Catalog

3.1 DIMENSIONS ANALYSIS : INTRODUCTION

Dimensional analysis.

Dimensional analysis is defined as a mathematical technique used in research work for design and conducting model tests.

It is particularly useful for:

- ✓ presenting and interpreting experimental data;
- ✓ attacking problems not amenable to a direct theoretical solution;
- ✓ checking equations;
- ✓ establishing the relative importance of particular physical phenomena
- ✓ physical modelling.

Fundamental dimensions

The fundamental units quantities such as length (L), mass (M), and time (T) are fixed dimensions known as fundamental dimensions.

Units.

Unit is defined as a yardstick to measure physical quantities like distance, area, volume, mass etc.

Derive the dimensions for velocity.

Velocity is the distance (L) travelled per unit time (T)

Velocity = Distance/ Time = [L/T] = LT^{-1}.

Dimensions of Derived Quantities.

Dimensions of common derived mechanical quantities are given in the following table.

S. No.	Physical Quantity	Symbol	Dimensions
	(a) **Fundamental**		
1.	Length	L	L
2.	Mass	M	M
3.	Time	T	T
S.No.	Physical Quantity	Symbol	Dimensions
	(b) **Geometric**		
4.	Area	A	L^2
5.	Volume	$\forall$	L^3
	(c) **Kinematic Quantities**		
6.	Velocity	v	LT^{-1}
7.	Angular Velocity	ω	T^{-1}
8.	Acceleration	a	LT^{-2}
9.	Angular Acceleration	α	T^{-2}
10.	Discharge	Q	L^3T^{-1}
11.	Acceleration due to Gravity	g	LT^{-2}
12.	Kinematic Viscosity	ν	L^2T^{-1}
	(d) **Dynamic Quantities**		
13.	Force	F	MLT^{-2}
14.	Weight	W	MLT^{-2}
15.	Density	ρ	ML^{-3}
16.	Specific Weight	w	$ML^{-2}T^{-2}$
17.	Dynamic Viscosity	μ	$ML^{-1}T^{-1}$
18.	Pressure Intensity	p	$ML^{-1}T^{-2}$
19.	Modulus of Elasticity	$\begin{cases} K \\ E \end{cases}$	$ML^{-1}T^{-2}$
20.	Surface Tension	σ	MT^{-2}
21.	Shear Stress	τ	$ML^{-1}T^{-2}$
22.	Work, Energy	W or E	ML^2T^{-2}
23.	Power	P	ML^2T^{-3}
24.	Torque	T	ML^2T^{-2}
25.	Momentum	M	MLT^{-1}

TABLE 3.1.1 Dimensions of Derived Quantities

DIMENSIONAL HOMOGENEITY

Dimensional homogeneity means the dimensions of each terms in an equation on both sides are the same.

If the dimensions of each term on both sides of an equation are the same the equation is known as dimensionally homogeneous equation.

Example:

$$S = ut + \frac{1}{2}at^2$$

$$[S] = L$$

$$[ut] = [LT^{-1}T] = [L]$$

$$\left[\frac{1}{2}at^2\right] = [LT^{-2}T^2] = [L]$$

It is a dimensionally homogeneous equation.

3.2 Methods of Dimensions Analysis

If the number of variables involved in a physical phenomenon are known, then the relation among the variables can be determined by the following two methods.

- Reyleigh's method
- Buckingham's Pi-theorem

Reyleigh's method

This method is used for determining the expression for a variable which depends upon maximum three or four variables only. If the number of independent variables is more than five then it becomes difficult to find expression for dependent variable.

Let $X_1, X_2, X_3 --------X_n$ are the variables involved in a physical problem. Let X_1 be the dependent variable and $X_2, X_3 --------X_n$ are independent variable upon which X_1 depends.

$$X_1 = f(X_2, X_3 --------X_n)$$

i.e $f_1(X_1, X_2, X_3 --------X_n) = 0$ -------------------------- (i)

Where K is a constant and a,b,c are the arbitrary powers

Buckingham's Pi-theorem

If there are n variables (independent and dependent) in a physical phenomenon and these variables contain m fundamental dimensions (M,L,T) then the variables are arranged into (n-m) dimensionless terms. Each term is called π term.

Let $X_1, X_2, X_3 --------X_n$, , are the variables involved in a physical problem. Let X1 be the dependent variable and $X_2, X_3 --------X_n$, are independent variable upon which X_1 depends.

$$X_1 = f(X_2, X_3 --------X_n)$$

i.e $f_1(X_1, X_2, X_3 --------X_n) = 0$ -------------------------- (i)

Equation (i) is dimensionally homogeneous equation. It contains n variables. If there are m fundamental dimensions then according to Buckingham' π – Theorem, eqn.(i) can be written in terms of number of dimensionless groups or π – terms in which number of π – terms is equal to (n-m). Hence eqn.(i) becomes

$f(\pi_1, \pi_2, \pi_3 --------\pi_{n-m}) = 0$ ------------------------------------ (ii)

Each π term is dimensionless and independent of the system. Division or multiplication by a constant does not change the character of the π – term. Each π – term contains m+1 variables, where m is number of fundamental dimensions and is also called repeating variables. Let in the above case X_2, X_3 ,X_4 are repeating variables if fundamental dimension m (M, L, T) = 3 then each π – term is written as

$$\pi_1 = X_2^{a_1} X_3^{b_1} X_4^{c_1} X_1$$
$$\pi_2 = X_2^{a_2} X_3^{b_2} X_4^{c_2} X_1$$

$$\pi_{n-m} = X_2^{a_{n-m}} X_3^{b_{n-m}} X_4^{c_{n-m}} X_1 \text{-------------------------} \quad (iii)$$

Each term is solved by the principle of dimensional homogeneity and values of a_1 ,b_1 ,c_1 etc are obtained. These values are substituted in the eqn. (iii) and values of $\pi_1 \; \pi_2 \; \pi_3$ – – – – – – – –π_{n-m} , , are obtained. These values are substituted in eqn. (ii). The final equation for the phenomenon is obtained by expressing any one of the π – terms as a function of others as

$$\pi_1 = \phi(\pi_2, \pi_3 \text{--------} \pi_{n-m}) = 0$$
$$\pi_2 = \phi(\pi_1, \pi_3 \text{--------} \pi_{n-m}) = 0$$

Method of selecting repeating variable:

1. As far as possible dependent variable should not be selected as repeating variable.

2. Repeating variables should be selected in such a way that one variable contains geometric property (such as length l , diameter d, height H etc), other variable contains flow properties (such as velocity, acceleration etc.) and the third variable contains fluid properties (such as viscosity, density etc)

3. Selected repeating variable should not form dimensionless group.

4. Repeating variables together must have same number of fundamental dimensions.

5. No two repeating variables should have the same dimension. For most of the fluid mechanics problems the choice for the repeating variable may be
(i) d,γ ,ρ (ii) l,γ ,ρ (iii) l,γ ,μ (iv) d,γ ,μ

PROBLEM 1: A partially submerged body is towed in water. The resistance R to its motion depends on the density ρ , viscosity μ of water, length L of the body, velocity V of the body and acceleration g due to gravity. Show that the resistance to the motion can be expressed in the form of

$$R = \rho L^2 V^2 \, \phi\left[\left(\frac{\mu}{\rho V L}\right) \cdot \left(\frac{lg}{V^2}\right)\right].$$

Soln. The resistance R depends on ρ,μ,L,V, g

$$R = K\rho^a \,.\, \mu^b \,.\, l^c \,.\, V^d \,.\, g^e \qquad \ldots(i)$$

Substituting the dimensions on both sides of the equation (*i*),

$$MLT^{-2} = K(ML^{-3})^a \,.\, (ML^{-1}T^{-1})^b \,.\, L^c \,.\, (LT^{-1})^d \,.\, (LT^{-2})^e$$

Equating the powers of *M*, *L*, *T* on both sides

Power of *M*, $\quad 1 = a + b$

Power of *L*, $\quad 1 = -3a - b + c + d + e$

Power of *T*, $\quad -2 = -b - d - 2e.$

There are 5 unknowns and 3 equations. Expressing the three unknowns in terms of two unknowns (μ and g) . Hence express a, c and d in terms of b and e. Solving we get

$$a = 1 - b$$
$$d = 2 - b - 2e$$
$$c = 1 + 3a + b - d - e = 1 + 3(1 - b) + b - (2 - b - 2e) - e$$
$$= 1 + 3 - 3b + b - 2 + b + 2e - e = 2 - b + e.$$

Substituting these values in equation (*i*), we get

$$R = K\rho^{1-b} \cdot \mu^b \cdot l^{2-b+e} \cdot V^{2-b-2e} \cdot g^e$$
$$= K\rho l^2 \cdot V^2 \cdot (\rho^{-b} \mu^b l^{-b} V^{-b}) \cdot (l^e \cdot V^{-2e} \cdot g^e)$$

$$= K\rho l^2 V^2 \cdot \left(\frac{\mu}{\rho V l}\right)^b \cdot \left(\frac{lg}{V^2}\right)^e$$

$$= K\rho l^2 V^2 \phi \left[\left(\frac{\mu}{\rho V l}\right) \cdot \left(\frac{lg}{V^2}\right)\right]. \quad \textbf{Ans.}$$

PROBLEM 2: The resisting force R of a supersonic plane during flight can be considered as dependent upon the length of the aircraft L, velocity V, air viscosity μ, air density , and bulk modulus of air k. Express the functional relationship between the variables and the resisting force.

Solution. The resisting force *R* depends upon

(*i*) density, *l*, (*ii*) velocity, *V*,
(*iii*) viscosity, μ, (*iv*) density, ρ,
(*v*) Bulk modulus, *K*.

$$\therefore \qquad R = Al^a \cdot V^b \cdot \mu^c \cdot \rho^d \cdot K^e \qquad ...(i)$$

Substituting the dimensions on both sides of the equation (*i*),

$$MLT^{-2} = AL^a \cdot (LT^{-1})^b \cdot (ML^{-1}T^{-1})^c \cdot (ML^{-3})^d \cdot (ML^{-1}T^{-2})^e$$

Equating the powers of *M*, *L*, *T* on both sides,

Power of *M*, $1 = c + d + e$

Power of *L*, $1 = a + b - c - 3d - e$

Power of *T*, $-2 = -b - c - 2e.$

There are five unknowns but equations are only three. Expressing the three unknowns in terms of two unknowns (μ and *K*).

∴ Express the values of *a*, *b* and *d* in terms of *c* and *e*.

Solving,
$$d = 1 - c - e$$
$$b = 2 - c - 2e$$
$$a = 1 - b + c + 3d + e = 1 - (2 - c - 2e) + c + 3(1 - c - e) + e$$
$$= 1 - 2 + c + 2e + c + 3 - 3c - 3e + e = 2 - c.$$

Substituting these values in (*i*), we get

$$R = A\, l^{2-c} \cdot V^{2-c-2e} \cdot \mu^c \cdot \rho^{1-c-e} \cdot K^e$$

$$= A\, l^2 . V^2 . \rho(l^{-c} V^{-c} \mu^c \rho^{-c}) . (V^{-2e} . \rho^{-e} . K^e)$$

$$= A\, l^2 V^2 \rho \left(\frac{\mu}{\rho V L}\right)^c \cdot \left(\frac{K}{\rho V^2}\right)^e$$

PROBLEM 3:Using Buckingham' π – Theorem show that velocity through circular orifice is given by

$$V = \sqrt{2gH}\, \phi\left(\frac{D}{H}, \frac{\mu}{\rho V H}\right),$$

where H is head causing flow, D is diameter of the orifice, μ is coefficient viscosity, ρ is mass density and g is acceleration due to gravity

∴ Total number of variable, $n = 6$ ...(*i*)

Writing dimension of each variable, we have

$$V = LT^{-1},\ H = L,\ D = L,\ \mu = ML^{-1}T^{-1},\ \rho = ML^{-3},\ g = LT^{-2}.$$

Thus number of fundamental dimensions, $m = 3$

∴ Number of π-terms $= n - m = 6 - 3 = 3.$

Equation (*i*) can be written as $f_1(\pi_1, \pi_2, \pi_3) = 0$ ...(*ii*)

Each π-term contains $m + 1$ variables, where $m = 3$ and is also equal to repeating variables. Here V is a dependent variable and hence should not be selected as repeating variable. Choosing H, g, ρ as repeating variable, we get three π-terms as

$$\pi_1 = H^{a_1} . g^{b_1} . \rho^{c_1} . V$$

$$\pi_2 = H^{a_2} . g^{b_2} . \rho^{c_2} . D$$

$$\pi_3 = H^{a_3} . g^{b_3} . \rho^{c_3} . \mu$$

First π-term $\pi_1 = H^{a_1} . g^{b_1} . \rho^{c_1} . V$

Substituting dimensions on both sides

$$M^0L^0T^0 = L^{a_1} . (LT^{-2})^{b_1} . (MT^{-3})^{c_1} . (LT^{-1})$$

Equating the powers of M, L, T on both sides,

Power of M, $0 = c,$ ∴ $c_1 = 0$

Power of L, $0 = a_1 + b_1 - 3c_1 + 1,$ ∴ $a_1 = -b_1 + 3c_1 - 1 = \frac{1}{2} - 1 = -\frac{1}{2}$

Power of T, $0 = -2b_1 - 1,$ ∴ $b_1 = -\frac{1}{2}$

Substituting the values of a_1, b_1 and c_1 in π_1,

$$\pi_1 = H^{-\frac{1}{2}} . g^{-\frac{1}{2}} . \rho^0 . V = \frac{V}{\sqrt{gH}}.$$

Second π-term $\pi_2 = H^{a_2} . g^{b_2} . \rho^{c_2} . D$

Substituting the dimensions on both sides,

$$M^0L^0T^0 = L^{a_2} . (LT^{-2})^{b_2} . (ML^{-3})^{c_2} . L$$

Equating the powers of M, L, T,

Power of M, $0 = c_2$ $\therefore$ $c_2 = 0$

Power of L, $0 = a_2 + b_2 - 3c_2 + 1$, $a_2 = -b_2 + 3c_2 - 1 = -1$

Power of T, $0 = -2b_2$, $\therefore$ $b_2 = 0$

Substituting the values of a_2, b_2, c_2 in π_2,

$$\pi_2 = H^{-1} . g^0 \rho^0 . D = \frac{D}{H}.$$

Third π-term $\pi_3 = H^{a_3} . g^{b_3} . \rho^{c_3} . \mu$

Substituting the dimensions on both sides

$$M^0 L^0 T^0 = L^{a_3} . (LT^{-2})^{b_3} . (ML^{-3})^{c_3} . ML^{-1}T^{-1}$$

Equating the powers of M, L, T on both sides

Power of M, $0 = c_3 + 1$, $\therefore$ $c_3 = -1$

Power of L, $0 = a_3 + b_3 - 3c_3 - 1$, $\therefore$ $a_3 = -b_3 + 3c_3 + 1 = \frac{1}{2} - 3 + 1 = -\frac{3}{2}$

Power of T, $0 = -2b_3 - 1$, $\therefore$ $b_3 = -\frac{1}{2}$

Substituting the values of a_3, b_3 and c_3 in π_3,

$$\pi_3 = H^{-3/2} . g^{-1/2} . \rho^{-1} . \mu = \frac{\mu}{H^{3/2}\rho\sqrt{g}}$$

$$= \frac{\mu}{H\rho\sqrt{gH}} = \frac{\mu V}{H\rho V\sqrt{gH}} \qquad \text{[Multiply and Divide by } V\text{]}$$

$$= \frac{\mu}{H\rho V} . \pi_1 \qquad \left\{\because \frac{V}{\sqrt{gH}} = \pi_1\right\}$$

Substituting the values of π_1, π_2 and π_3 in equation (*ii*),

$$f_1\left(\frac{V}{\sqrt{gH}}, \frac{D}{H}, \pi_1 \frac{\mu}{H\rho V}\right) = 0 \text{ or } \frac{V}{\sqrt{gH}} = \phi\left[\frac{D}{H}, \pi_1 \frac{\mu}{H\rho V}\right]$$

or $$V = \sqrt{2gH}\ \phi\left[\frac{D}{H}, \frac{\mu}{\rho VH}\right].\ \textbf{Ans.}$$

Multiplying by a constant does not change the character of π-terms.

3.1 DIMENSIONLESS NUMBERS

In fluid mechanics, Dimensionless numbers or non-dimensional numbers are those which are useful to determine the flow characteristics of a fluid. Inertia force always exists if there is any mass in motion. Dividing this inertia force with other forces like viscous force, gravity force, surface tension, elastic force, or pressure force, gives us the dimensionless numbers.

Dimensionless Numbers in Fluid Mechanics

Some important dimensionless numbers used in fluid mechanics and their importance is explained below.

1. Reynolds Number
2. Froude Number
3. Weber Number
4. Mach Number
5. Euler's Number

1. Reynolds number

Reynolds number is the ratio of inertia force to the viscous force. It describes the predominance of inertia forces to the viscous forces occurring in the flow systems.

$$R_e = \frac{\rho . v . d}{\mu}$$

Where,

ρ = Density of fluid (kg/m^3)

μ = viscosity of fluid (kg/m.s)

d = diameter of pipe (m)

v = velocity of flow (m/s)

Importance

Reynolds number is applicable for closed surface flows as well as for free surface flows. Some applications where Reynolds number is significant for finding the flow behavior are incompressible flow through small pipes, the motion of a submarine completely under water, flow through low-speed turbomachines, etc.

2. Froude number

Froude number is the ratio of inertia force to the gravitational force. Froude number is significant in case of free surface flows where the gravitational force is predominant compared to other forces.

$$F_r = \frac{v}{\sqrt{g.L}}$$

Where,

L = length of flow (m)

v = velocity of flow (m/s)

g = acceleration due to gravity (m/s^2)

Importance

Froude number is useful to describe the flow in open channels, flow over notches and weirs, the motion of a ship in turbulent sea conditions (ship resistance), flow over spillways, etc.

3. Weber number

Weber number is the ratio of inertia force to the surface tension. The formation of droplets or water bubbles in a fluid is normally due to surface tension. If Weber number is small, surface tension is larger and vice versa.

$$W_e = \frac{\rho . d . v^2}{\sigma}$$

Applications

Weber number is less than 1 when surface tension is predominant. It happens when the curvature of the liquid surface is small compared to its depth. This can be seen in different situations such as the flow of blood in veins and arteries, atomization of liquids, capillary flow of water in soils, thin layers of fluid passing over surface, etc.

4. Mach number

Mach number is the ratio of inertia force to the elastic force. If the Mach number is one, then the flow velocity is equal to the velocity of sound in the fluid. If it is less than one, then the flow is called subsonic flow, and if it is greater than one the flow is called supersonic flow.

$$M_a = \frac{v}{c}$$

Where,

v = Velocity of flow (m/s)

c = Velocity of sound in fluid (m/s)

Applications

Mach number is useful to describe problems in high flow velocities. It is also used in aerodynamics to describe the speed of jet plane or missile in terms of speed of sound.

5. Euler's number

Euler number is the ratio of pressure force to the inertia force.

$$E_u = \frac{F}{\rho . v^2 . L^2}$$

Where,

F = pressure force

ρ = Density of fluid (kg/m^3)

L = Characteristic length of flow (m)

v = velocity of flow (m/s)

Applications

Euler's number is significant in cases where pressure gradient exists such as flow through pipes, water hammer pressure in penstocks, discharge through orifices and mouthpieces, etc.

3.5 MODEL STUDIES

Model: Model is the small scale replica of the actual structure or machine. It is not necessary that models should be smaller than the prototypes (although in most of the cases it is), they may be larger than the prototypes.

Prototype: The actual structure or machine

Model analysis: Model analysis is the study of models of actual machine.

Advantages:

• The performance of the machine can be easily predicted, in advance.

• With the help of dimensional analysis, a relationship between the variables influencing a flow problem in terms of dimensional parameters is obtained. This relationship helps in conducting tests on the model.

• The merits of alternative designs can be predicted with the help of model testing. The most economical and safe design may be, finally, adopted.

Type of forces acting in the moving fluid

Inertial force: it is equal to the mass and acceleration of the moving fluid.

$$F_i = \rho A V^2$$

Viscous force: it is equal to the shear stress due to viscosity and surface area of the flow.It present in the flow problems where viscosity is having an important role to play.

$$F_v = \tau A = \mu \frac{du}{dy} A = \mu \frac{U}{d} A$$

Gravity force: product of mass and acceleration due to gravity.

$$F_g = \rho A L g$$

Pressure force: product of pressure intensity and flow area.

$$F_p = pA$$

Surface tension force: product of surface tension and the length of the surface of the flowing fluid.

$$F_s = \sigma d$$

Elastic force: product of elastic stress and area of the flow.

$$F_e = Elastic\ stress \times Area = KA$$

Classification of mode

• Undistorted models: are those models which are geometrically similar to their prototype. In other words the scale ratio for the linear dimensions of the model and its prototype are the same.

• Distorted models: are those models which are geometrically not similar to its prototype. In other words the scale ratio for the linear dimensions of the model and its prototype are not same.

For example river: If the horizontal and vertical scale ratios for the model and the prototype are same then it is undistorted model. In this case the depth of the water in the model becomes very small which may not be measured accurately.

Thus for cases distorted model is useful.

The followings are the advantages of distorted models

- ✓ The vertical dimension of the model can be accurately measured
- ✓ The cost of the model can be reduced
- ✓ Turbulent flow in the model can be maintained

Though there are some advantage of distorted models, however the results of such models cannot be directly transferred to prototype.

Scale Ratios for Distorted Models

Let: $(L_r)_H = \frac{L_P}{L_m} = \frac{B_P}{B_m}$ Scale ratio for horizontal direction

$(L_r)_V = \frac{h_P}{h_m}$ =Scale ratio for vertical direction

Scale Ratio for Velocity: $V_r = V_P / V_m = \frac{\sqrt{2gh_P}}{\sqrt{2gh_m}} = \sqrt{(L_r)_V}$

Scale Ratio for area of flow: $A_r = A_P / A_m = \frac{B_P h_P}{B_m h_m} = (L_r)_H (L_r)_V$

Scale Ratio for discharge: $Q_r = Q_P / Q_m = \frac{A_P V_P}{A_m V_m} = (L_r)_H (L_r)_V \sqrt{(L_r)_V} = (L_r)_H (L_r)_V^{3/2}$

3.4 SIMILITUDES AND MODEL LAWS

Similitude is basically defined as the similarity between model and its prototype in each and every respect. It suggests us that model and prototype will have similar properties or we can say that similitude explains that model and prototype will be completely similar.

Three types of similarities must exist between model and prototype and these similarities are as mentioned here.

Geometric similarity

Kinematic similarity

Dynamic similarity

Geometric similarity

Geometric similarity is the similarity of shape. Geometric similarity is said to exist between model and prototype, if the ratio of all respective linear dimension in model and prototype are equal.

Ratio of dimension of model and corresponding dimension of prototype will be termed as scale ratio i.e. Lr.

Let us assume the following linear dimension in model and prototype.

$$\frac{L_P}{L_m} = \frac{B_P}{B_m} = \frac{D_P}{D_m} = L_r \qquad \frac{A_P}{A_m} = L_r^2 \qquad \frac{V_P}{V_m} = L_r^3$$

where L_r is Scale Ratio

L_m = Length of model, L_P = Length of prototype

B_m = Breadth of model, B_P = Breadth prototype

D_m = Diameter of model, D_P = Diameter of prototype

A_m = Area of model, A_P = Area of prototype

V_m = Volume of model, V_P = Volume of prototype

Kinematic Similarity

The Kinemetic similarity is said to exist between model and prototype, if the ratios of velocity and acceleration at a point in model and at the respective point in the prototype are the same.

We must note it here that the direction of velocity and acceleration in the model and prototype must be identical.

$$\frac{V_P}{V_m} = V_r \qquad \frac{a_P}{a_m} = a_r$$

where V_r is Velocity Ratio where a_r is Acceleration Ratio

V_m = Velocity of fluid at a point in model

V_P = Velocity of fluid at respective point in prototype

a_m = Acceleration of fluid at a point in model

a_P = Acceleration of fluid at respective point in prototype

Dynamic Similarity

The dynamic similarity is said to exist between model and prototype, if the ratios of corresponding forces acting at the corresponding points are the same.

We must note it here that the direction of forces at the corresponding points in the model and prototype must be same.

F_m = Force at a point in model, F_P = Force at respective point in prototype

Model laws or similarity laws

For the dynamic similarity between the model and the prototype, ratio of corresponding forces acting on corresponding points in the model and the prototype should be same.

Ratios of the forces are dimensionless numbers. Therefore we can say that for the dynamic similarity between the model and the prototype, dimensionless numbers should be equal for the model and the prototype.

However, it is quite difficult to satisfy the condition that all the dimensionless numbers should be equal for the model and the prototype.

However for practical problems, it is observed that one force will be most significant as compared to others and that force is considered as predominant force. Therefore for dynamic similarity, predominant force will be considered in practical problems.

Th
phenomenon.

Hence, we can define the model laws or similarity laws as the law on which models are designed for the dynamic similarity.

There are following types of model laws

Reynold's Model law

Froude Model law

Euler Model law

Weber Model law

Mach Model law

Reynold's Model law

Reynold's model law could be defined as a model law or similarity law where models are designed on the basis of Reynold's numbers.

According to the Reynold's model law, for the dynamic similarity between the model and the prototype, Reynold's number should be equal for the model and the prototype.

In simple, we can say that Reynold's number for the model must be equal to the Reynold's number for the prototype.

As we know that Reynold's number is basically the ratio of inertia force and viscous force, therefore a fluid flow situation where viscous forces are alone predominant, models will be designed on the basis of Reynold's model law for the dynamic similarity between the model and the prototype.

$$(\mathrm{Re})_P = (\mathrm{Re})_m \quad or \quad \frac{V_P L_P}{\upsilon_P} = \frac{V_m L_m}{\upsilon_m}$$

Where,

V_m = Velocity of the fluid in the model

L_m = Length of the model

ν_m = Kinematic viscosity of the fluid in the model

V_P = Velocity of the fluid in the prototype

L_P = Length of the prototype

ν_P = Kinematic viscosity of the fluid in the prototype

Models based on the Reynold's model law

Pipe flow

Resistance experienced by submarines, airplanes etc.

Froude Model law

Froude model law could be defined as a model law or similarity law where models are designed on the basis of Froude numbers.

According to the Froude model law, for the dynamic similarity between the model and the prototype, Froude number should be equal for the model and the prototype.

In simple, we can say that Froude number for the model must be equal to the Froude number for the prototype.

As we know that Froude number is basically the ratio of inertia force and gravity force, therefore a fluid flow situation where gravity forces are alone predominant, models will be designed on the basis of Froude model law for the dynamic similarity between the model and the prototype.

$$(Fe)_P = (Fe)_m \quad or \quad \frac{V_P}{\sqrt{g_P L_P}} = \frac{V_m}{\sqrt{g_m L_m}}$$

Where,

V_m = Velocity of the fluid in the model

L_m = Length of the model

g_m = Acceleration due to gravity at a place where model is tested

V_P = Velocity of the fluid in the prototype

L_P = Length of the prototype

g_P = Acceleration due to gravity at a place where prototype is tested

Models based on the Froude model law

Free surface flows such as flow over spillways, weirs, sluices, channels etc,

Flow of jet from an orifice or from a nozzle,

Where waves are likely to be formed on surface

Where fluids of different densities flow over one another

Euler's Model law

Euler's model law could be defined as a model law or similarity law where models are designed on the basis of Euler's numbers.

According to the Euler's model law, for the dynamic similarity between the model and the prototype, Euler's number should be equal for the model and the prototype.

In simple, we can say that Euler's number for the model must be equal to the Euler's number for the prototype.

As we know that Euler's number is basically the ratio of pressure force and inertia force, therefore a fluid flow situation where pressure forces are alone predominant, models will be designed on the basis of Euler's model law for the dynamic similarity between the model and the prototype.

$$\frac{V_m}{\sqrt{p_m / \rho_m}} = \frac{V_p}{\sqrt{p_p / \rho_p}}$$

Where,

V_m = Velocity of the fluid in the model

P_m = Pressure of fluid in the model

ρ_m = Density of the fluid in the model

V_P = Velocity of the fluid in the prototype

P_P = Pressure of fluid in the prototype

ρ_P = Density of the fluid in the prototype

Models based on the Euler's model law

Euler's model law will be applicable for a fluid flow situation where flow is taking place in a closed pipe, in which case turbulence will be fully developed so that viscous forces will be negligible and gravity force and surface tension force will be absent.

Weber Model law

Weber model law could be defined as a model law or similarity law where models are designed on the basis of Weber numbers.

According to the Weber model law, for the dynamic similarity between the model and the prototype, Weber number should be equal for the model and the prototype.

In simple, we can say that Weber number for the model must be equal to the Weber number for the prototype.

As we know that Weber number is basically the ratio of inertia force and surface tension force, therefore a fluid flow situation where surface tension forces are alone predominant, models will be designed on the basis of Weber model law for the dynamic similarity between the model and the prototype.

$$\frac{V_m}{\sqrt{\sigma_m / \rho_m L_m}} = \frac{V_p}{\sqrt{\sigma_p / \rho_p L_p}}$$

Where,

V_m = Velocity of the fluid in the model

σ_m = Surface tension force in the model

ρ_m = Density of the fluid in the model

L_m = Length of surface in the model

V_P = Velocity of the fluid in the prototype

σ_P = Surface tension force in the prototype

ρ_P = Density of the fluid in the prototype

L_P = Length of surface in the prototype

Models based on the Weber model law

Capillary rise in narrow passage

Capillary movement of water in soil

Capillary waves in channels

Flow over weirs for small heads

Mach Model law

Mach model law could be defined as a model law or similarity law where models are designed on the basis of Mach numbers.

According to the Mach model law, for the dynamic similarity between the model and the prototype, Mach number should be equal for the model and the prototype.

In simple, we can say that Mach number for the model must be equal to the Mach number for the prototype.

As we know that Mach number is basically the ratio of inertia force and Elastic force, therefore a fluid flow situation where elastic forces are alone predominant, models will be designed on the basis of Mach model law for the dynamic similarity between the model and the prototype.

$$\frac{V_m}{\sqrt{K_m / \rho_m}} = \frac{V_P}{\sqrt{K_p / \rho_p}}$$

Where,

V_m = Velocity of the fluid in the model

K_m = Elastic stress for model

ρ_m = Density of the fluid in the model

V_P = Velocity of the fluid in the prototype

K_P = Elastic stress for prototype

ρ_P = Density of the fluid in the prototype

Models based on the Mach model law

Water hammer problems

Under water testing of torpedoes

Aerodynamic testing

Flow of aeroplane and projectile through air at supersonic speed

3.6 Solved Problems -Model Studies

PROBLEM 1: A Ship is 300m long moves in seawater, whose density is 1030 kg/m^3, A 1:100 model of this to be tested in a wind tunnel. The velocity of air in the wind tunnel around the model is 30m/s and the resistance of the model is 60N. Determine the velocity of ship in seawater and also the resistance of the ship in sea water. The density of air is given as 1.24 kg/m^3. Take the Kinematic viscosity of seawater and air as 0.012 stokes and 0.018 stokes respectively.

Solution. Given :

For Prototype,

Length, $L_P = 300$ m

Fluid = Sea-water

Density of water = 1030 kg/m^3

Kinematic viscosity, $\nu_P = 0.012$ stokes $= 0.012 \times 10^{-4}$ m^2/s

Let velocity of ship $= V_P$

Resistance $= F_P$

For model

Length, $L_m = \frac{1}{100} \times 300 = 3$ m

Velocity, $V_m = 30$ m/s

Resistance, $F_m = 60$ N

Density of air, $\rho_m = 1.24$ kg/m^3

Kinematic viscosity of air, $\nu_m = 0.018$ stokes $= .018 \times 10^{-4}$ m^2/s.

For dynamic similarity between the prototype and its model, the Reynolds number for both of them should be equal.

$$\therefore \quad \frac{V_P \times L_P}{\nu_P} = \frac{V_m \times L_m}{\nu_m} \quad \text{or} \quad V_P = \frac{\nu_P}{\nu_m} \times \frac{L_m}{L_P} \times V_m$$

$$= \frac{.012 \times 10^{-4}}{.018 \times 10^{-4}} \times \frac{3}{300} \times 30 = \frac{1}{1.5} \times \frac{1}{100} \times 30 = \mathbf{0.2\ m/s.}$$

Resistance = Mass × Acceleration

$$= \rho L^3 \times \frac{V}{t} = \rho L^2 \times \frac{V}{1} \times \frac{L}{t} = \rho L^2 V^2$$

Then

$$\frac{F_P}{F_m} = \frac{(\rho L^2 V^2)_P}{(\rho L^2 V^2)_m} = \frac{\rho_P}{\rho_m} \times \left(\frac{L_P}{L_m}\right)^2 \times \left(\frac{V_P}{V_m}\right)^2$$

But $\frac{\rho_P}{\rho_m} = \frac{1030}{1.24}$

$\therefore$ $\frac{F_P}{F_m} = \frac{1030}{1.24} \times \left(\frac{300}{3}\right)^2 \times \left(\frac{0.2}{30}\right)^2 = 369.17$

$\therefore$ $F_P = 369.17 \times F_m = 369.17 \times 60 = \mathbf{22150.2\ N.}$

PROBLEM 2: A 7.2 m height and 15m long spill way discharges 94 m^3/s discharge under a head of 2.0m. If a 1:9 scale model of this spillway is to be constructed, determine model dimensions, head over spillway model and the model discharge. If model experience a force of 7500N (764.53Kgf), determine force on the prototype.

Solution. Given :

For prototype : Height $h_P = 7.2$ m

Length, $L_P = 15$ m

Discharge, $Q_P = 94\ m^3/s$

Head, $H_P = 2.0$ m

Size of model = $\frac{1}{9}$ of the size of prototype.

$\therefore$ Linear scale ratio, $L_r = 9$

Force experienced by model, $F_P = 7500$ N

Find : (*i*) Model dimensions *i.e.*, height and length of model (h_m and L_m)

(*ii*) Head over model *i.e.*, H_m

(*iii*) Discharge through model *i.e.*, Q_m

(*iv*) Force on prototype (*i.e.*, F_p)

(i) Model dimensions (h_m and L_m)

$$\frac{h_P}{h_m} = \frac{L_P}{L_m} = L_r = 9$$

$\therefore$ $h_m = \frac{h_P}{9} = \frac{7.2}{9} = \mathbf{0.8\ m.}$

And $L_m = \frac{L_P}{9} = \frac{15}{9} = \mathbf{1.67\ m.}$

(ii) Head over model (H_m)

$$\frac{H_P}{H_m} = L_r = 9$$

$\therefore$ $H_m = \frac{H_P}{9} = \frac{2}{9} = \mathbf{0.222\ m.}$

(iii) Discharge through model (Q_m)

$$\frac{Q_P}{Q_m} = L_r^{2.5}$$

$$\therefore \quad Q_m = \frac{Q_P}{L_r^{2.5}} = \frac{94}{9^{2.5}} = \frac{94}{243} = \mathbf{0.387\ m^3/s.}$$

(iv) Force on the Prototype (F_P)

$$F_r = \frac{F_P}{F_m} = L_r^3$$

$$\therefore \quad F_P = F_m \times L_r^3 = 7500 \times 9^3 = \mathbf{5467500\ N.}$$

***PROBLEM 3**:* In the model test of a spillway the discharge and velocity of flow over the model were 2 m^3/s and 1.5 m/s respectively. Calculate the velocity and discharge over the prototype which is 36 times the model size.

Solution. Given :

Discharge over model, $Q_m = 2\ m^3/s$

Velocity over model, $V_m = 1.5\ m/s$

Linear scale ratio, $L_r = 36.$

For dynamic similarity, Froude model law is used.

$$\frac{V_P}{V_m} = \sqrt{L_r} = \sqrt{36} = 6.0$$

$$\therefore \quad V_P = \text{Velocity over prototype} = V_m \times 6.0 = 1.5 \times 6.0 = \mathbf{9\ m/s.}$$

For discharge,

$$\frac{Q_P}{Q_m} = L_r^{2.5} = (36)^{2.5}.$$

$$\therefore \quad Q_P = Q_m \times (36)^{2.5} = 2 \times 36^{2.5} = \mathbf{15552\ m^3/s.}$$

***PROBLEM 4**:* The characteristics of the spillway are to be studied by means of a geometrically similar model constructed to a scale of 1:10.

(i) If 28.3 cumecs, is the maximum rate of flow in prototype, what will be the corresponding flow in model?

(ii) If 2.4m/sec, 50mm and 3.5 Nm are values of velocity at a point on the spillway, height of hydraulic jump and energy dissipated per second in model, what will be the corresponding velocity height of hydraulic jump and energy dissipation per second in prototype?

Solution. Given :

$$\frac{\text{Linear dimension of model}}{\text{Linear dimension of prototype}} = \frac{1}{10}$$

∴ Scale ratio, $L_r = 10.$

(*i*) Discharge in prototype, $Q_P = 28.3 \text{ m}^3/\text{s}$

Let Q_m = Discharge in model

For discharge

$$\frac{Q_P}{Q_m} = L_r^{2.5}$$

$$\therefore \quad Q_m = \frac{Q_P}{L_r^{2.5}} = \frac{28.3}{10^{2.5}} = \mathbf{0.0895\ m^3/s.}$$

(*ii*) Velocity in the model, $V_m = 2.4$ m/s

Let V_P = Velocity in the prototype

For velocity

$$\frac{V_P}{V_m} = \sqrt{L_r}$$

$$\therefore \quad V_P = V_m \times \sqrt{L_r} = 2.4 \times \sqrt{10} = \mathbf{7.589\ m/s.}$$

(*iii*) Hydraulic jump in model, $H_m = 50$ mm

Let H_P = Hydraulic jump in prototype

Now scale ratio $= \dfrac{H_P}{H_m}$

$$\therefore \quad H_P = H_m \times \text{Scale ratio} = 50 \times 10 = \mathbf{500\ mm.}$$

(*iv*) Energy dissipated/s in model, $E_m = 3.5$ N m/s

Let E_P = Energy dissipated/s in prototype

Now using equation $\dfrac{E_P}{E_m} = L_r^{3.5}$

$$\therefore \quad E_P = E_m \times L_r^{3.5} = 3.5 \times 10^{3.5} = \mathbf{11067.9\ N\ m/s.}$$