Catalog

3.1 HORIZONTAL CONTROLS & ITS METHODS

The horizontal control consists of reference marks of known plan position, from which salient points of designed structures may be set out. For large structures primary and secondary control points are used. The primary control points are triangulation stations. The secondary control points are reference to the primary control stations.

Reference Grid

Reference grids are used for accurate setting out of works of large magnitude. The following types of reference grids are used:

1.Survey Grid

2.Site Grid

3.structural Grid

4.Secondary Grid

Survey grid is one which is drawn on a survey plan, from the original traverse. Original traverse stations form the control points of the grid. The site grid used by the designer is the one with the help of which actual setting out is done. As far as possible the site grid should be actually the survey grid. All the design points are related in terms of site grid coordinates. The structural grid is used when the structural components of the building are large in numbers and are so positioned that these components cannot be set out from the site grid with sufficient accuracy. The structural grid is set out from the site grid points. The secondary grid is established inside the structure, to establish internal details of the building, which are otherwise not visible directly from the structural grid.

3.2 VERTICAL CONTROL & ITS METHODS:

The vertical control consists of establishment of reference marks of known height relative to some special datum. All levels at the site are normally reduced to the nearby bench mark, usually known as master bench mark.

The setting of points in the vertical direction is usually done with the help of following rods:

1.Boning rods and travelers

2.Sight Rails

3.Slope rails or batter boards

4.Profile boards

A boning rod consist of an upright pole having a horizontal board at its top, forming a 'T 'shaped rod. Boning rods are made in set of three, and many consist of three 'T' shaped rods, each of equal size and shape, or two rods identical to each other and a third one consisting of longer rod with a detachable or movable 'T' piece. The third one is called traveling rod ortraveler.

Sight Rails:

A sight rail consists of horizontal cross piece nailed to a single upright or pair of uprights driven into the ground. The upper edge of the cross piece is set to a convenient height above the required plane of the structure, and should be above the ground to enable a man to conveniently align his eyes with the upper edge. A stepped sight rail or double sight rail is used in highly undulating or falling ground. Slope rails or Batter boards:

These are used for controlling the side slopes in embankment and in cuttings. These consist of two vertical poles with a sloping board nailed near their top. The slope rails define a plane parallel to the proposed slope of the embankment, but at suitable vertical distance above it. Travelers are used to control the slope during filling operation.

Profile boards:

These are similar to sight rails, but are used to define the corners, or sides of a building. A profile board is erected near each corner peg. Each unit of profile board consists of two verticals, one horizontal board and two cross boards. Nails or saw cuts are placed at the top of the profile boards to define the width of foundation and the line of the outside of the wall.

PROBLEMS

1. An instrument was set up at P and the angle of elevation to a vane 4 m above the foot of the staff held at Q was 9 º 30'. The horizontal distance between P and Q was known to be 2000 metres. Determine the R.L. of the staff station Q given that the R.L. of the instrument axis was 2650.38.

Solution:

Height of vane above the instrument axis = D tan ϴ= 2000 tan 9 º 30'

= 334.68 m

Correction for curvature and refraction

$C = 0.06735\ D^2$ m,

when D is in km

= 0.2694 = 0.27 m (+ ve)

Height of vane above the instrument axis

= 334.68 + 0.27 = 334.95

R.L. fo vane = 334.95 + 2650.38 = 2985.33 m

R.L. of Q = 2985.33 - 4 = 2981.33 m

2. An instrument was set up at P and the angle of depression to a vane 2 m above the foot of the staff held at Q was 5 º 36'. The horizontal distance between P and Q was known to be 3000 metres. Determine the R.L. of the staff station Q given that staff reading on a B.M. of elevation 436.050 was 2.865 metres.

Solution:

The difference in elevation between the vane and the instrument axis= D tan Θ

= 3000 tan 5 ° 36'. = 294.153

Combined correction due to cuvature and refraction

$C = 0.06735 D^2$ metres ,

when D is in km

= 0.606 m.

Since the observed angle is negative, the combined correction due to curvature and refraction is subtractive.

Difference in elevation between the vane and the instrument axis

= 294.153 - 0.606 = 293.547 = h.

R.L. of instrument axis = 436.050 + 2.865 = 438.915m

R.L. of the vane = R.L. of instrument aixs - h

= 438.915 - 293.547 = 145.368m

R.L. of Q = 145.368 - 2= 143.368 m.

3.In order to ascertain the elevation of the top (Q) of the signal on a hill, observations were made from two instrument stations P and R at a horizontal distance 100 metres apart, the station P and R being in the line with Q. The angles of elevation of Q at P and R were 28 ° 42' and 18 ° 6' respectively. The staff reading upon the bench mark of elevation 287.28 were respectively 2.870 and 3.750 when the instrument was at P and at R, the telescope being horizontal. Determine the elevation of the foot of the signal if the height of the signal above its base is 3 metres.

Solution:

Elevation of instrument axis at P = R.L. of B.M. + Staff reading

= 287.28 + 2.870 = 290.15 m

Elevation of instrument axis at R = R.L. of B.M. + staff reading

= 287.28 + 3.750 = 291.03 m

Difference in level of the instrument axes at the two stations S =291.03 - 290.15

= 0.88 m.

Θ_1 -- = 28 ° 42' and Θ_2 = 18 ° 6'

s cot Θ= 0.88 cot 18 ° 6' = 2.69 m= 152.1 m.

h-- = D tan Θ = 152.1 tan 28 ° 42' = 83.272 m

R.L. of foot of signal = R.L. of inst. aixs at P + h ht. of signal

= 290.15 + 83.272 - 3 = 370.422 m.

Check : (b + D) = 100 + 152.1 m = 252.1 m

h-- = (b + D) tan Θ - = 252.1 x tan 18 ° 6'= 82.399 m

R.L. of foot of signal = R.L. of inst. axis at R + h--+ ht. of signal

= 291.03 + 82.399 - 3 = 370.429 m.

3.3 CLASSIFICATION OF TRIANGULATION SYSTEM

The basis of the classification of triangulation figures is the accuracy with which the length and azimuth of a line of the triangulation are determined. Triangulation systems of different accuracies depend on the extent and the purpose of the survey. The accepted grades of triangulation are:

First order or Primary Triangulation
Second order or Secondary Triangulation
Third order or Tertiary Triangulation

1 FIRST ORDER OR PRIMARY TRIANGULATION:

The first order triangulation is of the highest order and is employed either to determine the earth's figure or to furnish the most precise control points to which secondary triangulation may be connected. The primary triangulation system embraces the vast area (usually the whole of the country). Every precaution is taken in making linear and angular measurements and in performing the reductions.

The following are the general specifications of the primary triangulation:

Average triangle closure : Less than 1 second
Maximum triangle closure : Not more than 3 seconds
Length of base line : 5 to 15 kilometers
Length of the sides of triangles : 30 to 150 kilometers
Actual error of base : 1 in 300,000
Probable error of base : 1 in 1,000,000
Discrepancy between two measures of a section: 10 mm kilometers
Probable error or computed distance : 1 in 60,000 to 1 in 250,000
Probable error in astronomic azimuth : 0.5 seconds

SECONDARY ORDER OR SECONDARY TRIANGULATION

The secondary triangulation consists of a number of points fixed within the framework of primary triangulation. The stations are fixed at close intervals so that the sizes of the triangles formed are smaller than the primary triangulation. The instruments and methods used are not of the same utmost refinement. The general specifications of the secondary triangulation are:

Average triangle closure : 3 sec

Maximum triangle closure : 8 sec

Length of base line : 1.5 to 5 km

Length of sides of triangles : 8 to 65 km

Actual error of base : 1 in 150,000

Probable error of base : 1 in 500,000

Discrepancy between two measures of a section: 20 mm kilometers

Probable error or computed distance : 1 in 20,000 to 1 in 50,000

Probable error in astronomic azimuth : 2.0 sec

THIRD ORDER OR TERTIARY TRIANGULATION:

The third-order triangulation consists of a number of points fixed within the framework of secondary triangulation, and forms the immediate control for detailed engineering and other surveys. The sizes of the triangles are small and instrument with moderate precision may be used. The specifications for a third-order triangulation are as follows:

Average triangle closure : 6 sec

Maximum triangle closure : 12 sec

Length of base line : 0.5 to 3 km

Length of sides of triangles : 1.5 to 10 km

Actual error of base : 1 in 75, 0000

Probable error of base : 1 in 250,000

Discrepancy between two Measures of a section : 25 mm kilometers

Probable error or computed distance : 1 in 5,000 to 1 in 20,000

Probable error in astronomic Azimuth: 5 sec.

Factors to be considered while selecting base line.

The measurement of base line forms the most important part of the triangulation operations. The base line is laid down with great accuracy of measurement and alignment as it forms the basis for the computations of triangulation system. The length of the base line depends upon the grades of the triangulation. Apart from main base line, several other check bases are also measured at some suitable intervals. In India, ten bases were used, the lengths of the nine bases vary from 6.4 to 7.8 miles and that of the tenth base is 1.7 miles.

Selection of Site for Base Line. Since the accuracy in the measurement of the base line depends upon the site conditions, the following points should be taken into consideration while selecting the site:

The site should be fairly level. If, however, the ground is sloping, the slope should be uniform and gentle. Undulating ground should, if possible be avoided.

The site should be free from obstructions throughout the whole of the length. The line clearing should be cheap in both labour and compensation.

The extremities of the base should be intervisible at ground level.

The ground should be reasonably firm and smooth. Water gaps should be few, and if possible not wider than the length of the long wire or tape.

The site should suit extension to primary triangulation. This is an important factor since the error in extension is likely to exceed the error in measurement.

In a flat and open country, there is ample choice in the selection of the site and the base may be so selected that it suits the triangulation stations. In rough country, however, the choice is limited and it may sometimes be necessary to select some of the triangulation stations that at suitable for the base line site.

Standards of Length. The ultimate standard to which all modern national standards are referred is the international meter established by the Bureau International der Poids at Measures and kept at the Pavilion de Breteuil, Sevres, with copies allotted to various national surveys. The meter is marked on three platinum- iridium bars kept under standard conditions. One great disadvantage of the standard of length that are made of metal are that they are subject to very small secular change in their dimensions. Accordingly, the meter has now been standardized in terms of wavelength of cadmium light.

3.4 ERROR

Types Of Error

Errors of measurement are of three kinds:

(i) mistakes,

(ii) systematic errors, and

(iii) accidental errors.

(i) Mistakes. Mistakes are errors that arise from inattention, inexperience, carelessness and poor judgment or confusion in the mind of the observer. If mistake is undetected, it produces a serious effect on the final result. Hence every value to be recorded in the field must be checked by some independent field observation.

(ii) Systematic Error. A systematic error is an error that under the same conditions will always be of the same size and sign. A systematic error always follows some definite mathematical or physical law, and a correction can be determined and applied. Such errors are of constant character and are regarded as positive or negative according as they make the result too great or too small. Their effect is therefore, cumulative. If undetected, systematic errors are very serious. Therefore:

All the surveying equipment's must be designed and used so that whenever possible systematic errors will be automatically eliminated and (2) all systematic errors that cannot be surely eliminated by this means must be evaluated and their relationship to the conditions that cause them must be determined. For example, in ordinary levelling, the levelling instrument must first be adjusted so that the line of sight is as nearly horizontal as possible when bubble is centered. Also the horizontal lengths for back sight and foresight from each instrument position should be kept as nearly equal as possible. In precise levelling, every day, the actual error of the instrument must be determined by careful peg test, the length of each sight is measured by stadia and a correction to the result is applied.

(iii) Accidental Error. Accidental errors are those which remain after mistakes and systematic errors have been eliminated and are caused by a combination of reasons beyond the ability of the observer to control. They tend sometimes in one direction and some times in the other, i.e., they are equally likely to make the apparent result too large or too small.

An accidental error of a single determination is the difference between (1) the true value of the quantity and (2) a determination that is free from mistakes and systematic errors. Accidental error represents limit of precision in the determination of a value. They obey the laws of chance and therefore, must be handled according to the mathematical laws of probability.

The theory of errors that is discussed in this chapter deals only with the accidental errors after all the known errors are eliminated and accounted for.

THE LAW OF ACCIDENTAL ERRORS

Investigations of observations of various types show that accidental errors follow a definite law, the law of probability. This law defines the occurrence of errors and can be expressed in the form of equation which is used to compute the probable value or the probable precision of a quantity. The most important features of accidental errors which usually occur are:

Small errors tend to be more frequent than the large ones; that is they are the most probable.

Positive and negative errors of the same size happen with equal frequency; that is, they are equally probable.

Large errors occur infrequently and are impossible.

3.5 PRINCIPLES OF LEAST SQUARES

It is found from the probability equation that the most probable values of a series of errors arising from observations of equal weight are those for which the sum of the squares is a minimum. The fundamental law of least squares is derived from this. According to the principle of least squares, the most probable value of an observed quantity available from a given set of observations is the one for which the sum of the squares of the residual errors is a minimum. When a quantity is being deduced from a series of observations, the residual errors will be the difference between the adopted value and the several observed values,

Let *V*1, *V*2, *V*3 etc. be the observed values x = most probable value

LAW OF WEIGHTS

From the method of least squares the following laws of weights are established:

(i) The weight of the arithmetic mean of the measurements of unit weight is equal to the number of observations.

For example, let an angle A be measured six times, the following being the values:

A	Weight	A	Weight
30 º 20' 8"		30 º 20' 10"	
30 º 20' 10"		30 º 20' 09"	
30 º 20' 07"		30 º 20' 10"	

Arithmetic mean= 30 º 20' + 1/6 (8' + 10' + 7' + 10' + 9' + 10')

= 30 º 20' 09"

Weight of arithmetic mean = number of observations = 6.

(ii) The weight of the weighted arithmetic mean is equal to the sum of the individual weights.

For example, let an angle A be measured six times, the following being the values :

A Weight A Weight

30 ^{o} 20' 8" 2 30 ^{o} 20' 10" 3

30 ^{o} 20' 10" 3 30 ^{o} 20' 09" 4

30 ^{o} 20' 07" 2 30 ^{o} 20' 10" 2

Sum of weights = 2 + 3 + 2 + 3 + 4 + 2 =16

Arithmetic mean = 30 ^{o} 20' + 1/16 (8'X2 + 10' X3+ 7'X2 + 10'X3 + 9' X4+ 10'X2)

= 30 ^{o} 20' 9".

Weight of arithmetic mean = 16.

(iii) The weight of algebric sum of two or more quantities is equal to the reciprocals of the individual weights.

For Example angle A = 30 ^{o} 20' 10" , Weight 2

B = 15 ^{o} 20' 08" , Weight 3

(iv) If a quantity of given weight is multiplied by a factor, the weight of the result is obtained by dividing its given weight by the square of the factor.

(v) If a quantity of given weight is divided by a factor, the weight of the result is obtained by multiplying its given weight by the square of the factor.

(vi) If a equation is multiplied by its own weight, the weight of the resulting equation is equal to the reciprocal of the weight of the equation.

(vii) The weight of the equation remains unchanged, if all the signs of the equation are changed or if the equation is added or subtracted from a constant.

DISTRIBUTION OF ERROR OF THE FIELD MEASUREMENT

Whenever observations are made in the field, it is always necessary to check for the closing error, if any. The closing error should be distributed to the observed quantities. For examples, the sum of the angles measured at a central angle should be 360 o , the error should be distributed to the observed angles after giving proper weight age to the observations. The following rules should be applied for the distribution of errors:

(i).The correction to be applied to an observation is inversely proportional to the weight of the observation.

(ii) The correction to be applied to an observation is directly proportional to the square of the probable error.

(iii) In case of line of levels, the correction to be applied is proportional to the length.

PROBLEMS

1.The following are the three angles x,y and z observed at a station P closing the horizon, along with their probable errors of measurement. Determine their corrected values.

Solution.

x = 78 º 12' 12" 2'

y = 136 º 48' 30" 4'

z = 144 º 59' 08" 5'

Sum of the three angles = 359 º 59' 50" Discrepancy = 10"

Hence each angle is to be increased, and the error of 10" is to be distributed in proportion to the square of the probable error.

Let c1, c2 and c3 be the correction to be applied to the angles x, y and z respectively.

c1 : c2 : c3 = $(2)^2 : (4)^2 : (5)^2$ = 4 : 16 : 25 -------------- (1)

Also, c1 + c2 + c3 = 10' ------------(2)

From (1), c2 = 16 /4 c1 = 4c1 And

c3 = 25/4 c1

Substituting these values of c2 and c3 in (2), we get

C 1 + 4c1 + 25/4 c1 = 10" or

c1 (1 + 4 + 25/4)= 10"

c1 = 10 x 4/45 = 0'.89

c2 = 4c1 = 3".36

And c3 = 25 /4 c1 = 5".55

Check: c1 + c2 + c3 = 0."89 + 3".56 + 5".55 = 10"'

Hence the corrected angles are

x= 78 º 12' 12" + 0'.89 = 78 º 12' 12".89

y = 136 º 48' 30" + 3".56 = 136 º 48' 33".56 and

z= 144 º 59' 08" + 5".55 = 144 º 59' 13".55

Sum = 360 º 00' 00"+ 00

2. An angle A was measured by different persons and the following are the values

Angle	Number of measurements
65 º 30' 10''	**2**
65 º 29' 50''	**3**
65 º 30' 00''	**3**
65 º 30' 20''	**4**
65 º 30' 10''	**3**

Find the most probable value of the angle.

Solution.

As stated earlier, the most probable value of an angle is equal to its weighted arithmetic mean.

65 º 30' 10" x 2 = 131 º 00' 20"

65 º 29' 50" x 3 = 196 º 29' 30"

65 º 30' 00" x 3 = 196 º 30' 00"

65 º 30' 20" x 4 = 262 º 01' 20"

65 º 30' 10" x 3 = 196 º 30' 30"

Sum = 982 º 31' 40" weight = 2 + 3 + 3 + 4 + 3 = 15

Weighted arithmetic mean

= 982 º 31' 40"/15 = 65 º 30' 6".67

Hence most probable value of the angle = 65 º 30' 6".67

3.6 NORMAL EQUATION

A normal equation is **formed by the multiplication of unknown coefficients** by which, the obtained equation is added and leads to the formation of normal equation. If the number of equations formed is equal to the number of unknowns the n the most probable value values can be found by the equations

PROBLEM

1. The telescope of a theodolite is fitted with stadia wires. It is required to find the most probable values of the constants C and K of tacheometer. The staff was kept vertical at three points in the field and with of sight horizontal the staff intercepts observed was as follows.

Distance of staff from tacheometer D(m)	Staff intercept S(m)
150	1.495
200	2.000
250	2.505

Solution:

The distance equation is D = KS + C

The observation equations are

150 = 1.495 K + C

200 = 2.000 K + C

250 = 2.505 K + C

If K and C are the most probable values, then the error of observations are:

150 - 1.495 K - C

200 - 2.000 K - C

250 - 2.505 K - C

By the theory of least squares

$(150 - 1.495 K - C)^2 + (200 - 2.000 K - C)^2 + (250 - 505 K - C)^2$ = minimum---(i)

For normal equation in K,

Differentiating equation (i) w.r.t. K,

2(-1.495)(150 -1.495 K -C) +2(-20.)(200 - 2.000 K- C)

+2(-2.505)(250 - 505 K - C) = 0

208.41667 - 2.085 K - C = 0----(2)

Normal equation in C Differentiating equation (i) w.r.t. C,

2(-1.0)(150 -1.495 K -C) +2(-1.0)(200 - 2.000 K- C)

+2(-1.0)(250 - 505 K - C) = 0

200 - 2 K - C = 0---- (3)

On solving Equations (2) and (3)

K = 99.0196

C = 1.9608

The distance equation is: D = 99.0196 S + 1.9608

3.7 METHODS OF CORRELATES

Method of correlation involves in determining the multiples or the individual constants, which can be further used for finding the probable values of unknowns. For finding these, a lot of conditions were established.

PROBLEM -1

The following angles were measured at a station O as to close the horizon.

$\angle AOB = 83°42'28''.75$	**weight 3**
$\angle BOC = 102°15'43''.26$	**weight 2**
$\angle COD = 94°38'27''.22$	**weight 4**
$\angle DOA = 79°23'23''.77$	**weight 2**

Adjust the angles by method of Correlates.

Solution:

$\angle AOB = 83°42'28''.75$	Weight 3
$\angle BOC = 102°15'43''.26$	Weight 2
$\angle COD = 94°38'27''.22$	Weight 4
$\angle DOA = 79°23'23''.77$	Weight 2

Sum = $360°00'03''.00$

Hence, the total correction E = 360° - (360°0'3")

= -3"

Let e_1, e_2, e_3 and e_4 be the individual corrections to the four angles respectively. Then by the condition equation, we get

$$e_1 + e_2 + e_3 + e_4 = -3'' \quad \text{--------- (1)}$$

Also, from the least square principle, $\Sigma(we^2)$ = a minimum

$$3e_1^2 + 2e_2^2 + 4e_3^2 + 2e_4^2 = \text{a minimum} \quad \text{------- (2)}$$

Differentiating (1) and (2), we get

$$\delta e_1 + \delta e_2 + \delta e_3 + \delta e_4 = 0 \quad \text{--------- (3)}$$

$3e_1\delta e_1 + 2e_2\delta e_2 + 4e_3\delta e_3 + 2e_4\delta e_4 = 0$ --------- (4)

Multiplying equation (3) by –λ and adding it to (4), we get

$\delta e_1(3e_1 - \lambda) + \delta e_2(2e_2-\lambda) + \delta e_3(4e_3-\lambda) + \delta e_4(2e_4-\lambda) = 0$-----------(5)

Since the coefficients of $\delta e_1, \delta e_2, \delta e_3, \delta e_4$ must vanish independently, we have

$3e_1 - \lambda = 0$ or $e_1 = \frac{\lambda}{3}$

$2e_1 - \lambda = 0$ or $e_2 = \frac{\lambda}{2}$ ---------- (6)

$4e_1 - \lambda = 0$ or $e_3 = \frac{\lambda}{4}$

$2e_1 - \lambda = 0$ or $e_4 = \frac{\lambda}{2}$

Substituting these values in (1), we get

$$\frac{\lambda}{3} + \frac{\lambda}{2} + \frac{\lambda}{4} + \frac{\lambda}{2} = -3$$

$$\lambda\left(\frac{19}{12}\right) = -3$$

$$\lambda = -\frac{3*12}{19}$$

Hence

$$e_1 = \frac{1}{3} * \frac{3*12}{19} = -\frac{12}{19} = -0.63''$$

$$e_2 = \frac{1}{2} * \frac{3*12}{19} = -\frac{18}{19} = -0.95''$$

$$e_3 = -\frac{1}{4} * \frac{3*12}{19} = -\frac{9}{19} = -0.47$$

$$e_4 = \frac{1}{2} * \frac{3*12}{19} = -\frac{18}{19} = -0.95''$$

Sum = -3.0"

BOC	= 102°15'43".26 – 0".95	= 102°15'42".31
COD	= 94°38'27".22 – 0".47	= 94°38'26".75
DOA	= 79°23'23".77 – 0".95	= 79°23'22".82
		360°00'00".00

The following round of angles was observed from central station to surrounding station of a triangulation survey.

A = 93°43'22" **weight 3**

B = 74°32'39" **weight 2**

C = 101°13'44" **weight 2**

D = 90°29'50" **weight 3**

In addition, one angle $\overline{(A+B)}$ was measured separately as comb angle with a mean value of 168°16'06" (wt 2).

Determine the most probable values of the angles A, B, C and D.

Solution:

A + B+C+D = 359°59'35"

Total correction E = 360° - (359° 59' 35")

= + 25°

Similarly, $\overline{(A+B)}$ = (A+B)

Hence correction E' = A + B - $\overline{(A+B)}$

= 168°16'01" – 168°16'06"

= -5"

Let e_1 ,e_2, e_3 ,e_4 and e_5 be the individual corrections to A, B, C, D $\overline{(A+B)}$ respectively. Then by the condition equation, we get

$e_1 + e_2 + e_3 + e_4 = -25"$ -------- (1(a))

$e_5 - e_1 - e_2 = -5"$ -------- (1(b))

Also, from the least square principle, $\Sigma(we^2)$ = a minimum

$3e_1^2 + 2e_2^2 + 2e_3^2 + 3e_4^2 + 2e_5^2$ = a minimum ------- (2)

Differentiating (1a) (1b) and (2), we get

$\delta e_1 + \delta e_2 + \delta e_3 + \delta e_4 = 0$ ---------- (3a)

$\delta e_5 - \delta e_1 - \delta e_2 = 0$ ---------- (3b)

ering

Solving (I) and (II) simultaneously, we get

$$\lambda_1 = +\frac{210}{11}$$

$$\lambda_2 = +\frac{90}{11}$$

$$3e_1\delta e_1 + 2e_2\delta e_2 + 2e_3\delta e_3 + 3e_4\delta e_4 + 2e_5\delta e_5 = 0 \qquad \text{---------- (4)}$$

Multiplying equation (3a) by $-\lambda_1$, (3b) by $-\lambda_2$ and adding it to (3), we get

$$\delta e_1(3e_1 - \lambda_1 + \lambda_2) + \delta e_2(2e_2 - \lambda_1 + \lambda_2) + \delta e_3(2e_3 - \lambda_1) + \delta e_4(3e_4 - \lambda_1)$$

$$+\delta e_5(-\lambda_2 + 2e_5) = 0 \qquad \text{---------- (5)}$$

Since the coefficients of $\delta e_1, \delta e_2, \delta e_3, \delta e_4$ etc. must vanish independently, we have

$$-\lambda_1 + \lambda_2 + 3e_1 = 0 \quad \text{or} \quad e_1 = \frac{\lambda_1}{3} - \frac{\lambda_2}{3}$$

$$-\lambda_1 + \lambda_2 + 2e_2 = 0 \quad \text{or} \quad e_2 = \frac{\lambda_1}{2} - \frac{\lambda_2}{2}$$

$$-\lambda_2 + 2e_3 = 0 \quad \text{or} \quad e_3 = \frac{\lambda_1}{2} \qquad \text{------------- (6)}$$

$$-\lambda_1 + 3e_4 = 0 \quad \text{or} \quad e_4 = \frac{\lambda_1}{3}$$

$$-\lambda_2 + 2e_5 = 0 \quad \text{or} \quad e_5 = -\frac{\lambda_2}{2}$$

Substituting these values of e_1, e_2, e_3, e_4 and e_5 in Equations (1a) and (1b)

$$\frac{\lambda_1}{3} - \frac{\lambda_2}{3} + \frac{\lambda_1}{2} - \frac{\lambda_2}{2} + \frac{\lambda_1}{2} + \frac{\lambda_1}{3} = 25 \quad from(1a)$$

$$\text{or} \quad 5\frac{\lambda_1}{3} - \frac{5}{6}\lambda_2 = 25$$

$$\frac{\lambda_1}{3} - \frac{1}{6}\lambda_2 = 5 \qquad \text{--------- (I)}$$

$$\frac{\lambda_2}{2} - \frac{\lambda_1}{3} + \frac{\lambda_2}{3} - \frac{\lambda_1}{2} + \frac{\lambda_2}{32} = -5 \quad from(1b)$$

$$4\frac{\lambda_2}{3} - \frac{5}{6}\lambda_1 = -5 \qquad \text{---------- (II)}$$

Hence $e_1 = \frac{1}{3} \cdot \frac{210}{11} - \frac{1}{3} \cdot \frac{90}{11} = +\frac{40''}{11} = +3''.64$

$$e_2 = \frac{1}{2} \cdot \frac{210}{11} - \frac{1}{2} \cdot \frac{90}{11} = +\frac{60''}{11} = +5''.45$$

$$e_3 = \frac{1}{2} \cdot \frac{210}{11} = +\frac{105''}{11} = +9''.55$$

$$e_4 = \frac{1}{3} \cdot \frac{210}{11} = +\frac{70''}{11} = +6''.36$$

Total = +25".00

Also

$$e_5 = \frac{1}{2} \cdot \frac{90}{11} + 4''09.$$

Hence the corrected angles are

A = 93°43'22" + 3".64 = 93°43'25".64

B = 74°32'39" + 5".45 = 74°32'44".45

C = 103°13'44" + 9".55 = 101°13'53".55

D = 90°29'50" + 6".36 = 90°29'56".36

Sum = 360°00'00".00