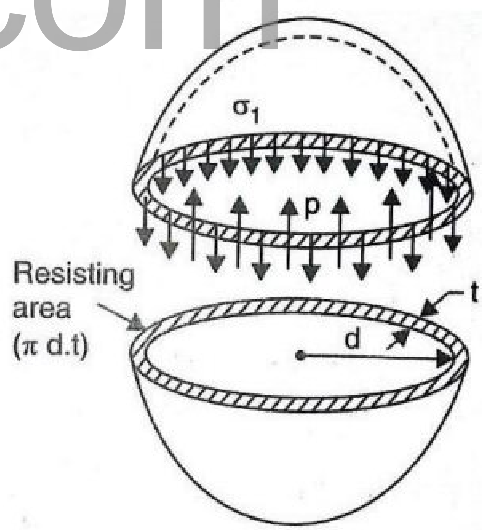
Initial compressive stress due to wire winding = **19.63 N/mm²**

Resultant stress = **0.27 N/mm² (compressive)**

= **89.97N/mm² (tensile)**

5.10. THIN SPHERICAL SHELLS

A thin spherical shell of internal diameter d and thickness t and subjected to an internal fluid pressure p. The fluid inside the shell has a tendency to split the shell into two hemisphere along x axis



The force P which has a tendency to split the shell

$$= p \times \frac{\pi}{4} \times d^2$$

The area resisting this force=π.d.L

∴ Hoop or circumferential stress induced in the material of the shell is given by

$$\sigma_1 = \frac{\text{Force P}}{\text{Area resisting the force P}}$$

$$= \frac{p \times \frac{\pi}{4} \times d^2}{\pi.d.t} = \frac{pd}{4t}$$

The stress σ_1 is tensile in nature The fluid inside the shell is also having tendency to split the shell into two hemispheres along y-y axis. Then it can be shown that the tensile hoop stress will also be equal to $\frac{pd}{4t}$. Let the stress is σ_2

∴ $\sigma_2 = \frac{pd}{4t}$ The stress σ_2 will be right angles to σ_1

Problem 5.19: A vessel in the shape of a spherical shell of 1.20m internal diameter and 12mm shell thickness is subjected to pressure of 1.6N/mm2. Determine the stress induced in the material of the shell.

Given data:

Internal diameter $d = 1.2m = 1.2\times10^3$ mm

shell thickness $t = 12mm$

Fluid pressure $p = 1.6\ N/mm^2$

To find:

Stress induced in the shell (σ_1)=?

Solution:

The stress induced in the material of spherical shell is given by

$$\sigma_1 = \frac{pd}{4t} = \frac{1.6\times1.2\times10^3}{4\times12} = 40\ N/mm^2$$

Result:

Stress induced in the shell (σ_1)= 40 N/mm^2

Problem 5.20: A spherical vessel 1.5m diameter is subjected to an internal pressure of $2N/mm^2$, find the thickness of the plate required if maximum stress is not to exceed $150N/mm^2$ and joint efficiency is 75%

Given data:

Diameter of the shell $d = 1.5m = 1.5\times10^3$ mm

Fluid pressure $p = 2\ N/mm^2$

Stress in material $\sigma_1 = 150\ N/mm^2$

Joint efficiency $\eta = 75\% = 0.75$

To find:

The thickness of the plate =?

Solution:

The stress induced is given by

$$\sigma_1 = \frac{pd}{4t}$$

>>> $$t = \frac{pd}{4\times\sigma_1} = \frac{2\times1.5\times10^3}{4\times150} = 6.67\ mm$$

Result: The thickness of the plate = 6.67mm

5.11 CHANGE IN DIMENSIONS OF A THIN SPHERICAL SHELL DUE TO AN INTERNAL PRESSURE

In previous article, we have seen that the stresses at any point are equal to $\frac{pd}{4t}$ to like. There is no shear stress at any point in the shell.

Maximum shear stress = $\frac{\sigma_1 - \sigma_2}{2} = \frac{\frac{pd}{4t} - \frac{pd}{4t}}{2} = 0$

These stresses σ_1 and σ_2 are acting at right angles to each other.

∴ Strain in any one direction is given by

$$e_1 = \frac{\sigma_1}{E} - \mu\frac{\sigma_1}{E}$$

$$= \frac{\sigma_1}{E}(1-\mu)$$

$$= \frac{pd}{4tE}(1-\mu)$$

We know that strain in any direction is also $= \frac{\delta d}{d}$

∴ $\frac{\delta d}{d} = \frac{pd}{4tE}(1-\mu)$

Then change in diameter $\delta d = \frac{pd}{4tE}(1-\mu)d$

Volumetric strain

The ratio of change of volume to the original volume is known as volumetric strain. If v=original volume and dv=change in volume. Then volumetric strain = $\frac{dV}{V}$

Let V=Original volume $= \frac{\pi}{6} \times d^3$

Taking the differential of the above equations we get

$$dV = \frac{\pi}{6} \times 3d^2 \times d(d)$$

$$\frac{dV}{V} = \frac{\frac{\pi}{6} \times 3d^2 \times d(d)}{\frac{\pi}{6} \times d\Lambda 3}$$

$$= 3 \times \frac{d(d)}{d}$$

But from change in diameter equation we have

$$\frac{\delta d}{d} \text{ or } \frac{d(d)}{d} = \frac{pd}{4tE}(1-\mu)$$

Substituting this value in $\frac{dv}{v}$ we get,

$$\frac{dV}{V} = \frac{3pd}{4tE}(1-\mu)$$

Problem 5.21: A spherical shell of internal diameter 0.9m and of thickness 10mm is subjected to an internal pressure of 1.4 N/mm^2. Determine the increases in diameter and increases in volume. Take E=2×10^5 N/mm^2 and μ=1/3

Given data:

Internal diameter	$d = 0.9m = 0.9\times10^3 mm$
Thickness of shell	$t = 10mm$
Fluid pressure	$p = 1.4\ N/mm^2$

To find:

Increases in diameter and increases in volume dd, dV =?

Solution:

wkt $\delta d = \frac{pd}{4tE}(1-\mu)d$

$$= \frac{1.4\times(0.9\times10^3)}{4\times10\times2\times10^5}\left[1-\frac{1}{3}\right]\times 0.9\times10^3$$

$$= 0.0945mm$$

Now volumetric strain

$$\frac{dV}{V} = \frac{3pd}{4tE}(1-\mu)$$

$$dV = \frac{3pd}{4tE}(1-\mu)V \qquad \left(\because V = \frac{\pi}{6}\times d^3\right)$$

$$= \frac{3\times1.4\times(0.9\times10^3)}{4\times10\times2\times10^5}\left(1-\frac{1}{3}\right)\times\frac{\pi}{6}\times(0.9\times10^3)^3$$

=12028.5mm3

Result:

Increases in diameter dd = 0.0945mm

Increases in volume dV = 12028.5mm^3

5.12 ROTATIONAL STRESSES IN THIN CYLINDER

Thin cylinder rotating at an angular velocity about the axis

Let r=mean radius of the cylinder

t=thickness of the cylinder

ω=angular speed of the cylinder

ρ=density of the material of the cylinder

Due to rotational of cylinder centrifugal force will be acting on the walls of the cylinder. This centrifugal force will produce a circumferential stress σ. For a thin cylinder, this hoop stresses σ may be assumed constant.

Consider a small element ABCD of the rotating cylinder. Let this element makes an angle δθ at the centre. Consider unit length of this element perpendicular to the plane of paper

The forces acting on the element are;

(i)Centrifugal force (mv^2/r or $m\omega^2 r$) acting radially outwards. Here m is the mass of the element per unit length.

m = mass of element

= ρ ×volume of element

= ρ ×(area of element)×unit length

= ρ ×[(r×δθ)×t]×1

= ρ ×r×δθ×t

∴ Centifigual force = $m\omega^2 r$

$= (\rho\, r\delta\theta t)\omega^2 r$

$= \rho\, r2 \times \omega 2 \times \delta\theta \times t$

(ii)Tensile force due to hoop stress (σ) on the face AB. This force is equal to (σ×t×1) and acts as perpendicular to face AB

(iii)Tensile force due to hoop stress (σ) on the face CD. This force is equal t0 (σ×t×1) and acts perpendicular to face CD.

The horizontal component $\sigma \times t \times \cos\frac{\delta\theta}{2}$ on the face AB and CD are equal and opposite. The radial component $\sigma \times t \times \sin\frac{\delta\theta}{2}$ are acting towards centre and they will be added.

Resolving the forces radially for equilibrium, we get

Centrifugal force $= \sigma t \times \sin\frac{\delta\theta}{2} + \sigma t \times \sin\frac{\delta\theta}{2}$

or $$\rho\, r^2 \times \omega^2 \times \delta\theta \times t = 2 \times \sigma \times t \times \sin\frac{\delta\theta}{2}$$

$$= 2 \times \sigma \times t \times \delta\theta/2$$

or $$\rho\, r^2 \times \omega^2 = \sigma$$

or $$\sigma = \rho\, r^2 \times \omega^2$$

Problem 5.22: A rim type flywheel is rotating at a speed of 2400 rpm If the mean diameter of the flywheel is 750mm and density of the material of the wheel is 8000kg/m^3, then find the hoop stress produced in the rim due to rotation. If E=200GN/m^2 then what will be the changes in diameter of the flywheel due to rotation

Given data:

Speed $N = 2400$rpm

Mean diameter $d = 750$ mm $= 0.75$ m

Density $\rho = 8000$kg/m^3

$E = 200$GN/m$^2 = 200 \times 10^9$ N/m^2

To find:

Hoop stress produced in the rim due to rotation =?

Solution:

Wkt, angular speed $\omega = 2\pi N/60 = 2 \times \pi \times 2400/60$

$\omega = 80\pi$ rad/sec

$r = 750/2 = 0.375$m

Using hoop stress produced in the rim due to rotation equation we get,

$$\sigma = \rho \times r^2 \times \omega^2$$

$$= 8000 \times 0.3752 \times (80\pi)^2$$

$$= 71.0485 \times 10^6 \text{ N/m}^2$$

$$= 71.0485 \text{MN/m}^2$$

Change in diameter calculation

Due to hoop stress circumferential strain

$$e = \frac{\sigma}{E} \quad \text{and} \quad e = \frac{\delta d}{d}$$

equating the above equation, we get

$$\frac{\delta d}{d} = \frac{\sigma}{E}$$

>>> Change in diameter $\delta d = \frac{\sigma \times d}{E}$

$$\delta d = \frac{71.0485 \times 106 \ \times 0.75}{200 \times 10^9}$$

$$= 0.002664\text{m}$$

$$= 0.2664\text{mm}$$

Result:

Hoop stress produced in the rim due to rotation = **71.0485MN/m^2**

Change in diameter δd = **0.2664mm**

Problem 5.23: Find the speed of rotation of a wheel of diameter 750mm if the hoop stress is not to exceed 120MN/m^2. The wheel has a thin rim and density of the wheel is 7200kg/m^3

Given data:

Diameter $d = 750\text{mm}$

$r = 750/2 = 375\text{mm} = 0.375\text{m}$

Max. hoop stress $\sigma = 120\text{MN/m}^2 = 120 \times 10^6 \text{ N/m}^2$

$\rho = 7200\text{kg/m}^3$

To find:

Speed of rotation of a wheel (N) =?

Solution:

Using hoop stress produced in the rim due to rotation equation we get,

$$\sigma = \rho \times r^2 \times \omega^2$$

$$120 \times 10^6 = 7200 \times 0.375^2 \times \omega^2$$

$$\omega = \sqrt{\frac{120 \times 10^6}{7200 \times 0.375^2}}$$

$$= 344.26 \text{ rad/s}$$

But $\omega = 2\pi N/60$

$$N = 60 \times 344.26/2\pi$$

$$N = 3287.4 \text{ rpm}$$

Result:

Speed of rotation of a wheel (N) = **3287.4 rpm**

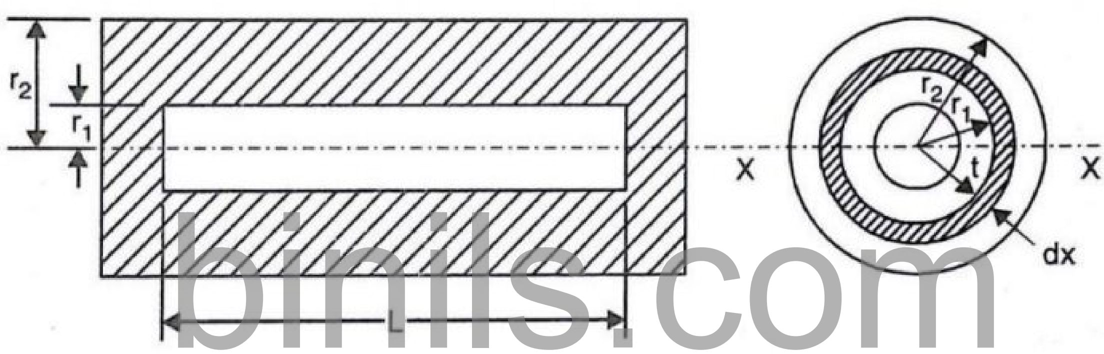
5.13. INTRODUCTION OF THICK CYLINDER

In the last chapter, we have mentioned that if the ratio of thickness to internal diameter of a cylindrical shell is less than about 1/20,the cylinder shell is know as thin cylinders. For them it may be assumed with reasonable accuracy that the hoop and longitudinal stresses are constant over the thickness and the radial stress is small and can be neglected. If the ratio of thickness to internal diameter is more than 1/20,then cylinder shell is know as thick cylinders.

The hoop stress in case of a thick cylinder will not be uniform across the thickness. Actually the hoop stress will vary from a maximum value at the inner circumference to a minimum value at the outer circumference.

5.14. STRESSES IN A THICK CYLINDERICAL SHELL

Fig. shows a thick cylinder subjected to an internal fluid pressure.



Let r_2 = External radius of the cylinder,

r_1 = Internal radius of the cylinder, and

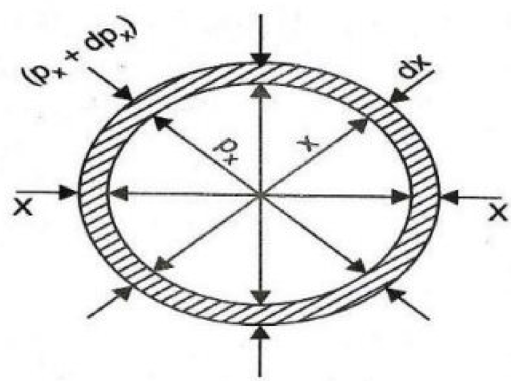
L = Length of cylinder.

Consider an elementary ring of the cylinder of radius x and thickness dx as shown in the figure

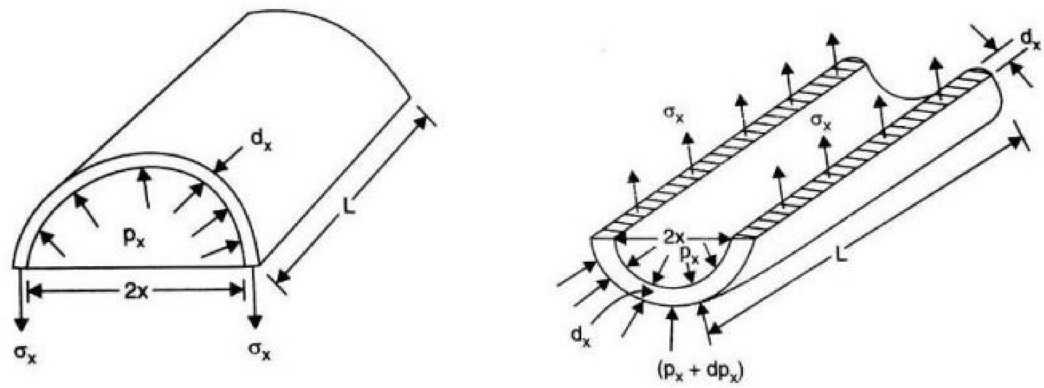
Let p_x = Radial pressure on the inner surface of the ring

$p_x + dp_x$ = Radial pressure on the outer surface of the ring

σ_x = Hoop stress induced in the ring.



Take a longitudinal section x-x and consider the equilibrium of half of the ring of figure.



Bursting force

$=p_x$ (2xL) – (p_x+ dp_x) × 2(x + dx). L

= 2L [p_x. x – (p_x.x + p_x.dx + x . dp_x + dp_x. dx)]

=2L [- p_x.dx - x . dp_x]

= - 2L (p_x.dx + x . dp_x) …. (1)

Resisting force = Hoop stress × Area on which it acts = σ_x × 2dx.L …. (2)

Equating the resisting force to the bursting force, we get

σ_x × 2dx.L = - 2L (p_x.dx + x . dp_x)

Or $\sigma_x = -p_x - x\,\frac{dp_x}{dx}$ …. (3)

The longitudinal strain at any point in the section is constant and is independent of the radius. This mean that cross- sections remain plane after straining and this is true for sections, remote from any end fixing. As longitudinal strain is constant, hence longitudinal stress will also be constant.

Let σ_2 = Longitudinal stress.

Hence at any point at a distance x from the centre, three principle stresses are acting :

They are :

1. the radial compressive stress, p_x
2. the hoop (or circumferential) tensile stress, σ_x
3. the longitudinal tensile strain σ_2.

The longitudinal strain (e_2) at this point is given by,

$$e_2 = \frac{\sigma_2}{E} - \frac{\mu\sigma_x}{E} + \frac{\mu p_x}{E} = \text{constant}$$

But longitudinal strain is constant

$$\frac{\sigma_2}{E} - \frac{\mu\sigma_x}{E} + \frac{\mu p_x}{E} = \text{constant}$$

But σ_2 is also constant, and for the material of the cylinder E and μ are constant.

$$\sigma_x - p_x = \text{constant}$$

$$= 2a \text{ where a is constant}$$

$$\sigma_x = p_x + 2a \qquad \text{.... (4)}$$

Equating the two values of σ_x given by equation (3) and (4), we get

$$p_x + 2a = -p_x - x\,\frac{dp_x}{dx}$$

$$x\,\frac{dp_x}{dx} = -p_x - p_x - 2a = -2p_x - 2a$$

$$\frac{dp_x}{dx} = -\frac{2p_x}{x} - \frac{2a}{x} = \frac{-2(p_x+a)}{x}$$

$$\frac{dp_x}{(p_x+a)} = -\frac{2d_x}{x}$$

Integrating the above equation, we get

$$\log_e(p_x + a) = -\log_e x^2 + \log_e b$$

$$= \log_e \frac{b}{x^2}$$

$$p_x + a = \frac{b}{x^2}$$

$$p_x = \frac{b}{x^2} - a \qquad \text{.... (5.1)}$$

Substituting the values of p_x in equation (4), we get

$$\sigma_x = \frac{b}{x^2} - a + 2a = \frac{b}{x^2} + a \qquad \text{.... (5.2)}$$

Equation (5.1) gives the radial pressure p_x and equation (5.2) gives the hoop stress at any radius x. These two equations are called Lame's equations. The constants 'a' and 'b' are obtained from boundary conditions, which are :

i. at x = r₁, $p_x = p_0$ or the pressure of fluid inside the cylinder, and
ii. at x = r₂, $p_x = 0$ or atmosphere pressure.

After knowing the value of 'a' and 'b' , the hoop stress can be calculated at any radius

Problem 5.24: Determine the maximum and minimum hoop stress across the section of a pipe of 400 mm internal diameter and 100 mm thick, when the pipe contains a fluid at a pressure of 8 N/mm^2. Also sketch the radial pressure distribution and hoop stress distribution across the section.

Sol. Given:

Internal dia, = 400 mm

Internal radius, $r_1 = \frac{400}{2} = 200$ mm

Thickness, = 100 mm

External dia, = 400 +2 × 100 = 600 mm

External radius, $r_2 = \frac{600}{2} = 300$ mm

Fluid pressure, $p_0 = 8$ N/mm^2

or at $x = r_1$, $p_x = p_0 = 8$ N/mm^2

The radial pressure (p_x) is given by equation (18.1) as

$$p_x = \frac{b}{x^2} - a \quad \text{.... (1)}$$

Now apply the boundary conditions to the above equation. The boundary conditions are : At $x = r_1 = 200$ mm , $p_x = 8$ N/mm^2

1) At $x = r_2 = 300$ mm , $p_x = 0$

Substituting these boundary conditions in equation (1), we get

$$8 = \frac{b}{200^2} - a = \frac{b}{40000} - a \quad \text{.... (2)}$$

$$0 = \frac{b}{300^2} - a = \frac{b}{90000} - a \quad \text{.... (3)}$$

and

Subtracting equation (3) from equation (2), we get

$$8 = \frac{b}{40000} - \frac{b}{90000} = \frac{9b-4b}{360000} = \frac{5b}{360000}$$

$$b = \frac{360000 \times 8}{5} = 576000$$

Substituting this value in equation (3), we get

$$0 = \frac{576000}{90000} - a \text{ or } a = \frac{576000}{9000} = 6.4$$

The values of 'a' and 'b' are substituted in the hoop stress.

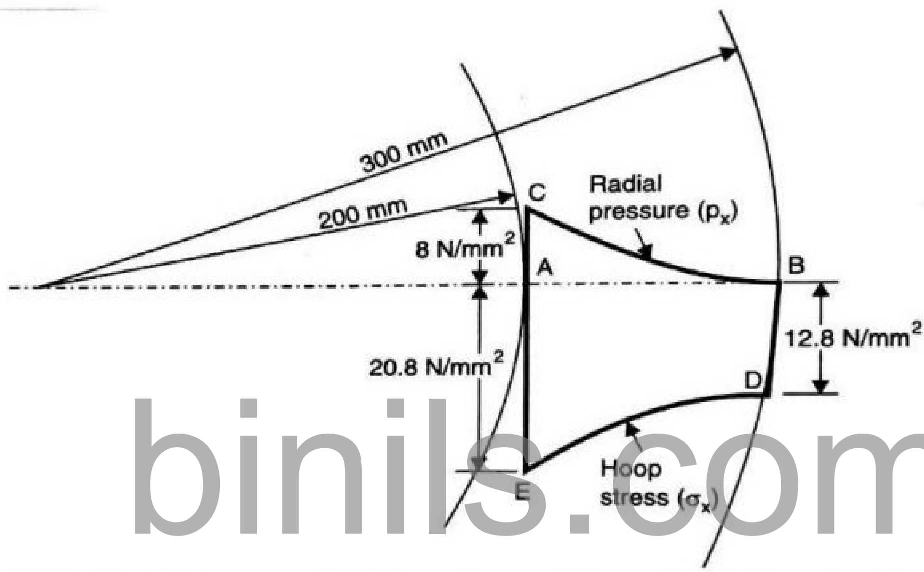
Now hoop stress at any radius x is given by equation as

$$\sigma_x = \frac{b}{x^2} + a = \frac{576000}{x^2} + 6.4$$

At x = 200 mm, $\sigma_{200} = \frac{576000}{200^2} + 6.4 = 14.4 + 6.4 =$ **20.8 N/mm².Ans.**

At x = 300 mm, $\sigma_{300} = \frac{576000}{300^2} + 6.4 = 6.4 + 6.4 =$ **12.8 N/mm².Ans.**

Figure shows the radial pressure distribution and hoop stress distribution across the section. AB is taken a horizontal line. AC = 8 N/mm² . The variation between B and



C is parabolic. The curve BC shows the variation of radial pressure across AB.

The curve DE which is also parabolic, shows the variation of hoop stress across AB. Values BD = 12.8 N/mm². The radial pressure is compressive whereas the hoop stress is tensile.

Problem 5.25: Find the thickness of metal necessary for a cylindrical shell of internal diameter 160 mm to withstand an internal pressure of 8 N/mm². The maximum hoop stress in the section is not to exceed 35 N/mm².

Sol. Given :

Internal dia, = 160 mm

Internal radius, $r_1 = \frac{160}{2} = 80$ mm

Internal pressure, = 8 N/mm²

This means at x =80 mm, p_x= 8 N/mm²

Maximum hoop stress, $\sigma_x = 35$ N/mm^2

The maximum hoop stress is at the inner radius of the shell.

Let r_2 = External radius.

The radial pressure and hoop stress at any radius x are given by equation (18.1) and (18.2) as

$$p_x = \frac{b}{x^2} - a \quad \text{.... (1)}$$

and

$$\sigma_x = \frac{b}{x^2} + a \quad \text{.... (2)}$$

Let us now apply the boundary conditions. The boundary conditions are :

At x = 80 mm, $p_x = 8$ N/mm^2 and $\sigma_x = 35$ N/mm^2

Substituting x = 80 mm and $p_x = 8$ N/mm^2 in equation (1),we get

$$8 = \frac{b}{80^2} - a = \frac{b}{6400} - a \quad \text{.... (3)}$$

Substituting x = 80 mm and $\sigma_x = 35$ N/mm^2 in equation (2),we get

$$35 = \frac{b}{80^2} + a = \frac{b}{6400} + a \quad \text{.... (4)}$$

Subtracting equation (3) from equation (4),we get

$$27 = 2a \quad \text{or} \quad a = \frac{27}{2} = 13.5$$

Substituting the value of a in equation (3),we get

$$8 = \frac{b}{6400} - 13.5$$

$$b = (8 + 13.5) \times 6400 = 21.5 \times 6400$$

Substituting the values of 'a' and 'b' in equation (1),

$$p_x = \frac{21.5 \times 6400}{x^2} - 13.5$$

But at the outer surface, the pressure is zero. Hence at x = r_2, $p_x = 0$.Substituting these values in the above equation , we get

$$0 = \frac{21.5 \times 6400}{r_2^2} - 13.5$$

$$r_2^2 = \frac{21.5 \times 6400}{13.5} \quad \text{or} \quad r_2 = \sqrt{\frac{21.5 \times 6400}{13.5}} = 100.96 \text{ mm}$$

Thickness of the shell, t = $r_2 - r_1$

= 100.96 – 80 = **20.96 mm.Ans.**

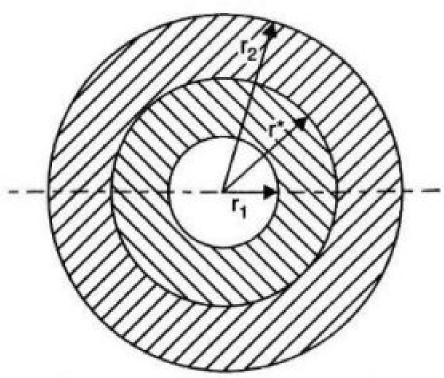
5.15. STRESSES IN COMPOUND THICK CYLINDERS

From the problem, we find that the hoop stress is maximum at the inner radius and it decreases towards the outer radius. The hoop stress is tensile in nature and it is caused by the internal fluid pressure inside the cylinder. This maximum hoop stress at the inner radius is always greater than the internal fluid pressure. Hence the maximum fluid pressure inside the cylinder is limited corresponding to the condition that the hoop stress at the inner radius reaches the permissible value. In case of cylinders which have to carry high internal fluid pressure, some methods of reducing the hoop stress have to be devised.

One method is to wind* strong steel wire under tension on the cylinder. The effect of the wire is to put the cylinder wall under an initial compressive stress.

Another method is to shrink one cylinder over the other. Due to this , the inner cylinder will be put into initial compression whereas the outer cylinder will be put into initial tension. If now the compound cylinder is subjected to internal fluid pressure, both the inner and outer cylinders will be subjected to hoop tensile stress. The net effect of the Initial stresses due to shrinking and those due to internal fluid pressure is to make the resultant stresses more or less uniform.

Figure shows a compound thick cylinder made up of two cylinders.



Let r_2 = Outer radius of compound cylinder

r_1 = Inner radius of compound cylinder

r^*= Radius at the junction of the two cylinders

(i.e.., outer radius of inner cylinder or inner radius of outer

cylinder)

p^* = Radial pressure at the junction of the two cylinders.

Let us now apply Lame's equation for the initial conditions (i.e., after shrinking the outer cylinderover the inner cylinder and fluid under pressure is not admitted into the inner cylinder).

1) For outer cylinder

The Lame's equation at a radius x for outer cylinder are given by

$$p_x = \frac{b_1}{x^2} - a_1 \quad \text{.... (1)} \qquad \sigma_x = \frac{b_1}{x^2} + a_1 \text{.... (2)}$$

where a_1, b_1 are constants for outer cylinder.

At $x = r_2$, $p_x = 0$. And at $x = r^*$, $p_x = p*$

Substituting these conditions in equation (1), we get

$$0 = \frac{b_1}{r_2^2} - a_1 \quad \text{.... (3)} \qquad p^* = \frac{b_1}{r*^2} + a_1 \text{ .... (4)}$$

From equation (3) and (4), the constants a_1 and b_1 can be determined. These values are substituted in equation (2). And then hoop stresses in the outer cylinder due to shrinking can be obtained.

2) For inner cylinder

The Lame's equations for inner cylinder at a radius x are given by

$$p_x = \frac{b_2}{x^2} - a_2 \;, \sigma_x = \frac{b_2}{x^2} + a_2$$

Where a_2, b_2 are constants for inner cylinder.

At $x = r_1$, $p_x = 0$ as fluid under pressure is not admitted into the inner cylinder. And at $x = r^*$, $p_x = p*$.

Substituting these values in the above value of p_x, we get

$$0 = \frac{b_2}{x^2} - a_2 \quad \text{.... (5)} \quad \text{and} \quad p^* = \frac{b_2}{r*^2} + a_2 \quad \text{.... (6)}$$

From equation (5) and (6) , the constants a_2 and b_2 can be determined. These values are substituted in σ_x. And then hoop stresses are obtained.

Hoop stresses in compound cylinder due to internal fluid pressure alone

When the fluid under pressure is admitted into the compound cylinder, the hoop stresses are set in the compound cylinder. To find these stresses, the inner cylinder and outer cylinder will together be considered as one thick shell, Let p = internal fluid pressure. Now the Lame's equations are applied, which are given by

$$p_x = \frac{B}{x^2} - A \quad \text{.... (7) and} \quad \sigma_x = \frac{B}{x^2} + A \quad \text{.... (8)}$$

where A and B are constants for single thick shell due to internal fluid pressure.

At $x = r_2$, $p_x = 0$.

Substituting these values in equation (7), we get

$$0 = \frac{B}{r_2^2} - A \qquad \text{.... (9)}$$

At $x = r_1, p_x = p$.

Substituting these values in equation (7), we get

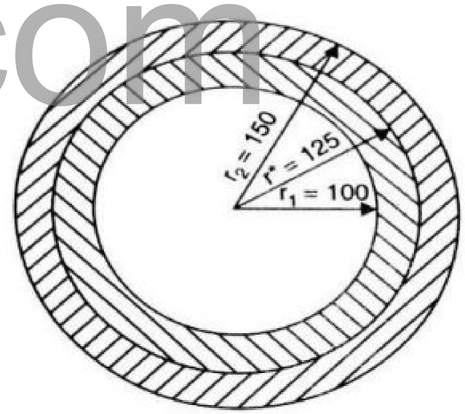
$$p = \frac{B}{r_1^2} - A \qquad \text{.... (10)}$$

From equation (9) and (10), the constants A and B can be determined. These values are substituted in equation (8). And then hoop stresses across the section can be obtained.

The resultant hoop stresses will be the algebraic sum of the hoop stresses caused due to shrinking and those due to internal fluid pressure.

Problem 5.26: A compound cylinder is made by shrinking a cylinder of external diameter 300 mm and internal diameter of 250 mm over another cylinder of external diameter 250 mm and internal diameter 200 mm. The radial pressure at the junction after shrinking is 8 N/mm². Find the final stresses set up across the section, when the compound cylinder is subjected to an internal fluid pressure of 84.5 N/mm².

Sol. Given :



For outer cylinder :

External diameter = 300 mm

External radius , $r_2 = \frac{300}{2} = 150$ mm

Internal diameter, = 250 mm

Radius at the junction, $r^* = \frac{250}{2} = 125$ mm

For inner cylinder :

Internal diameter, = 200 mm

Internal radius, $r_1 = \frac{200}{2} = 100$ mm

Radial pressure due to shrinking at the junction,

$$P^* = 8 \text{ N/mm}^2$$

Fluid pressure in the compound cylinder, $p = 84.5$ N/mm².

i. Stresses due to shrinking in the outer and inner cylinders before the fluid pressure is admitted.

(a) Lame's equations for outer cylinders are :

$$p_x = \frac{b_1}{x^2} - a_1 \quad \text{.... (1)} \quad \text{and} \quad \sigma_x = \frac{b_1}{x^2} + a_1 \quad \text{.... (2)}$$

At x = 150 mm, $p_x = 0$.

Substituting these values in equation (1),

$$0 = \frac{b_1}{150^2} - a_1 = \frac{b_1}{22500} - a_1 \quad \text{.... (3)}$$

At x = r* = 125 mm, $p_x = p* = 8$ N/mm².

Substituting these values in equation (1), we get

$$8 = \frac{b_1}{125^2} - a_1 = \frac{b_1}{15625} - a_1 \quad \text{.... (4)}$$

Subtracting equation (3) from equation (4), we get

$$8 = -\frac{b_1}{22500} + \frac{b_1}{15625} = \frac{(-15625+22500)b_1}{22500 \times 15625}$$

$$b_1 = \frac{8 \times 22500 \times 15625}{(-15625 + 22500)} = 409090.9$$

Substituting the value of b_1 in equation (3), we get

$$0 = \frac{409090.9}{22500} - a_1 \quad \text{or} \quad a_1 = \frac{409090.9}{22500} = 18.18$$

Substituting the value of a_1 and b_1 in equation (2), we get

$$\sigma_x = \frac{409090.9}{x^2} + 18.18$$

The above equation gives the hoop stress in the outer cylinder due to shrinking. The hoop stress at the outer and inner surface of the outer cylinder is obtained by substituting x = 150 mm and x = 125 mm respectively in the above equation.

$$\sigma_{150} = \frac{409090.9}{150^2} + 18.18 = 36.36 \text{ N/mm}^2 \text{ (tensile)}$$

and $$\sigma_{125} = \frac{409090.9}{125^2} + 18.18 = 44.36 \text{ N/mm}^2 \text{ (tensile)}.$$

(b) Lame's equation for the inner cylinder are :

$$p_x = \frac{b_2}{x^2} - a_2 \quad \text{.... (5)} \quad \text{and} \quad \sigma_x = \frac{b_2}{x^2} + a_2 \quad \text{.... (6)}$$

At x = r_1 = 100 mm , $p_x = 0$ (There is no fluid under pressure.)

Substituting these values in equation (5), we get

$$0 = \frac{b_2}{100^2} - a_2 = \frac{b_2}{10000} - a_2 \quad \text{.... (7)}$$

At x = r* =125 mm, p_x = P* = 8 N/mm^2. Substituting these values in equation (5), we get

$$8 = \frac{b_2}{125^2} - a_2 = \frac{b_2}{15625} - a_2 \qquad \text{.... (8)}$$

Subtracting equation (7) from equation (8), we get

$$8 = \frac{b_2}{15625} - \frac{b_2}{10000}$$

$$= \frac{b_2(10000-15625)}{15625 \times 10000} = \frac{-5625\, b_2}{15625 \times 10000}$$

$$b_2 = -\frac{8 \times 15625 \times 10000}{5625} = -222222.2$$

Substituting the value of b_2 in equation (8), we get

$$0 = -\frac{222222.2}{10000} - a_2$$

$$a_2 = -22.22$$

Substituting the values of a_2 and b_2 in equation (7), we get

$$\sigma_x = -\frac{222222.2}{x^2} - 22.22$$

Hence the hoop stress for the inner cylinder is obtained by substituting x = 125 mm respectively in the above equation.

$$\sigma_{125} = -\frac{222222.2}{125^2} - 22.22$$

$$= -14.22 - 22.22 = -36.44 \text{ N/mm}^2 \text{ (Compressive)}$$

and

$$\sigma_{100} = -\frac{222222.2}{100^2} - 22.22$$

$$= -22.22 - 22.22 = -44.44 \text{ N/mm}^2 \text{ (Compressive)}$$

ii. Stresses due to fluid pressure alone

When the fluid under pressure is admitted inside the compound cylinder, the two cylinders together will be considered as one single unit. The hoop stresses are calculated by Lame's equations, which are

$$p_x = \frac{B}{x^2} - A \quad \text{.... (9) and} \quad \sigma_x = \frac{B}{x^2} + A \quad \text{.... (10)}$$

Where A and B are constants.

At x = 100 mm , p_x = p = 84.5 N/mm^2. Substituting the values in equation (9),we get

$$84.5 = \frac{B}{100^2} - A = \frac{B}{10000} - A \qquad \text{.... (11)}$$

At x = 150 mm, $p_x = 0$. Substituting the values in equation (9),we get

$$0 = \frac{B}{150^2} - A = \frac{B}{22500} - A \qquad \text{.... (12)}$$

Subtracting equation (12) from equation (11), we get

$$84.5 = \frac{B}{10000} - \frac{B}{22500}$$

$$= \frac{B(22500 - 10000)}{1000 \times 22500} = \frac{12500 \times B}{10000 \times 22500}$$

$$B = \frac{84.5 \times 10000 \times 22500}{12500} = 1521000$$

Substituting this value in equation (12), we get

$$0 = \frac{1521000}{22500} - A \qquad \text{or } A = \frac{1521000}{22500} = 67.6$$

Substituting the values of A and B in equation (x), we get

$$\sigma_x = \frac{1521000}{x^2} + 67.6$$

Hence the hoop stresses due to internal fluid pressure alone are given by,

$$\sigma_{100} = \frac{1521000}{100^2} + 67.6 = 219.7 \text{ N/mm}^2 \text{ (tensile)}$$

$$\sigma_{125} = \frac{1521000}{125^2} + 67.6 = 97.344 + 67.6 = 164.94 \text{ N/mm}^2$$

$$\sigma_{150} = \frac{1521000}{150^2} + 67.6 = 67.6 + 67.6 = 135.2 \text{ N/mm}^2$$

The resultant stresses will be the algebraic sum of the initial stresses due to shrinking and those due to internal fluid pressure.

Inner cylinder

$F_{100} = \sigma_{100}$ due to shrinkage + σ_{100} due to internal fluid pressure

$= -44.44 + 219.7 = 175.26$ N/mm^2 (tensile). **Ans.**

$F_{125} = \sigma_{125}$ due to shrinkage + σ_{125} due to internal fluid pressure

$= -36.44 + 164.94 = 128.5$ N/mm^2 (tensile). **Ans.**

Outer cylinder

$F_{125} = \sigma_{125}$ due to shrinkage + σ_{125} due to internal fluid pressure

$= 44.36 + 164.94 = 209.3$ N/mm^2 (tensile). **Ans.**

$F_{150} = \sigma_{150}$ due to shrinkage + σ_{150} due to internal fluid pressure

$= 36.36 + 135.2 = 171.56$ N/mm^2 (tensile). **Ans.**

5.16. INITIAL DIFFERENCE IN RADII AT THE JUNCTION OF A COMPOUND CYLINDER FOR SHRINKAGE

By shrinking the outer cylinder over the inner cylinder, some compressive stresses are produced in the inner cylinder . In order to shrink the outer cylinder over the inner cylinder, the inner diameter of the outer cylinder should be slightly less than the outer diameter of the inner cylinder. Now the outer cylinder shrinks over the inner cylinder. Thus inner cylinder is put into compression and outer cylinder is put into tension. After shrinking , the outer radius of inner cylinder decreases whereas the inner radius of outer cylinder increases from the initial values.

Let r_2 = Outer radius of the outer cylinder

r_1 = Inner radius of the inner cylinder

r* = Radius of junction after shrinking or it is common radius after shrinking

p* = Radial pressure at the junction after shrinking.

Before shrinking, the outer radius of the inner cylinder is slightly more than r* and inner radius of the cylinder is slightly less than r*.

For the outer and inner cylinder Lame's equation are used. These equations are

$$p_x = \frac{b}{x^2} - a \quad \text{and} \quad \sigma_x = \frac{b}{x^2} + a$$

The values of constants a and b will be different for each cylinder.

Let the constants for inner cylinder be a_2,b_2 and for outer cylinder a_1,b_1.

The radial pressure at the junction (i.e.,p*) is same for outer cylinder and inner cylinder.

At the junction ,x = r* and $p_x = p*$. Hence radial pressure at the junction.

$$P* = p* = \frac{b_1}{r*^2} - a_1 = \frac{b_2}{r*^2} + a_1 \qquad \text{.... (A)}$$

or $$\frac{b_1 - b_2}{r*^2} = (a_1 - a_2) \qquad \text{.... (B)}$$

or $$(b_1 - b_2) = r*^2(a_1 - a_2)$$

Now the hoop strain (or circumferential strain) in the cylinder at any point

$$= \frac{\sigma_x}{E} + \frac{p_x}{mE} \qquad \text{.... (C)}$$

But circumferential strain

$$= \frac{Increase\ in\ circumference}{Original\ Circumference}$$

$$= \frac{2\pi(r+dr) - 2\pi r}{2\pi r} = \frac{dr}{r} \qquad \text{.... (D)}$$

= Radial strain

Hence equating the two values of circumferential strain given by equation (C) and (D),we get

$$\frac{dr}{r} = \frac{\sigma_x}{E} + \frac{p_x}{mE} \quad \text{.... (1)}$$

On shrinking, at the junction there is extension in the inner radius of the outer cylinder and compression in the outer radius of the inner cylinder.

At the junction where x = r*, increase in the inner radius of outer cylinder

$$= r^* \left(\frac{\sigma_x}{E} + \frac{p_x}{mE}\right) \quad \text{.... (2)}$$

But for outer cylinder at the junction, we have

$$\sigma_x = \frac{b_1}{r_*^2} + a_1 \text{ and } \quad p_x = \frac{b_1}{r_*^2} - a_1$$

Where a_1 and b_1 are constants for outer cylinders.

Substituting the values of σ_x and p_x in equation (2), we get

Increase in the inner radius of outer cylinder

$$= r^* \left[\frac{1}{E}\left(\sigma_x + \frac{p_x}{m}\right)\right] = r^* \left[\frac{1}{E}\left(\frac{b_1}{r_*^2} + a_1\right) + \frac{1}{mE}\left(\frac{b_1}{r_*^2} - a_1\right)\right]$$

Similarly, decrease in the outer radius of the inner cylinder is obtained from equation (1) as

$$= - r^* \left(\frac{\sigma_x}{E} + \frac{p_x}{mE}\right) \quad \text{(−ve sign is due to decrease) .... (3)}$$

But for inner cylinder at the junction , we have

$$\sigma_x = \frac{b_2}{r_*^2} + a_2 \text{ and } \quad p_x = \frac{b_2}{r_*^2} - a_2$$

Substituting these values in equation (3), we get

Decrease in the outer radius of inner cylinder

$$= -r^* \left[\frac{1}{E}\left(\frac{b_2}{r_*^2} + a_2\right) + \frac{1}{mE}\left(\frac{b_2}{r_*^2} - a_2\right)\right] \quad \text{....}$$

(4)

But the original difference in the outer radius of the inner cylinder and inner radius of the outer cylinder.

= Increase in inner radius of outer cylinder + Decrease in outer radius of the inner cylinder

$$= r^* \left[\frac{1}{E}\left(\frac{b_1}{r_*^2} + a_1\right) + \frac{1}{mE}\left(\frac{b_1}{r_*^2} - a_1\right)\right] - r^* \left[\frac{1}{E}\left(\frac{b_2}{r_*^2} + a_2\right) + \frac{1}{mE}\left(\frac{b_2}{r_*^2} - a_2\right)\right]$$

$$= \frac{r_*}{E}\left[\left(\frac{b_1}{r_*^2} + a_1\right) - \left(\frac{b_2}{r_*^2} + a_2\right)\right] + \frac{r_*}{mE}\left[\left(\frac{b_1}{r_*^2} - a_1\right) - \left(\frac{b_2}{r_*^2} - a_2\right)\right]$$

But from equation (A),

$$\frac{b_1}{r_*^2} - a_1 = \frac{b_2}{r_*^2} - a_2.$$

Hence second part of the above equation is zero. Hence above equation becomes as

Original difference of radii at the junction

$$= \frac{r_*}{E}\left[\left(\frac{b_1}{r_*^2} + a_1\right) - \left(\frac{b_2}{r_*^2} + a_2\right)\right]$$

$$= \frac{r_*}{E}\left[\frac{(b_1 - b_2)}{r_*^2} + (a_1 - a_2)\right]$$

$$= \frac{r_*}{E}[(a_1 - a_2) + (a_1 - a_2)] \left[From\ equation\ (B), \frac{b_1 - b_2}{r_*^2} = a_1 - a_2\right]$$

$$= \frac{2r_*}{E}(a_1 - a_2) \qquad \ldots (18.3)$$

The values of a_1 and a_2 are obtained from the given conditions. The value of a_1 is for outer cylinder whereas of a_2 is for inner cylinder.

Problem 5.27: A steel cylinder of 300 mm external diameter is to be shrunk to another steel cylinder of 150 mm internal diameter. After shrinking, the diameter at the junction is 250 mm and radial pressure at the common junction is 28 N/mm^2. Find the original difference in radii at the junction. Take E = 2 × 10^5 N/mm^2.

Sol. Given:

External dia. of outer cylinder = 300 mm

Radius, r_2 = 150 mm

Internal dia. of inner cylinder = 150 mm

Radius, r_1 = 75 mm

Diameter at the junction = 250 mm

Radius, r* = 125 mm

Radial pressure at the junction, p* = 28 N/mm^2

Value of E = 2 × 10^5 N/mm^2

Using equation (18.3), we get

Original difference of radii at the junction

$$= \frac{2r*}{E}(a_1 - a_2) \qquad \text{.... (1)}$$

First find the values of a_1 and a_2 from the given conditions. These are the constants for outer cylinder and inner cylinder respectively. They are obtained by using Lame's equation.

For outer cylinder $p_x = \frac{b_1}{x^2} - a_1$

1) At junction, x = r* = 125 mm and $p_x = p* = 28$ N/mm^2
2) At x =150 mm , $p_x = 0$.

Substituting these two conditions in the above equation, we get

$$28 = \frac{b_1}{125^2} - a_1 = \frac{b_1}{15625} - a_1 \qquad \text{.... (2)}$$

and $$0 = \frac{b_1}{150^2} - a_1 = \frac{b_1}{22500} - a_1 \qquad \text{.... (3)}$$

Solving equation (2) and (3), we get

b_1 = 1432000 and a_1 = 63.6.

For inner cylinder

$$p_x = \frac{b_2}{x^2} - a_2$$

1) At junction, x = r* = 125 mm and $p_x = p* = 28$ N/mm^2
2) At x =75 mm , $p_x = 0$.

Substituting these two conditions in the above equation, we get

$$28 = \frac{b_1}{125^2} - a_1 = \frac{b_1}{15625} - a_1 \qquad \text{.... (4)}$$

and $$0 = \frac{b_1}{75^2} - a_1 = \frac{b_1}{5625} - a_1 \qquad \text{.... (5)}$$

Solving equation (4) and (5), we get

b_2 = – 246100 and a_1 = – 43.75

Now substituting the values of a_2 and a_1 in equation (1),we get

Difference of radii at the junction

$$= \frac{2 \times 125}{2 \times 10^5}[63.6 - (-43.75)]$$

$$= \frac{125}{10^5} \times 107.35 = 0.13 \text{ mm. } \textbf{Ans.}$$

Problem 5.28: A steel tube of 200 mm external diameter is to be shrunk onto another steel tube of 60 mm internal diameter. The diameter at the junction after shrinking is 120 mm. Before shrinking on , the difference of diameters at the junction is 0.08 mm. Calculate the radial pressure at the junction and the hoop stresses developed in the two tubes after shrinking on. Take E as 2×10^5 N/mm^2.

Sol. Given :

External dia. of outer tube = 200 mm

Radius , r_2 = 100 mm

Internal dia. of inner tube = 60 mm

Radius, r_1 = 30 mm

The diameter at the junction after shrinking = 120 mm

Radius, r^*= 60 mm

Before shrinking on, the difference of dia. at the junction = 0.08 mm

Difference of original radii = 0.04 mm

Value of E = 2×10^5 N/mm^2

Let p* = Radial pressure at the junction

Using equation (18.3),

Original difference of radii at junction

$$= \frac{2r*}{E}(a_1 - a_2)$$

or $$0.04 = \frac{2 \times 60}{2 \times 10^5}(a_1 - a_2) \quad \text{or} \quad \frac{0.04 \times 2 \times 10^5}{2 \times 60} = (a_1 - a_2)$$

or $$(a_1 - a_2) = \frac{200}{3} \quad \text{.... (1)}$$

Now using Lame's equation for outer tube

$$p_x = \frac{b_1}{x^2} - a_1 \quad \text{.... (2)} \quad \text{and} \quad \sigma_x = \frac{b_1}{x^2} + a_1 \quad \text{.... (3)}$$

At x = 100 mm, $p_x = 0$.

Substituting these values in equation (2),

$$0 = \frac{b_1}{100^2} - a_1 \quad = \frac{b_1}{10000} - a_1 \quad \text{.... (4)}$$

At x = 60 mm, $p_x = p*$.

Substituting these values in equation (2),

$$p^* = \frac{b_1}{60^2} - a_1 = \frac{b_1}{3600} - a_1 \quad \text{.... (5)}$$

Now applying Lame's equation for inner tube

$$p_x = \frac{b_2}{x^2} - a_2 \quad \text{.... (6)} \quad \text{and} \quad \sigma_x = \frac{b_2}{x^2} + a_2 \quad \text{.... (7)}$$

At x = 30 mm, $p_x = 0$.

Substituting these values in equation (6),

$$0 = \frac{b_2}{30^2} - a_2 = \frac{b_2}{900} - a_2 \quad \text{.... (8)}$$

At x = 60 mm, $p_x = p*$.

Substituting these values in equation (6),

$$p^* = \frac{b_2}{60^2} - a_2 = \frac{b_2}{3600} - a_2 \quad \text{.... (9)}$$

Equating the two values of p*, given by equation (5) and (9)

$$\frac{b_2}{3600} - a_2 = \frac{b_1}{3600} - a_1$$

or $$\frac{b_2 - b_1}{3600} = a_2 - a_1 \quad \text{.... (10)}$$

But from equation (4), $b_1 = 10000\, a_1$

and, from equation (8), $b_2 = 900\, a_2$

Substituting these values in equation (x), we get

$$\frac{900a_2 - 10000\, a_1}{3600} = a_2 - a_1$$

or $$900a_2 - 10000\, a_1 = 3600a_2 - 3600a_1$$

or $$900a_2 - 3600a_2 = -3600a_1 + 10000\, a_1$$

or $$-2700\, a_2 = 6400\, a_1$$

or $$a_1 = -\frac{2700}{6400} a_2 = -\frac{27}{64} a_2 \quad \text{.... (11)}$$

Substituting these values of a_1 in equation (1), we get

$$-\frac{27}{64} a_2 - a_2 = \frac{200}{3}$$

or $$-\frac{(27a_2 + 64\, a_2)}{64} = \frac{200}{3} \text{ or } a_2 = -\frac{200 \times 64}{3 \times 91} = -46.88$$

Substituting these values in equation (11), we get

$$a_1 = + \frac{27}{64} \times 46.88 = + 19.77$$

$$b_1 = 10000 \times a_1 = 10000 \times (19.77) = 197700$$

$$b_2 = 900 \times a_2 = - 900 \times 46.88 = -42192$$

and

1) Radial pressure at the junction (p*)

Substituting these values of a_2 *and* b_2 in equation (9), we get

$$p^* = \frac{b_2}{3600} - a_2 = \frac{42192}{3600} + 46.88$$

$$= \frac{197700}{3600} - 19.77 = 54.916 - 19.77$$

$$= 35.146 \text{ N/mm}^2. \textbf{ Ans.}$$

2) Hoop stresses in the two tubes after shrinking on

The hoop stresses can be calculated from equations (3) and (7)

(a) For outer tube

$$\sigma_x = \frac{b_1}{x^2} + a_1 = \frac{197700}{x^2} + 19.77 \quad (b_1 = 197700, a_1 = 19.77)$$

$$\sigma_{100} = \frac{197700}{100^2} + 19.77$$

$$= 39.54 \text{ N/mm}^2 \text{ (tensile)}. \textbf{ Ans.}$$

$$\sigma_{60} = \frac{197700}{60^2} + 19.77$$

$$= 74.68 \text{ N/mm}^2 \text{ (tensile)}. \textbf{ Ans.}$$

and

(b) For inner tube

$$\sigma_x = \frac{b_2}{x^2} + a_2 = - \frac{42192}{x^2} - 46.88 \quad (b_2 = -42192, a_2 = -46.88)$$

$$\therefore \quad \sigma_{60} = - \frac{42192}{60^2} - 46.88$$

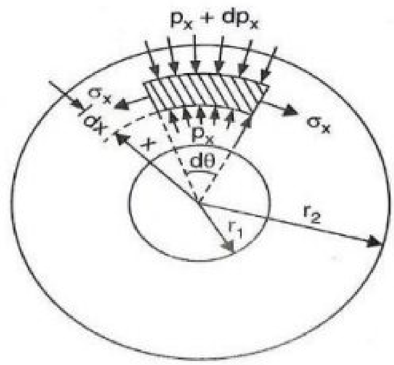
$$= - 58.6 \text{ N/mm}^2 \text{ (compressive)}. \textbf{ Ans.}$$

and $\sigma_{30} = - \frac{42192}{30^2} - 46.88$

$$= - 93.76 \text{ N/mm}^2 \text{ (compressive)}. \textbf{ Ans.}$$

5.17. THICK SPHERICAL SHELLS

Figure shows a spherical shell subjected to an internal fluid pressure p.



Let r_2 = External radius

r_1 = Internal radius.

Consider an elemental disc of the spherical shell of thickness dx at a radius x. Let this elemental disc subtend an angle $d\theta$ at the centre.

Due to internal fluid pressure, let the radius x increase to $(x + u)$ and increase in thickness dx be du.

Let e_y = Circumferential strain and

e_x = Radial strain

Now increase in radius = u

∴ Final radius = $x + u$

∴ Circumferential strain,

$$e_y = \frac{Final\ circumference - Original\ circumference}{Original\ circumference}$$

$$= \frac{2\pi(x+u) - 2\pi x}{2\pi x} = \frac{u}{x}. \quad \text{.... (1)}$$

Now original thickness of element = dx

Final thickness of element = dx + du

∴ Radial strain,

$$e_x = \frac{Final\ thickness\ of\ element - Original\ thickness}{Original\ thickness}$$

$$= \frac{(dx+du) - dx}{dx} = \frac{du}{dx}. \quad \text{.... (2)}$$

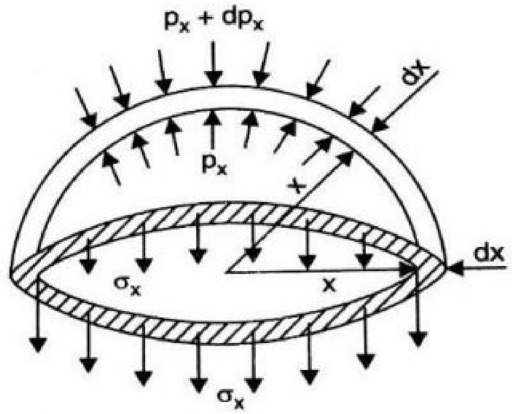
But from equation (1),

$$u = x \,.\, e_y$$

∴ Radial strain,

$$e_x = \frac{d}{dx}(x \,.\, e_y) = e_y + x \,.\frac{de_y}{dx} \quad \text{.... (3)}$$

Now consider an elemental spherical shell of radius x and thickness dx as shown in Fig. 18.7. Let p_x and $p_x + dp_x$ be the radial pressure at radii x and $x + dx$ respectively. And σ_x is the circumferential tensile stress which is equal in all direction in a spherical shell.



Consider the equilibrium of half of the elementary spherical shell on which the following external force are acting:

1) An upward force of $\pi x^2 \cdot p_x$ due to internal radial pressure p
2) A downward force of $\pi(x + dx)^2 \cdot (p_x + dp_x)$ due to radial pressure $p_x + dp_x$.
3) A downward resisting force $\sigma_x(2\pi x \cdot dx)$.

Equating the upward and downward forces, we get

$$\pi x^2 p_x = \pi(x + dx)^2 \cdot (p_x + dp_x) + 2\pi x \cdot dx \cdot \sigma_x$$

$$= \pi(x^2 + dx^2 + 2x \cdot dx)(p_x + dp_x) + 2\pi x \cdot dx \cdot \sigma_x$$

$$x^2 \cdot p_x = (x^2 + dx^2 + 2x \cdot dx)(p_x + dp_x) + 2x \cdot dx \cdot \sigma_x$$

$$=(x^2 \cdot p_x + dx^2 \cdot p_x + 2x \cdot dx \cdot p_x + x^2 \cdot dp_x + dx^2 \cdot dp_x + 2x \cdot dx \cdot dp_x)$$

$$+ 2x \cdot dx \cdot \sigma_x$$

Neglecting squares and products of dx and dp_x, we get

$$x^2 \cdot p_x = x^2 \cdot p_x + 0 + 2x \cdot dx \cdot p_x + x^2 \cdot dp_x + 0 + 0 + 2x \cdot dx \cdot \sigma_x$$

$$0 = 2x \cdot dx \cdot p_x + x^2 \cdot dp_x + 2x \cdot dx \cdot \sigma_x$$

or $\quad 2x \cdot dx \cdot \sigma_x = -2x \cdot dx \cdot p_x - x^2 \cdot dp_x$

or $\quad 2 \cdot \sigma_x = -2 \cdot p_x - x \cdot \frac{dp_x}{dx}$ (Divided both sides by $x \cdot dx$)

or $\quad \sigma_x = -p_x - \frac{x}{2} \cdot \frac{dp_x}{dx}$ …. (A)

Differentiating the above equation w.r.t. x., we get

$$\frac{d}{dx}(\sigma_x) = \frac{d}{dx}(-px) - \frac{1}{2}\frac{d}{dx}\left(x.\frac{dp_x}{dx}\right) = -\frac{dp_x}{dx} - \frac{1}{2}\left(x.\frac{d^2p_x}{dx^2} + \frac{dp_x}{dx}\right) \quad \ldots.(4)$$

At any point in the elementary spherical shell, there are three principle stresses:

1) The radial pressure p_x , which is compressive
2) Circumferential (or hoop stress) σ_x, which is tensile and
3) Circumferential (or hoop stress) σ_x, which is tensile of the same magnitude as of (2) and on a plane at right angles to the plane of σ_x of (2).

Now radial strain,

$$e_x = \frac{p_x}{E} + \frac{\sigma_x}{mE} + \frac{\sigma_x}{mE} \qquad \text{Here } \frac{1}{m} = \text{Poisson's ratio} = \mu$$

$$= \frac{p_x}{E} + \frac{2\sigma_x}{mE} \qquad \text{(compressive)}$$

$$= -\left(\frac{p_x}{E} + \frac{2\sigma_x}{mE}\right) \qquad \text{(tensile)} \qquad \ldots.(5)$$

and circumferential strain,

$$e_y = \frac{\sigma_x}{E} - \frac{\sigma_x}{mE} + \frac{p_x}{mE} \qquad \text{(tensile)}$$

$$= \frac{1}{E}\left(\sigma_x - \frac{\sigma_x}{m} + \frac{p_x}{m}\right) = \frac{1}{E}\left[\sigma_x\left(\frac{m-1}{m}\right) + \frac{p_x}{m}\right] \qquad \text{(tensile)} \qquad \ldots.(6)$$

Substituting the values of e_x and e_y from equation (5) and (6) in equation (3),we get

$$-\left(\frac{p_x}{E} + \frac{2\sigma_x}{mE}\right) = \frac{1}{E}\left[\sigma_x\left(\frac{m-1}{m}\right) + \frac{p_x}{m}\right] + x.\frac{d}{dx}\left[\frac{1}{E}\left\{\sigma_x\left(\frac{m-1}{m}\right) + \frac{p_x}{m}\right\}\right]$$

$$-\frac{1}{E}\left(p_x + \frac{2\sigma_x}{mE}\right) = \frac{1}{E}\left[\frac{\sigma_x(m-1)}{m} + \frac{p_x}{m}\right] + \frac{x}{E}\left[\left(\frac{m-1}{m}\right).\frac{d\sigma_x}{dx} + \frac{1}{m}\frac{dp_x}{dx}\right]$$

$$-\left(p_x + \frac{2\sigma_x}{mE}\right) = \left(\frac{\sigma_x(m-1)}{m} + \frac{p_x}{m}\right) + \frac{x(m-1)}{m}.\frac{d\sigma_x}{dx} + \frac{x}{m}\frac{dp_x}{dx}$$

$$-mp_x - 2\sigma_x = (m-1)\sigma_x + p_x + x(m-1)\frac{d\sigma_x}{dx} + x\frac{dp_x}{dx}$$

$$-p_x(m+1) - \sigma_x(2+m-1) = x(m-1)\frac{d\sigma_x}{dx} + x\frac{dp_x}{dx}$$

$$-(m+1)(p_x + \sigma_x) = x(m-1)\frac{d\sigma_x}{dx} + x\frac{dp_x}{dx}$$

$$(m+1)(p_x + \sigma_x) + x(m-1)\frac{d\sigma_x}{dx} + x\frac{dp_x}{dx} = 0.$$

Now substituting the value of σ_x and $\frac{d}{dx}(\sigma_x)$ from equation (A) and (4) in the above equation, we get

$$(m+1)\left(p_x + -p_x - \frac{x}{2}.\frac{dp_x}{dx}\right) + x(m-1) \times \left[-\frac{dp_x}{dx} - \frac{1}{2}\left(x\frac{d^2p_x}{dx^2} + \frac{dp_x}{dx}\right)\right] + x\frac{dp_x}{dx} = 0$$

$$(m+1)\left(-\frac{x}{2}.\frac{dp_x}{dx}\right) + x(m-1)\left(-\frac{3}{2}\frac{dp_x}{dx} - \frac{1}{2}x\frac{d^2p_x}{dx^2}\right) + x\frac{dp_x}{dx} = 0$$

$$\frac{dp_x}{dx}\left[-\frac{x}{2}(m+1) - \frac{3x}{2}(m-1) + x\right] - \frac{x^2(m-1)}{2}\frac{d^2p_x}{dx^2} = 0$$

$$x.\frac{dp_x}{dx}\left[\frac{-m-1-3m+3+2}{2}\right] - \frac{x^2}{m}(m-1)\frac{d^2p_x}{dx^2} = 0$$

$$\frac{dp_x}{dx}\left(\frac{-4m+4}{2}\right) - \frac{x}{2}(m-1)\frac{d^2p_x}{dx^2} = 0 \qquad \text{(Cancelling } x\text{)}$$

$$-\frac{4}{2}\frac{dp_x}{dx}(m-1) - \frac{x}{2}(m-1)\frac{d^2p_x}{dx^2} = 0$$

$$\frac{4dp_x}{dx} + x\frac{d^2p_x}{dx^2} = 0 \qquad \left[Cancelling - \frac{(m-1)}{2}\right]$$

Substituting $\frac{dp_x}{dx} = Z$ in the above equation, we get

$$4Z + x.\frac{d}{dx}\left(\frac{dp_x}{dx}\right) = 0$$

$$4Z + x.\frac{dZ}{dx} = 0$$

$$4Z = -x.\frac{dZ}{dx}$$

$$\frac{dZ}{dx} = -4\frac{dx}{x}$$

Integrating the above equation, we get

$$\log_e Z = -4 \log_e x + \log_e C_1$$

where C_1 is the constant of integration.

The above equation can also be written as

$$\log_e Z = \log_e x^{-4} + \log_e C_1 = \log_e C_1 \times x^{-4}$$

$$= \log_e\left(\frac{C_1}{x^4}\right) \quad \text{or} \quad Z = \frac{C_1}{x^4}$$

But $Z = \frac{dp_x}{dx}$

$\therefore$ $\frac{dp_x}{dx} = \frac{C_1}{x^4}$ or $dp_x = \frac{C_1}{x^4}dx$

Integrating the above equation, we get

$$p_x = -\frac{C_1}{3x^3} + C_2 \quad \text{..... (7)}$$

where C_2 is another constant of integration.

Substituting this value of p_x in equation (A), we get

$$\sigma_x = -\left(-\frac{C_1}{3x^3} + c_2\right) - \frac{x}{2}\frac{dp_x}{dx}$$

$$= \frac{C_1}{3x^3} - c_2 - \frac{x}{2} \cdot \frac{C_1}{x^4} \qquad \left(\frac{dp_x}{dx} = \frac{C_1}{x^4}\right)$$

$$= \frac{C_1}{3x^3} - c_2 - \frac{C_1}{2x^3} = -\frac{C_1}{6x^3} - c_2 \quad \text{.... (8)}$$

If we substitute $C_1 = -6b$ and $C_2 = -a$ in equation (7) and (8), we get

$$p_x = -\frac{(-6b)}{3x^3} + (-a) = \frac{2b}{x^3} - a \quad \text{.... (9)}$$

$$\sigma_x = -\frac{(-6b)}{6x^3} - (-a) = \frac{b}{x^3} + a \quad \text{.... (10)}$$

and

The constants a and b are obtained from initial given conditions.

For example, (i) at x = r_1, $p_x = 0$ and at x = r_2, $p_x = p$.

Substituting these values in equation (9), we get

$$0 = \frac{2b}{r_1{}^3} - a \quad \text{.... (11)} \quad \text{and} \quad p = \frac{2b}{r_2{}^3} - a \quad \text{.... (12)}$$

Solving equations (11) and (12), we get

$$a = \frac{pr_2^3}{r_1^3 - r_2^3} \quad \text{and} \quad b = \frac{pr_1^3 r_2^3}{2(r_1^3 - r_2^3)}.$$

Problem 5.29: A thick spherical shell of 200 mm internal diameter is subjected to an internal fluid pressure of 7 N/mm^2. If the permissible tensile stress in the shell material is 8 N/mm^2, find the thickness of the shell.

Sol. Given :

Internal dia, = 200 mm

∴Internal radius, r_1= 100 mm

Internal fluid pressure, p = 7 N/mm^2

Permissible tensile stress, σ_x= 8 N/mm^2.

The radial pressure and hoop stress at any radius of spherical shell are given by

$$p_x = \frac{2b}{x^3} - a \text{ .... (1)} \quad \text{and} \quad \sigma_x = \frac{b}{x^3} + a \quad \text{.... (2)}$$

The hoop stress, σ_x will be maximum at the internal radius . Hence permissible tensile stress of 8 N/mm^2 is the hoop stress at the internal radius.

At x =100 mm, $p_x = 7$ N/mm^2.

Substituting these values in equation (1), we get

$$7 = \frac{2b}{100^3} - a = \frac{2b}{1000000} - a \quad \text{.... (3)}$$

At x =100 mm, $\sigma_x = 8$ N/mm^2.

Substituting these values in equation (2), we get

$$8 = \frac{b}{100^3} + a = \frac{b}{1000000} - a \quad \text{.... (4)}$$

Adding equations (3) and (4), we get

$$15 = \frac{3b}{1000000}$$

$$b = \frac{1000000 \times 15}{3} = 5000000.$$

Substituting the value of b in equation (4), we get

$$8 = \frac{5000000}{1000000} - a = 5 + a$$

$$\therefore \quad a = 8 - 5 = 3$$

Substituting the value of a and b in equation (1), we get

$$p_x = \frac{2 \times 5000000}{x^3} - 3$$

Let r_2 = External radius of the shell.

At outside, the pressure

$$p_x = 0 \text{ or at } x = r_2,\ p_x = 0.$$

Substituting these values in equation (5), we get

$$0 = \frac{2 \times 5000000}{r_2{}^3} - 3 \quad \text{or}$$

$$r_2^3 = \frac{10000000}{3}$$

$$\therefore \quad r_2 = \left(\frac{10^7}{3}\right)^{1/3} = (3.333)^{1/3} \times 10^2 = 149.3 \text{ mm}$$

∴ Thickness of the shell,

$$t = r_2 - r_1 = 149.3 - 100$$

$$= 49.3\ mm.\textbf{Ans.}$$

Problem 5.30: For the problem 5.29, find the minimum value of the hoop stress.

Sol. Given :

The data from problem 18.6 is :

$r_1 = 100$ mm , $r_2 = 149.3$ mm

p = 7 N/mm², σ_x at the internal radius = 8 N/mm²

Values of constants are ;

a = 3, b = 5000000

The hoop stress at any radius of spherical shell is given by

$$\sigma_x = \frac{b}{x^3} + a = \frac{5000000}{x^3} + 3$$

The hoop stress will be minimum at the external radius i.e., at x = r_2 = 149.3 mm.

Substituting this value of x in the above equation, we get

$$\sigma_x = \frac{5000000}{(149.3)^3} + 3 = \frac{5000000}{\left(\frac{10000000}{3}\right)} + 3 = 1.5 + 3 = 4.5 \text{ N/mm}^2. \textbf{ Ans.}$$

IMPORTANT TERMS

Circumferential Stress (OR) Hoop Stress (σ_c)	$\sigma_c = \frac{p\ x\ d}{2t}$	p = intensity of pressure inside the cylinder d = inner diameter of cylinder shell t = thickness of cylinder shell η_l= efficiency of longitudinal joint η_c= efficiency of circumferential joint
Longitudinal Stress (σ_l)	$\sigma_l = \frac{p\ x\ d}{4t}$	
Circumferential Stress with efficiency (σ_c)	$\sigma_c = \frac{p\ x\ d}{2\ t\ x\ \eta_l}$	
Longitudinal Stress with efficiency (σ_l)	$\sigma_l = \frac{p\ x\ d}{4\ t\ x\ \eta_c}$	
Circumferential Strain	$e_c = \frac{\delta d}{d} = \frac{pd}{2\ t\ E}\left(1 - \frac{1}{2}\mu\right)$	μ= poission's ratio E = young's modulus l = length of shell
Longitudinal Strain	$e_l = \frac{\delta l}{l} = \frac{pd}{2\ t\ E}\left(\frac{1}{2} - \mu\right)$	
Change in diameter	$\delta d = \frac{pd}{2\ t\ E}\left(1 - \frac{1}{2}\mu\right)d$	
Change in Length	$\delta l = \frac{pd}{2\ t\ E}\left(\frac{1}{2} - \mu\right)l$	
Volumetric Strain	$e_v = \frac{\delta V}{V} = \frac{pd}{2\ t\ E}\left(\frac{5}{2} - 2\mu\right)$ $e_v = 2e_c - e_l$	$Volume\ (V) = \frac{\pi d^2}{4} x\ l$
Change in Volume	$\delta V = \frac{pd}{2\ t\ E}\left(\frac{5}{2} - 2\mu\right)V\ (or)$ $= (2e_c - e_l)V$	

Major Principal Stress	$= \frac{\sigma_c + \sigma_l}{2} + \sqrt{\left(\frac{\sigma_c - \sigma_l}{2}\right)^2 + \tau^2}$	τ = shear stress
Minor Principal Stress	$= \frac{\sigma_c + \sigma_l}{2} - \sqrt{\left(\frac{\sigma_c - \sigma_l}{2}\right)^2 + \tau^2}$	
Maximum Shear stress	$\frac{1}{2}(Major\ Principal\ Stress - Minor\ Principal\ Stress)$	ρ = density of the material r = mean radius of the cylinder ω = angular speed of the cylinder $= \frac{2\pi N}{60}$
Rotational Stress	$\sigma_r = \rho r^2 \omega^2$	
SPHERICAL SHELLS		
Circumferential Stress (OR) Hoop Stress (σ_c)	$\sigma_c = \frac{p\ x\ d}{4t}\ OR = \frac{p\ x\ d}{4t\ \eta}$	p = intensity of pressure inside the spherical shell d = inner diameter of spherical shell t = thickness of spherical shell E = young's modulus l = length of shell μ= poission's ratio
Circumferential Strain	$e_c = \frac{\delta d}{d} = \frac{pd}{4\ t\ E}(1 - \mu)$	
Volumetric Strain	$e_v = \frac{\delta V}{V} = \frac{3pd}{4\ t\ E}(1 - \mu)$ $Volume\ (V) = \frac{4}{3} x\ \pi r^3$	

THEORETICAL QUESTIONS

1. Define thin cylinders. Name the stresses set up in a thin cylinder subjected to int fluid pressure.
2. Prove that the circumference stress and longitudinal stress

   σ_1 = pd/2t, σ_2 = pd/4t where p = int fluid pressure
   D = int dia of thin cylinder
   T = thickness of wall of thin cylinder

   Derive an expression for circumferential stress and longitudinal stress for a thin shell subjected to an int pressure.
3. A)Derive the expression for hoop stress and longitudinal stress in a thin cylinder with ends closed by rigid flanges and subjected to an internal fluid pressure p. Take the int dia and shell thickness of the cylinder to be d and t respectively. B)Derive from the first principles of expressions for circumferential and longitudinal stresses in a thin cylinder closed at both ends and subjected to int fluid pressure.
4. Show that in thin cylinder shells subjected to int fluid pressure , the circumferential stress is twice the longitudinal stress.
5. While resighing a cylindrical vessel, which stress should be used for calculating the thickness of the cylindrical vessel.
6. Prove that max shear stress at any point in a thin cylinder, subjected to int fluid pressure is given by,

   Max shear stress = pd/8t

Where p = int fluid pressure
D = int dia of thin cylinder
T = wall thickness of cylinder

7. Find the expression for circumferential stress and longitudinal stress for a longitudinal joint and circumferential joint.
8. Prove that the circumferential strain and longitudinal strain produced in thin cylinder when subjected to int flud pressure are given by
d_1 = pd/2tE(1-1/2*μ)
e_2 = pd/2tE(1/2 – μ)
Where p = int fluid pressure
D = int dia of thin cylinder
T = thickness of wall of thin cylinder
μ = poisons ratio
9. A cylindrical shell is subjected to int fluid pressure find an expression for change in dia and change in length of cylinder
10. Prove that volumetric strain in case of a thin cylinder subjected to int fluid pressure is equal to two times the circumferential strain plus longitudinal strain. show that when a thin walled cylindrical vessel of dia D, length L and thickness t is subjected to an int pressure p, the change in volume = π*p*L*D^3(5-4*μ)/16tE
11. Find an expression for the change in volume of a thin cylindrical shell subjected to int fluid pressure
12. Write down expression for major principal stress, minor principle stress when a thin cylindrical shell is subjected to int fluid pressure and a torque
13. Show that when a thin walled spherical vessel of dia d and thickness t is subjected to int fluid pressure p the increase in volume equal to
$$\frac{\pi}{8} * pd^4/tE(1 - \frac{1}{\mu})$$
Where E = elastic modulus
μ = poisons ratio
14. Differentiate between a thin cylinder and thick cylinder. Find an expression for the radial pressure and hoop stress at any point in case of thick cylinder.
15. What do you mean by lame's equation. How will you derive these equations.
16. The hoop stress is min at the outer surface and is max at the inner surface of the thick cylinder. prove this statement. Sketch the radial pressure distribution and hoop stress distribution across the section of cylinder.
17. What do you mean by a thick compound cylinder. How will you determine the hoop stress in a thick compound cylinder.
18. What are the different methods of reducing hoop stress. Explain the terms: wire winding of thin cylinders and shrinking one cylinder over another cylinder.
19. Prove that the original difference in radii at the junction of a compound cylinder for shrinking is given by

$$Dr = 2r^*(a_1 - a_2)/E$$

Where r* = common radius after shrinking

E = young's modulus

A = constants

Derive an expression for the radial pressure and hoop stress for a thick spherical shell.

NUMERICAL PROBLEMS

1. A cylindrical pipe of dia 2m and thickness 2cm is subjected to an int fluid pressure of 1.5 N/mm^2. determine longitudinal stress and circumferential stress. Ans = 37.5 N/mm^2, 75 N/mm^2
2. A cylinder of int dia of 3m and of thickness 6m contains a gas. If the tensile stress in the material is not to exceed 70 N/mm^2, determine the int pressure of gas. Ans = 2.8 N/mm^2
3. A cylinder of int dia 0.60m contains air at a pressure of 7.5 N/mm^2. If max permissible stress induced in the material is 75 N/mm^2 find the thickness of cylinder. Ans = 3cm
4. A thin cylinder of int dia 2m contains a fluid at an int pressure of 3 N/mm^2. Determine the max thickness of cylinder if longitudinal stress is not to exceed 303 N/mm^2 and the circumferential stress is not to exceed 403 N/mm^2. Ans = 7.5 cm
5. A water main 90cm dia contains water at a pressure head of 110m. If the weight density of water is 9810 N/mm^2, find the thickness of metal required for the water main. Given the permissible stress as 223 N/mm^2. Ans = 2.25 cm
6. A boiler is subjected to an internal steam pressure of 3 N/mm^2. The thickness of boiler plate is 2.5 cm and the permissible tensile stress is 125 N/mm^2. Find out max dia when efficiency of longitudinal joint is 90% and that of circumferential joint is 35%. Ans = 145.83cm
7. A boiler shell is to be made of 20mm thick plate having a limiting tensile stress of 125N/mm^2. If the efficient of longitudinal and circumferential joints are 80% and 30%. Determine max permissibe dia of shell for an int pressure of 2.5 N/mm^2 and permissible intensity of int pressure when the shell dia is 1.6m. ans = 120 cm, 1.875 N/mm^2
8. A cylinder of thickness 2cm has to withstand max int pressure of 2 N/mm^2. If the ultimate tensile stress in the material of cylinder is 292 N/mm^2, factor of safety 4 and joint efficiency 80%, determine the dia of cylinder. Ans = 116.8cm
9. A thin cylindrical sell of 120 cm dia, 1.5cm thick and 6m long is subjected to int fluid pressure of 2.5 N/mm^2. If poisons ratio is 0.3. find change in dia, change in length,change in volume. Ans = 0.051m, 0.06cm, 6449.7cm
10. A cylindrical shell 100cm long 20cm int dia having thickness of metal as 10mm is filled with fluid at atm pressure. If an additional 20cm of fluid is pumped into cylinder find the pressure exerted by fluid on cylinder and the hoop stress induced.poisons ratio = 0.3. ans = 10.05N/mm^2, 100.52 N/mm^2

11. A cylindrical vessel whose ends are closed by means of rigid flange plates is made of steel plates 4mm thick. The length and int dia of vessel are 100cm and 30 cm. Determine the longitudinal and hoop stress in cylindrical shell due to int fluid pressure of 2 N/mm^2. Also calculate the increase in length, dia and volume of vessel.poissons ratio = 0.3. ans = 37.53 N/mm^2, 75 N/mm^2, 0.075 cm, 0.0095cm, 50.36cm
12. A thin cylindrical tube 100mm int dia and 5mm thick is closed at the ends and is subjected to an int pressure of 5 N/mm^2. A torque of 22000Nm is also applied to the tube.find the hoop stress, longitudinal stress,max and min principle stresses and max shear stress. Ans = 50N/mm^2, 25 N/mm^2, 28N/mm^2, 68.16 N/mm^2, 6.84 N/mm^2, 30.66N/mm^2
13. A copper cylinder 100 cm long, 50 cm ext dia and wall thickness 5mm has its both ends closed by rigid blank flanges. It is initially full of oil at atm pressure. Calculate the additional volume of oil which must be pumped into it in order to raise the oil pressure to 4 N/mm^2 above the atm pressure. Ans = 486.3cm
14. A vessel in the shape of a spherical shell of 1.4dia and 4.5mm thickness is subjected to a pressure of 1.8 N/mm^2. Determine the stress induced in the material of the vessel. Ans = 140 N/mm^2
15. A thin spherical shell of 1.20mm int dia is subjected to int pressure of 1.6 N/mm^2. If the permissible stress in the plate material is 80 N/mm^2 and joint efficiency is 75% find the min thickness. Ans = 8mm
16. A thin spherical shell of int dia 1.5m and of thickness 8mm is subjected to an int pressure of 1.5 N/mm^2. Determine the increase in dia and increase in volume.poisons ratio = 0.3. ans = 0.369mm, 1304*$10^3 mm^3$
17. Determine the max hoop stress across the section of pipe of ext dia 600 mm and int dia 440 mm, when the pipe is subjected to an int fluid pressure of 0 N/mm^2. Ans = 99.9 N/mm^2
18. Find the thickness of metal necessary for a cylinder shell of int dia 150mm to withstand an int pressure of 50 N/mm^2. The max hoop stress in section is not to exceed 150 N/mm^2. Ans = 31mm
19. A compound cylinder is made by shrinking a cylinder of ext dia 200mm and int dia 160mm over another cylinder of ext dia 160mm and int dia 120mm. The radial pressure at the junction after shrinking is 8 N/mm^2. Find the final stress set up across the section when the compound cylinder is subjected to an int fluid pressure of 60 N/mm^2. Ans = inner F_{60} = 90.9 and F_{80} = 57.9 N/mm^2, outer F_{80} = 122.9 and F_{100} = 25.9 N/mm^2
20. A steel cylinder of 200mm ext dia is to be shrunk to another steel cylinder of 100mm int dia. After shrinking the dia at junction is 150 mm and radial pressure

at the junction is 12.5N/mm. Find the original difference in radii at the junction. Ans = 0.02025mm

21. A steel tube of 240mm ext dia is to be shrunk on another steel tube of 80mm int dia. After shrinking the dia at junction is 160mm. Before shrinking on the difference of dia at the junction was 0.08mm. calculate the radii pressure at the junction and hoop stress developed in the two tubes after shrinking.
22. A thick spherical shell of 400mm int dia is subjected to an int fluid pressure of 1.5 N/mm^2. If the permissible tensile stress in the shell material is 3 N/mm^2. Find the necessary thickness of shell. Ans = 52mm