Catalog

1.1 FLUID - DEFINITION

Introduction: In general matter can be distinguished by the physical forms known as solid, liquid, and gas. The liquid and gaseous phases are usually combined and given a common name of fluid. Solids differ from fluids on account of their molecular structure (spacing of molecules and ease with which they can move). The intermolecular forces are large in a solid, smaller in a liquid and extremely small in gas.

Fluid mechanics is the study of fluids at rest or in motion. It has traditionally been applied in such area as the design of pumps, compressor, design of dam and canal, design of piping and ducting in chemical plants, the aerodynamics of airplanes and automobiles. In recent years fluid mechanics is truly a 'high-tech' discipline and many exciting areas have been developed like the aerodynamics of multistory buildings, fluid mechanics of atmosphere, sports, and micro fluids.

Definition of Fluid: A *fluid* is a substance which deforms continuously under the action of shearing forces, however small they may be. Conversely, it follows that: If a fluid is at rest, there can be no shearing forces acting and, therefore, all forces in the fluid must be perpendicular to the planes upon which they act.

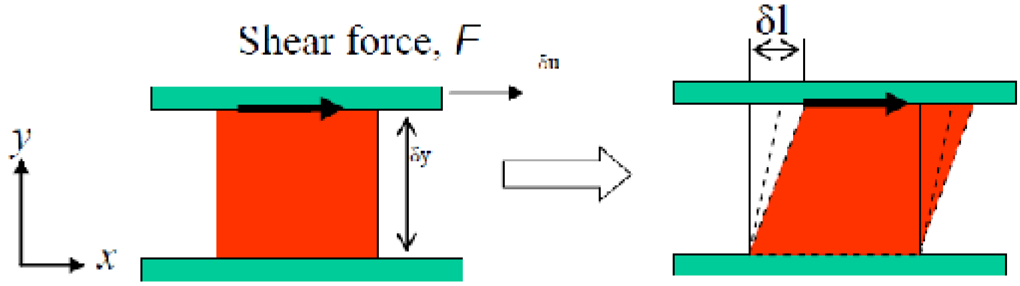


Figure 1.1.1 Deformation of a Solid and a Fluid Exposed to an applied Force

[Source: "https://en.wikiversity.org/wiki/Fluid_Mechanics_for_Mechanical_Engineers/Introduction"*]*

Fluid deforms continuously under the action of a shear force

$$\tau_{yx} = \frac{dF_x}{dA_y} = f(\text{Deformation Rate})$$

Shear stress in a moving fluid:

Although there can be no shear stress in a fluid at rest, shear stresses are developed when the fluid is in motion, if the particles of the fluid move relative to each other so that they have different velocities, causing the original shape of the fluid to become distorted. If, on the other hand, the velocity of the fluid is same at every point, no shear stresses will be produced, since the fluid particles are at rest relative to each other.

Differences between solids and fluids: The differences between the behaviour of solids and fluids under an applied force are as follows:

i. For a solid, the strain is a function of the applied stress, providing that the elastic limit is not exceeded. For a fluid, the rate of strain is proportional to the applied stress.

ii. The strain in a solid is independent of the time over which the force is applied and, if the elastic limit is not exceeded, the deformation disappears when the force is removed. A fluid continues to flow as long as the force is applied and will not recover its original form when the force is removed.

Differences between liquids and gases:

Although liquids and gases both share the common characteristics of fluids, they have many distinctive characteristics of their own. A liquid is difficult to compress and, for many purposes, may be regarded as incompressible. A given mass of liquid occupies a fixed volume, irrespective of the size or shape of its container, and a free surface is formed if the volume of the container is greater than that of the liquid.

A gas is comparatively easy to compress (Fig.1). Changes of volume with pressure are large, cannot normally be neglected and are related to changes of temperature. A given mass of gas has no fixed volume and will expand continuously unless restrained by a containing vessel. It will completely fill any vessel in which it is placed and, therefore, does not form a free surface.

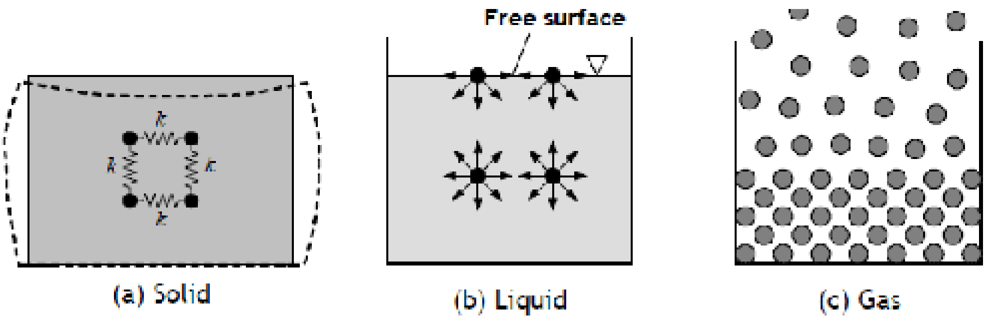


Figure 1.1.2 Comparison of Solid, Liquid and Gas

*[Source: "*https://en.wikiversity.org/wiki/Fluid_Mechanics_for_Mechanical_Engineers/Introduction*"]*

1.2 Systems of Units

The official International System of Units (System International Units). Strong efforts are underway for its universal adoption as the exclusive system for all engineering and science, but older systems, particularly the CGS and FPS engineering gravitational systems are still in use and probably will be around for some time. The chemical engineer finds many physiochemical data given in CGS units; that many calculations are most conveniently made in fps units; and that SI units are increasingly encountered in science and engineering. Thus it becomes necessary to be expert in the use of all three systems.

SI system:

Primary quantities:

Quantity	*Unit*
Mass in Kilogram	kg
Length in Meter	m
Time in Second	s or as sec
Temperature in Kelvin	K
Mole	mol

Derived quantities:

Quantity	*Unit*
Force in Newton (1 N = 1 kg.m/s^2)	N
Pressure in Pascal (1 Pa = 1 N/m^2)	N/m^2
Work, energy in Joule (1 J =1 N.m)	J
Power in Watt (1 W = 1 J/s)	W

CGS Units:

The older centimeter-gram-second (CGS) system has the following units for derived quantities:

Quantity	*Unit*
Force in dyne (1 dyn = 1 g.cm/s^2)	dyn
Work, energy in erg (1 erg = 1 dyn.cm = 1 x 10^{-7} J)	erg
Heat Energy in calorie (1 cal = 4.184 J)	cal

Dimensions: Dimensions of the primary quantities:

Fundamental dimension	***Symbol***
Length	L
Mass	M
Time	t
Temperature	T

Dimensions of derived quantities can be expressed in terms of the fundamental dimensions.

Quantity	***Representative symbol***		***Dimensions***
Angular velocity	ω		t^{-1}
Area	A		L^2
Density	ρ		M/L^3
Force	F		ML/t^2
Kinematic viscosity	ν		L^2/t
Linear velocity	v		L/t

1.2 FLUID PROPERTIES:

1.Density or Mass density(ρ) : Density or mass density of a fluid is defined as the ratio of the mass of a fluid to its volume. Thus mass per unit volume of a fluid is called density.

$$\therefore \quad \rho = \frac{Mass}{Volume}$$

$$\rho = \frac{M}{V} \text{ or } \frac{dM}{dV}$$

The unit of density in S.I. unit is kg/m^3. The value of density for water is 1000kg/m. With the increase in temperature volume of fluid increases and hence mass density decreases in case of fluids as the pressure increases volume decreases and hence mass density increases.

2.Specific weight or weight density (γ): Specific weight or weight density of a fluid isthe ratio between the weight of a fluid to its volume. The weight per unit volume of a fluid is called weight density.

$$\therefore \quad \gamma = \frac{Weight}{Volume} = \frac{W}{V} \text{ or } \frac{dW}{dV}$$

The unit of specific weight in S.I. units is N/m^3. The value of specific weight or weightdensity of water is $9810 N/m^3$.

With increase in temperature volume increases and hence specific weight decreases.

With increases in pressure volume decreases and hence specific weight increases.

Note: Relationship between mass density and weight density:

$$\text{We have } \gamma = \frac{Weight}{Volume}$$

$$\gamma = \frac{mass \ x \ g}{Volume}$$

$$\gamma = \rho \text{ x } g$$

3.Specific Volume ($\forall$): Specific volume of a fluid is defined as the volume of a fluidoccupied by a unit mass or volume per unit mass of a fluid.

$$\therefore \quad \forall = \frac{Volume}{mass} = \frac{V}{M} \; or \; \frac{dV}{dM}$$

As the temperature increases volume increases and hence specific volume increases. As the pressure increases volume decreases and hence specific volume decreases.

4.Specific Gravity(S): Specific gravity is defined as the ratio of the weight density of afluid to the weight density of a standard fluid.

$$S = \frac{\rho_{fluid}}{\rho_{standard\ fluid}}$$

Unit: It is a dimensionless quantity and has no unit.

In case of liquids water at 4°C is considered as standard liquid. $\rho_{water} = 1000\ kg/m^3$

Problem1: Calculate specific weight, mass density, specific volume and specific gravity of a liquid having a volume of $4m^3$ and weighing 29.43 kN. Assume missing data suitably.

$\gamma = ?$

$\rho = ?$

$\forall = ?$

$S = ?$

$V = 4\,m^3$

$W = 29.43\ kN$

$= 29.43 \times 10^3 N$

$$\gamma = \frac{W}{V} = \frac{29.43X10^3}{4}$$

$$\gamma = 7357.58\ N/m^3$$

To find ρ - Method 1:

$W = mg$

$29.43 \times 10^3 = m \times 9.81$

$m = 3000\ kg$

$$\therefore \rho = \frac{m}{v} = \frac{3000}{4}$$

Method 2:

$\gamma = \rho g$

$7357.5 = \rho\, 9.81$

$\rho = 750\ kg/m^3$

$\rho = 750 \ kg/m^3$

$\rho = \frac{M}{V}$

i) $\forall = \frac{V}{M}$

$\forall = \frac{V}{M}$

$= \frac{4}{3000}$

$\forall = \frac{1}{\rho} = \frac{1}{750}$

$\forall = 1.33 \ x \ 10^{-3} m^3 / kg$

$\forall = 1.33X10^{-3} m^3 / kg$

$S = \frac{\gamma}{\gamma_{Standard}}$

$S = \frac{\rho}{\rho_{Standaard}}$

$= \frac{7357.5}{9810}$ or $S = \frac{750}{1000}$

$S = 0.75$

$S = 0.75$

Problem2: Calculate specific weight, density, specific volume and specific gravity and if one liter of Petrol weighs 6.867N.

$\gamma = \frac{W}{V}$

$V = 1 \, Litre$

$= \frac{6.867}{10^{-3}}$

$V = 10^{-3} m^3$

$\gamma = 6867 N / m^3$

$W = 6.867N$

$S = \frac{\gamma}{\gamma_{Standard}}$

$\rho = s \ g$

$= \frac{6867}{9810}$

$6867 = \rho \, x \, 9.81$

$S = 0.7$

$\rho = 700 kg / m^3$

$$\forall = \frac{V}{M}$$

$$= \frac{10^{-3}}{0.7}$$

$$\forall = 1.4x10^{-3} m^3 / kg$$

$$M = 6.867 \div 9.81$$

$$M = 0.7 \text{ kg}$$

***Problem* 3:** Specific gravity of a liquid is 0.7 Find i) Mass density ii) specific weight. Also find the mass and weight of 10 Liters of liquid.

$$S = \frac{\gamma}{\gamma_{Standard}}$$

$$\gamma = \rho g$$

$$S = 0.7$$
$$V = ?$$
$$\rho = ?$$
$$M = ?$$
$$W = ?$$
$$V = 10 \text{ litre}$$
$$= 10x10^{-3} \text{m}^3$$

$$0.7 = \frac{\gamma}{9810}$$

$$6867 = \rho \text{ x } 9.81$$

$$\rho = 700 \text{ kg/m}^3$$

$$\gamma = 6867 N / m^3$$

$$S = \frac{\rho}{\rho_{Standard}}$$

$$0.7 = \frac{\rho}{1000}$$

$$\rho = 700 \text{kg/m}^3$$

$$\rho = \frac{M}{V}$$

$$700 = \frac{M}{10x10^{-3}}$$

$$\rho = \frac{M}{V}$$

$$700 = \frac{M}{10x10^{-3}}$$

$$M = 7 \text{kg}$$

5. Viscosity: Viscosity is the property by virtue of which fluid offers resistance against the flow or shear deformation. In other words, it is the reluctance of the fluid to flow. Viscous force is that force of resistance offered by a layer of fluid for the motion of another layer over it.

In case of liquids, viscosity is due to cohesive force between the molecules of adjacent layers of liquid. In case of gases, molecular activity between adjacent layers is the cause of viscosity.

Newton's law of viscosity:

Let us consider a liquid between the fixed plate and the movable plate at a distance 'Y' apart, 'A' is the contact area (Wetted area) of the movable plate, 'F' is the force required to move the plate with a velocity 'U' According to Newton's law shear stress is proportional to shear strain.

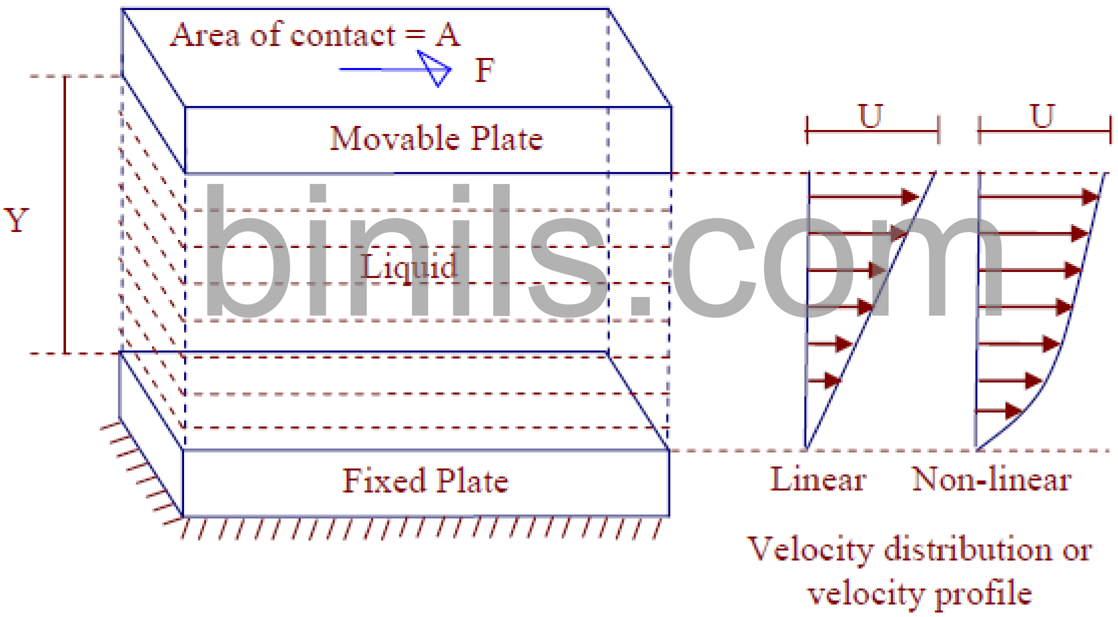


Figure 1.3.1 Definition diagram of Liquid viscosity

*[Source: "*https://en.wikiversity.org/wiki/Fluid_Mechanics_for_Mechanical_Engineers/fluid Properties*"]*

- $F \alpha A$
- $F \alpha \frac{1}{Y}$
- $F \alpha U$

$$\therefore F \alpha \frac{AU}{Y}$$

$$F = \mu . \frac{AU}{Y}$$

'μ' is the constant of proportionality called Dynamic Viscosity or Absolute Viscosity or Coefficient of Viscosity or Viscosity of the fluid.

$$\frac{F}{A} = \mu.\frac{U}{Y} \longrightarrow \therefore \tau = \mu\frac{U}{Y}$$

'τ' is the force required; Per Unit area called 'Shear Stress'. The above equation is called Newton's law of viscosity.

Velocity gradient or rate of shear strain:

It is the difference in velocity per unit distance between any two layers.

If the velocity profile is linear then velocity gradient is given by U/Y . If the velocity profile is non – linear then it is given by du/dy

Unit of force (F): N

- Unit of distance between the twp plates (Y): m
- Unit of velocity (U): m/s
- Unit of velocity gradient : $\frac{U}{Y} = \frac{m/s}{m} = /s = s^{-1}$
- Unit of dynamic viscosity (τ): $\tau = \mu\frac{u}{y}$

$$\mu = \frac{\tau y}{U}$$

$$\Rightarrow \frac{N/m^2 . m}{m/s}$$

$$\mu \Rightarrow \frac{N - sec}{m^2} \quad or \quad \mu \Rightarrow P_a - S$$

NOTE: In CGS system unit of dynamic viscosity is $\frac{dyne . S}{Cm^2}$ and is called poise (P).

If the value of μ is given in poise, multiply it by 0.1 to get it in $\frac{NS}{m^2}$.

1 Centipoises = 10^{-2} Poise.

- ***Effect of Pressure on Viscosity of fluids:***

  Pressure has very little or no effect on the viscosity of fluids.

- ***Effect of Temperature on Viscosity of fluids:***

  ❖ *Effect of temperature on viscosity of liquids***:** Viscosity of liquids is due to cohesive force between the molecules of adjacent layers. As the temperature increases cohesive force decreases and hence viscosity decreases.

- *Effect of temperature on viscosity of gases:* Viscosity of gases is due to molecular activity between adjacent layers. As the temperature increases molecular activity increases and hence viscosity increases.

Kinematics Viscosity: It is the ratio of dynamic viscosity of the fluid to its mass density.

$$\therefore \text{Kinematic Viscosity} = \frac{\mu}{\rho}$$

Unit of Kinematics Viscosity

$$KV \Rightarrow \frac{\mu}{\rho}$$

$$\Rightarrow \frac{NS/m^2}{kg/m^3}$$

$$= \frac{NS}{m^2} x \frac{m^3}{kg}$$

$$F = ma$$

$$= \left(\frac{kg\,m}{s^2}\right) x \frac{s}{m^2} x \frac{m^3}{kg} = m^2/s$$

$$N = Kg.m/s^2$$

$\therefore$ Kinematic Viscosity $= m^2/s$

NOTE: Unit of kinematics Viscosity in CGS system is cm^2/s and is called stoke (S)

If the value of KV is given in stoke, multiply it by 10^{-4} to convert it into m^2/s.

***Problem* 4:** Viscosity of water is 0.01poise. Find its kinematics viscosity if specific gravity is 0.998.

Kinematics viscosity = ?

$S = 0.998$

$$S = \frac{\rho}{\rho_{standrad}}$$

$$0.998 = \frac{\rho}{1000}$$

$$\rho = 998\ kg/m^3$$

$\mu = 0.01P$

$= 0.01x0.1$

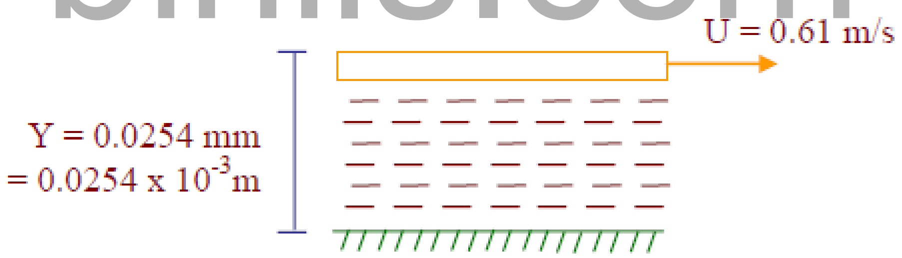
$$\mu = 0.001 \frac{NS}{m^2}$$

$$\therefore \text{Kinmetic Vis cosity} = \frac{\mu}{\rho}$$

$$= \frac{0.001}{998}$$

$$KV = 1 \text{ x } 10^{6} m^2/s$$

***Problem* 5:** A Plate at a distance 0.0254mm from a fixed plate moves at 0.61m/s and requires a force of 1.962N/m² area of plate. Determine dynamic viscosity of liquid between the plates.



$$\tau = 1.962\ N/m^2$$

$\mu = ?$

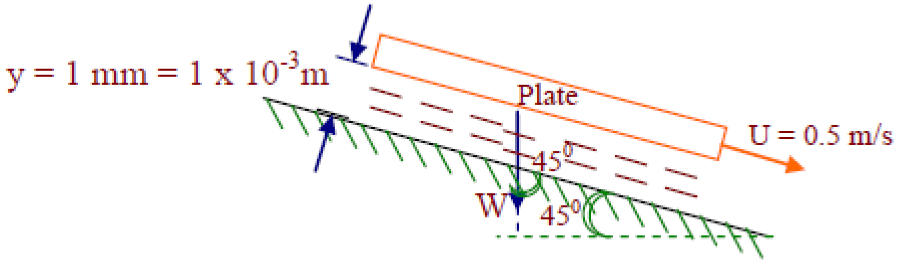
Assuming linear velocity distribution

$$\tau = \mu \frac{U}{Y}$$

$$1.962 = \mu \text{ x } \frac{0.61}{0.0254 \text{ x } 10^{-3}}$$

$$\mu = 8.17 \text{ x } 10^{-5} \frac{NS}{m^2}$$

***Problem* 6 :** A plate having an area of $1m^2$ is dragged down an inclined plane at 45^0 to horizontal with a velocity of 0.5m/s due to its own weight. Three is a cushion of liquid 1mm thick between the inclined plane and the plate. If viscosity of oil is 0.1 PaS find the weight of the plate.



$A = 1m^2$

$U = 0.5 m/s$

$Y = 1 x 10^{-3} m$

$\mu = 0.1 NS/m^2$

$W = ?$

$F = W \times \cos 45^0$

$= W \times 0.707$

$F = 0.707W$

$$\tau = \frac{F}{A}$$

$$\tau = \frac{0.707W}{1}$$

$$\tau = 0.707 W N/m^2$$

Assuming linear velocity distribution,

$$\tau = \mu . \frac{U}{Y}$$

$$0.707W = 0.1 \times \frac{0.5}{1 \times 10^{-3}}$$

$$W = 70.72 N$$

***Problem* 7:** A flat plate is sliding at a constant velocity of 5 m/s on a large horizontal table. A thin layer of oil (of absolute viscosity = 0.40 N-s/m^2) separates the plate from the table. Calculate the thickness of the oil film (mm) to limit the shear stress in the oil layer to 1 kPa.

Given : τ = 1 kPa = 1000 N/m2; U = 5m/s; μ = 0.4 N-s/m^2

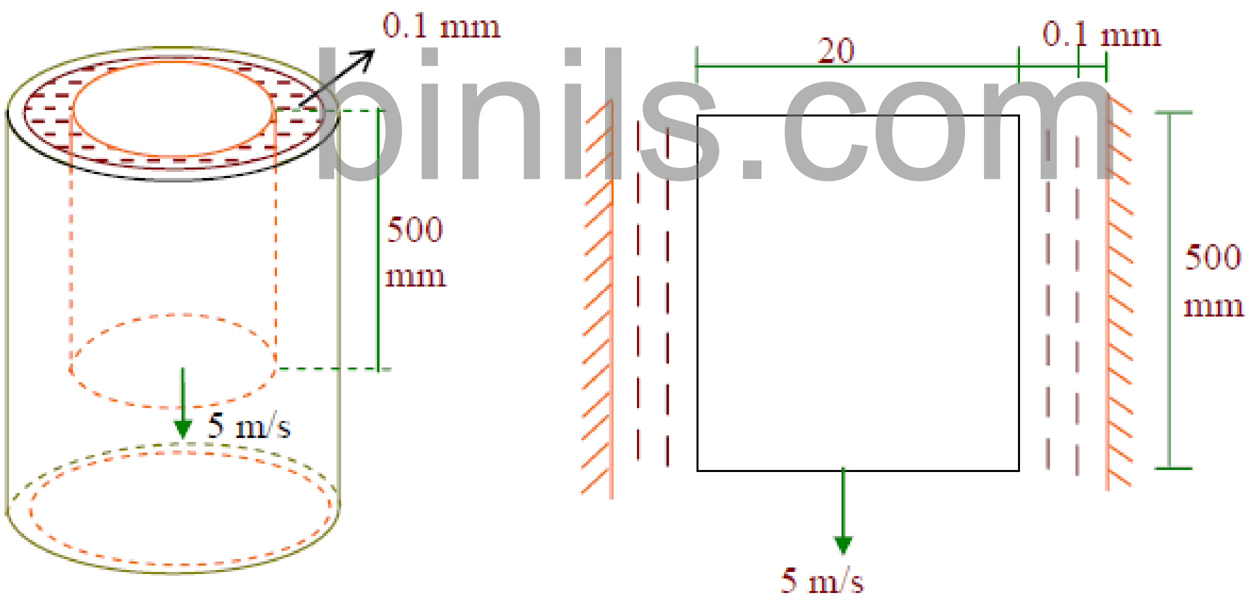
Applying Newton's Viscosity law for the oil film -

$$\tau = \mu \frac{du}{dy} = \mu \frac{U}{y}$$

$$1000 = 0.4 \frac{5}{y}$$

$$y = 2 \times 10^{-3} = 2 \text{ mm}$$

***Problem* 8:** A shaft of ϕ 20mm and mass 15kg slides vertically in a sleeve with a velocity of 5 m/s. The gap between the shaft and the sleeve is 0.1mm and is filled with oil. Calculate the viscosity of oil if the length of the shaft is 500mm.



$D = 20mm = 20x10^{-3}m$

$M = 15\ kg$

$W = 15x\ 9.81$

$W = 147.15N$

$y = 0.1mm$

$y = 0.1\ x\ 10^{-3}mm$

$U = 5m/s$

$F = W$

$F = 147.15N$

$\mu = ?$

$A = \Pi\ D\ L$

$A = \Pi\ x\ 20\ x\ 10^{-3}\ x\ 0.5$

$A = 0.031\ m^2$

$$\tau = \mu.\frac{U}{Y}$$

$$4746.7 = \mu\ x\ \frac{5}{0.1x10^{-3}}$$

$$\mu = 0.095\ \frac{NS}{m^2}$$

$$\tau = \frac{F}{A}$$

$$= \frac{147.15}{0.031}$$

$$\tau = 4746.7N/m^2$$

***Problem* 9 :** If the equation of velocity profile over 2 plate is $V= 2y^{2/3}$ in which 'V' is the velocity in m/s and 'y' is the distance in 'm' . Determine shear stress at (i) y = 0 (ii) y = 75mm. Take $\mu = 8.35P$.

a. at y = 0
b. at y = 75mm
   $= 75\ x\ 10^{-3}m$

$\tau = 8.35P$

$= 8.35 \text{ x } 0.1 \ \frac{NS}{m^2}$

$= 0.835 \ \frac{NS}{m^2}$

$V = 2y^{2/3}$

$\frac{dv}{dy} = 2x\frac{2}{3}y^{2/3-1}$

$= \frac{4}{3}y^{-1/3}$

$at, y = 0, \frac{dv}{dy} = 3\frac{4}{\sqrt[3]{0}} = \infty$

$at, y = 75x10^{-3}m, \frac{dv}{dy} = 3\frac{4}{\sqrt[3]{75x10^{-3}}}$

$\frac{dv}{dy} = 3.16/s$

$\tau = \mu.\frac{dv}{dy}$

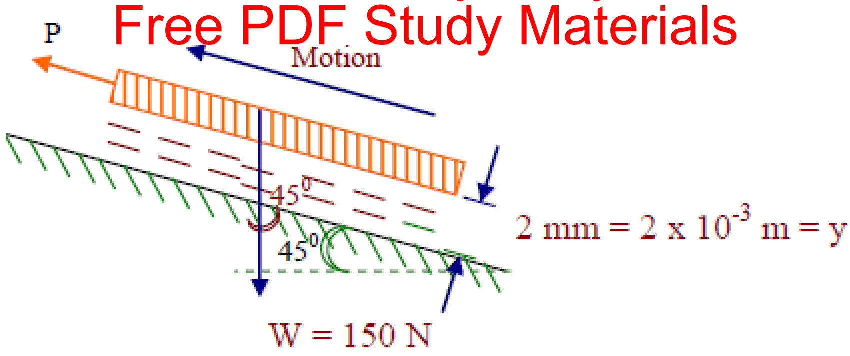
$at, y = 0, \ \tau = 0.835x\infty$

$\tau = \infty$

$at, y = 75x10^{-3}m, \tau = 0.835 \ x \ 3.16$

$\tau = 2.64 \ N/m^2$

***Problem* 10 :** A circular disc of 0.3m dia and weight 50 N is kept on an inclined surface with a slope of 45^0. The space between the disc and the surface is 2 mm and is filled with oil of dynamics viscosity $1N/Sm^2$.What force will be required to pull the disk up the inclined plane with a velocity of 0.5m/s.



D = 0.3m

$$A = \frac{\Pi \; x \; 0.3m^2}{4}$$

$A = 0.07m^2$

W = 50N

$$\mu = 1 \; \frac{NS}{m^2}$$

$y = 2x10^{-3} m$

$U = 0.5 m/s$

$$F = P - 50\cos 45$$

$$F = (P - 35{,}35)$$

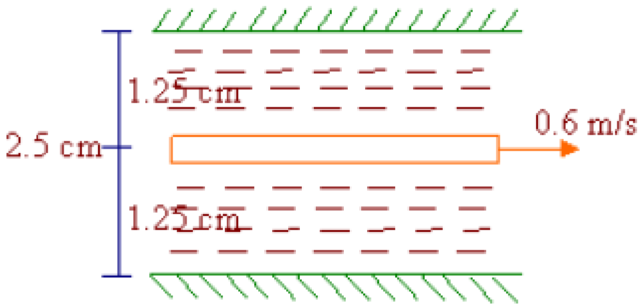
$$\upsilon = \frac{(P - 35.35)}{0.07} N/m^2$$

$$\tau = \mu . \frac{U}{Y}$$

$$\left(\frac{P - 35{,}35}{0.07}\right) = 1x \; \frac{0.5}{2x10^{-3}}$$

$$P = 52.85N$$

***Problem* 10 :** Two large surfaces are 2.5 cm apart. This space is filled with glycerin of absolute viscosity 0.82 NS/m^2. Find what force is required to drag a plate of area 0.5m^2 between the two surfaces at a speed of 0.6m/s. (i) When the plate is equidistant from the surfaces, (ii) when the plate is at 1cm from one of the surfaces.



$$U = \frac{\Pi DN}{60}$$

$$= \frac{\Pi \text{ x } 0.4 \text{ x } 190}{60}$$

$$U = 3.979 \text{ m/s}$$

$$\tau = \mu . \frac{U}{Y}$$

$$= 0.6 \text{ x } \frac{3.979}{1.5 \text{ x } 10^{-3}}$$

$$\tau = 1.592 \text{ x } 10^{3} \text{ N/m}^{2}$$

$$\frac{F}{A} = 1.59 \text{ x } 10^{3}$$

$$F = 1.591 \text{x } 10^{3} \text{ x } 0.11$$

$$F = 175.01 \text{ N}$$

$$T = F \text{ x } R$$

$$= 175.01 \text{x} 0.2$$

$$T = 35 Nm$$

$$P = \frac{2\Pi NT}{60{,}000}$$

$$P = 0.6964 KW$$

$$P = 696.4 W$$

Let F_1 be the force required to overcome viscosity resistance of liquid above the plate and F_2 be the force required to overcome viscous resistance of liquid below the plate. In this case $F_1 = F_2$. Since the liquid is same on either side or the plate is equidistant from the surfaces.

$$\tau_1 = \mu_1 \frac{U}{Y}$$

$$\tau_1 = 0.82 \text{x} \frac{0.6}{0.0125}$$

$$\tau_1 = 39.36 N/m^2$$

$$\frac{F_1}{A} = 39.36$$

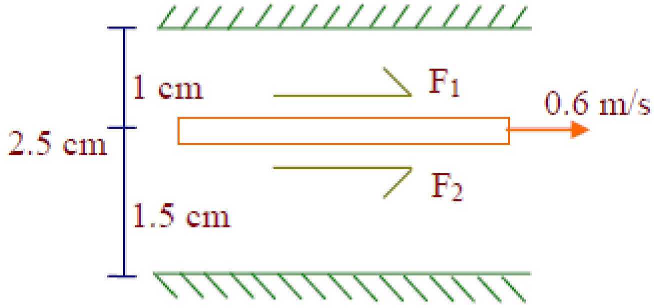
$$F_1 = 19.68N$$

Total force required to drag the plate $=F_1 + F_2$ = 19.68+19.68

F= 39.36N

Case (ii) when the plate is at 1cm from one of the surfaces

Here $F_1 \neq F_2$



F/A=49.2

F_1=49.2x0.5

F_1=24.6N

F_2 /A=32.8

F_2=32.8x0.5

F_2=16.4N

Total Force F = $F_1 + F_2$ = 24.6 + 16.4

F = 41N

6.* Capillarity *:

Capillarity is the phenomena by which liquids will rise or fall in a tube of small diameter dipped in them. Capillarity is due to cohesion adhesion and surface tension of liquids. If adhesion is more than cohesion then there will be capillary rise. If cohesion is greater than adhesion then will be capillary fall or depression. The surface tensile force supports capillary rise or depression.

$$h = \frac{4\sigma \cos\theta}{\gamma D}$$

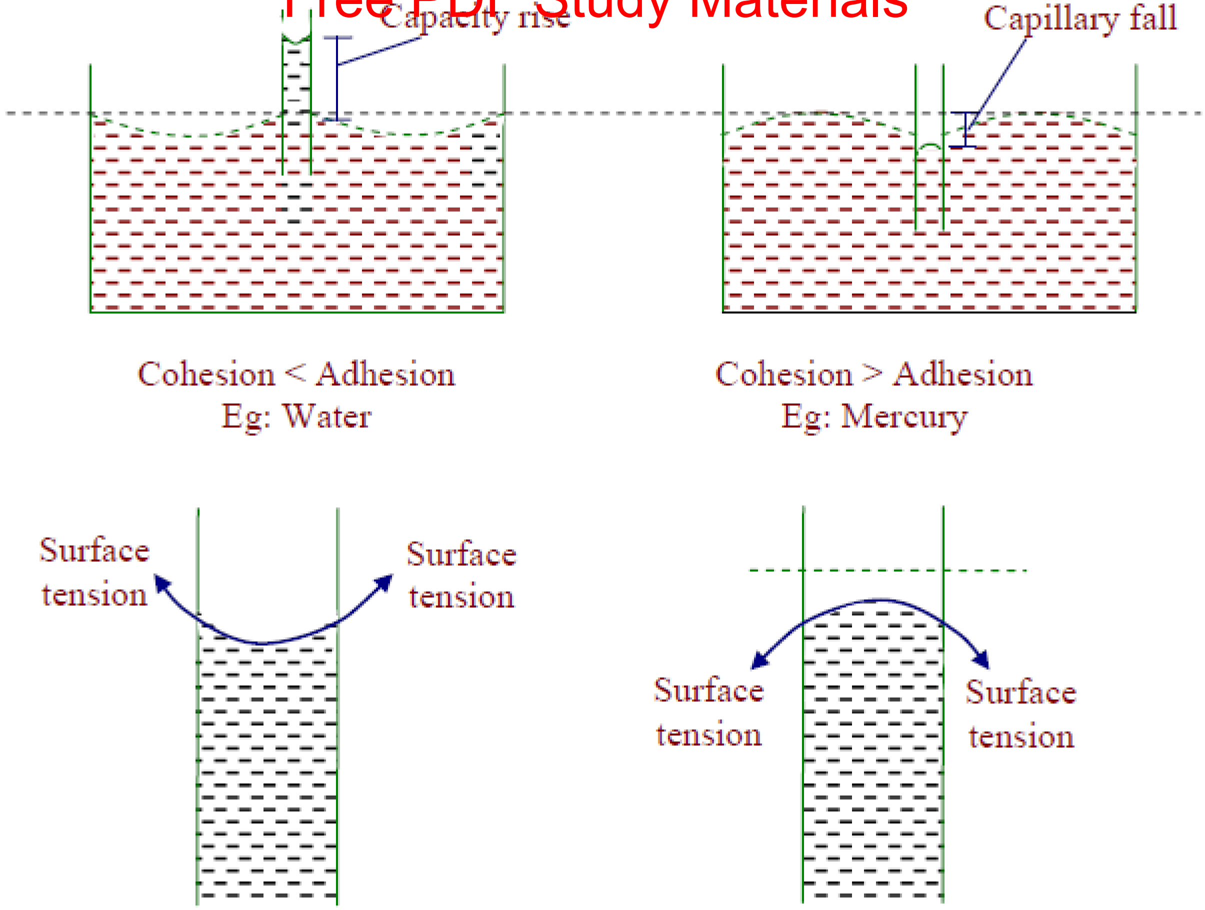


Figure 1.3.2 Capillarity

[Source: "https://en.wikiversity.org/wiki/Fluid_Mechanics_for_Mechanical_Engineers/fluid Properties"*]*

***Problem* 11 :** Capillary tube having an inside diameter 5mm is dipped in water at 20^0. Determine the heat of water which will rise in tube. Take σ=0.0736N/m at 20^0 C.

$$h = \frac{4\sigma\cos\theta}{\gamma D}$$

$$= \frac{4 \text{ x } 0.0736 \text{ x } \cos\theta}{9810 \text{ x } 5 \text{ x } 10^{-3}}$$

$\theta = 0^0$ (assumed)

$\gamma = 9810 N/m^3$

$$h = 6 \text{ x } 10^{-3} m$$

***Problem* 12 :** Calculate capillary rise in a glass tube when immersed in Hg at 20⁰C. Assume σ for Hg at 20⁰C as 0.51N/m. The diameter of the tube is 5mm. θ = 130^0c.

$$S = \frac{\gamma}{\gamma_{Standard}}$$

$$h = \frac{4\sigma\cos\theta}{\gamma D}$$

$$13.6 = \frac{\gamma}{9810}$$

$$h = -1.965 \text{ x} 10^{-3} m$$

$\gamma = 133.416 \times 10^3 N/m^3$

-ve sign indicates capillary depression.

***Problem* 13:** Calculate the capillary effect in millimeters a glass tube of 4mm diameter, when immersed in (a) water (b) mercury. The temperature of the liquid is 20^0 C and the values of the surface tension of water and mercury at 20^0 C in contact with air are 0.073575 and 0.51 N/m respectively. The angle of contact for water is zero that for mercury 130^0. Take specific weight of water as 9790 N / m^3..

Given:

Diameter of tube $\Rightarrow$ d = 4 mm = $4\times10^{-3} m$

Capillary effect (rise or depression) $\Rightarrow h = \dfrac{4\sigma \cos\theta}{p \times g \times d}$

$\sigma =$ Surface tension in kg f/m

$\theta =$ Angle of contact and p = density

Capillary effect for water

$\sigma = 0.073575\ N/m, \quad \theta = 0^0$

$p = 998\ kg/m^3\ @\ 20^0 c$

$$h = \frac{4\times0.73575\times Cos0^0}{998\times9.81\times4\times10^{-3}} = 7.51\times10^{-3} m$$

= 7.51 mm.

Capillary effect for mercury:

$\sigma = 0.51\ N/m, \qquad \theta = 130^0$

$p = sp\ gr \times 1000 = 13.6\times1000 = 13600 kg/m^3$

= - 2.46 mm.

-Ve indicates capillary depression.

7.Surface Tension:

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two two immiscible liquids such that the contact surface behaves like a membrane under tension

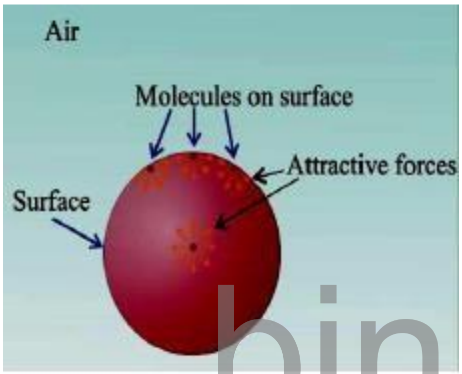
Excess Pressure inside a Water Droplet:

Pressure inside a Liquid droplet: Liquid droplets tend to assume a spherical shape since a sphere has the smallest surface area per unit volume.

The pressure inside a drop of fluid can be calculated using a free-body diagram of a spherical shape of radius R cut in half, as shown in Figure below and the force developed around the edge of the cut sphere is $2\pi R\sigma$. This force must be balance with the difference between the internal pressure pi and the external pressure Δp acting on the circular area of the cut. Thus,

$$2\pi R\sigma = \Delta p \pi R^2$$

$$\Delta p = (p_{internal} - p_{external}) = \frac{2\times\sigma}{R} = \frac{4\times\sigma}{D}$$



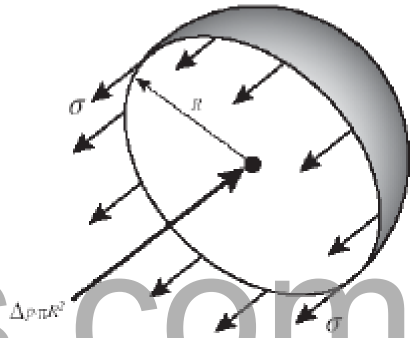


Figure 1.3.3 Surface Tension inside a Water Droplet

*[Source: "*https://en.wikiversity.org/wiki/Fluid_Mechanics_for_Mechanical_Engineers/fluid Properties*"]*

The excess pressure within a Soap bubble:

The fact that air has to be blown into a drop of soap solution to make a bubble should suggest that the pressure within the bubble is greater than that outside. This is in fact the case: this excess pressure creates a force that is just balanced by the inward pull of the soap film of the bubble due to its surface tension.

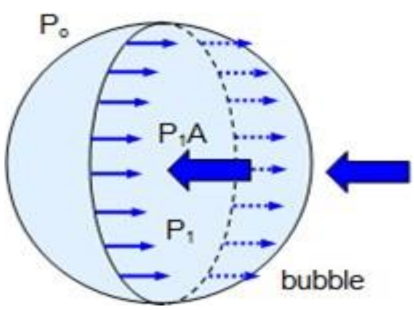


Figure 1.3.4 Surface Tension within a Soap bubble

*[Source: "*https://en.wikiversity.org/wiki/Fluid_Mechanics_for_Mechanical_Engineers/fluid Properties*"]*

Consider a soap bubble of radius r as shown in Figure 1. Let the external pressure be

P_o and the internal pressure P_1. The excess pressure ΔP within the bubble is therefore given by: Excess pressure $\Delta P = (P_1 - P_0)$

Consider the left-hand half of the bubble. The force acting from right to left due to the internal excess pressure can be shown to be PA, where A is the area of a section through the centre of the bubble. If the bubble is in equilibrium this force is balanced by a force due to surface tension acting from left to right. This force is $2 \times 2\pi r\sigma$ (the factor of 2 is necessary because the soap film has two sides) where 'σ' is the coefficient of surface tension of the soap film. Therefore

$2 \times 2\pi r\sigma = \Delta pA = \Delta p\pi r^2$ giving:

Excess pressure in a soap bubble (P) = $4\sigma/r$

8.Compressibility:

Compressibility is the reciprocal of the bulk modulus of elasticity, Kwhich is defined as the ratio of compressive stress to volumetric strain.

Bulk Modulus (K):

When a solid or fluid (liquid or gas) is subjected to a uniform pressure all over the surface, such that the shape remains the same, then there is a change in volume.

Then the ratio of normal stress to the volumetric strain within the elastic limits is called as Bulk modulus. This is denoted by K.

$$K = \frac{\text{Normal stress}}{\text{volumetric strain}}$$

$$K = \frac{F/A}{-\Delta V/V} = \frac{-pV}{\Delta V}$$

where p = increase in pressure; V = original volume; ΔV = change in volume

The negative sign shows that with increase in pressure p, the volume decreases by ΔV i.e. if p is positive, ΔV is negative. The reciprocal of bulk modulus is called compressibility.

$$C = Compressibility = \frac{1}{K} = \frac{\Delta V}{pV}$$

S.I. unit of compressibility is $N^{-1}m^2$ and C.G.S. unit is $dyne^{-1}\ cm^2$.

***Problem* 13:** The surface tension of water in contact with air at 20°C is 0.0725 N/m. The pressure inside a droplet of water is to be 0.02 N/cm^2 greater than the outside pressure. Calculate the diameter of the droplet of water.

Given: Surface Tension of Water σ =0.0725 N/m, Δp= 0.02 N/cm^2 = 0.02×10^{-4}N/m^2

Let 'D' be the diameter of jet

$$\Delta p = \frac{4\sigma}{D}$$

$$0.02 \times 10^{-4} = \frac{4 \times 0.0725}{D}$$

$$D = 0.00145\text{m} = 1.45\text{mm}$$

***Problem* 14:** Find the surface tension in a soap bubble of 40mm diameter when inside pressure is 2.5 N/m^2 above the atmosphere.

Given: D = 40mm = 0.04 m, Δp = 2.5 N/m^2

Let 'σ' be the surface tension of soap bubble

$$\Delta p = \frac{8\sigma}{D}$$

$$2.5 = \frac{4\sigma}{0.04}$$

$$\sigma = 0.0125 \text{ N/m}$$

9.Vapour Pressure

Vapour pressure is a measure of the tendency of a material to change into the gaseous or vapour state, and it increases with temperature. The temperature at which the vapour pressure at the surface of a liquid becomes equal to the pressure exerted by the surroundings is called the boiling point of the liquid.

Vapor pressure is important to fluid flows because, in general, pressure in a flow decreases as velocity increases. This can lead to ***cavitation***, which is generally destructive and undesirable. In particular, at high speeds the local pressure of a liquid sometimes drops below the vapor pressure of the liquid. In such a case, ***cavitation*** occurs. In other words, a "cavity" or bubble of vapor appears because the liquid vaporizes or boils at the location where the pressure dips below the local vapor pressure.

Cavitation is not desirable for several reasons. First, it causes noise (as the cavitation bubbles collapse when they migrate into regions of higher pressure). Second, it can lead to inefficiencies and reduction of heat transfer in pumps and turbines (turbo machines). Finally, the collapse of these cavitation bubbles causes pitting and corrosion of blades and other surfaces nearby. The left figure below shows a cavitating propeller in a water tunnel, and the right figure shows cavitation damage on a blade.

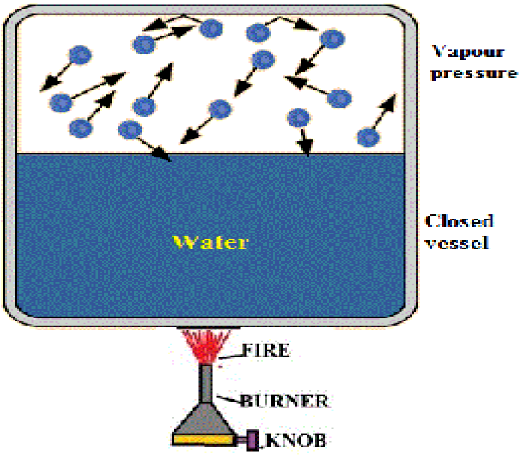


Figure 1.3.5 Vapour Pressure

[Source: "https://www.hkdivedi.com/2017/12/vapour-pressure-and-cavitation.html"]

1.4 Fluid Statics: Concept of Fluid Static Pressure

Fluid is a state of matter which exhibits the property of flow. When a certain mass of fluids is held in static equilibrium by confining it within solid boundaries, it exerts force along direction perpendicular to the boundary in contact. This force is called fluid pressure (compression).

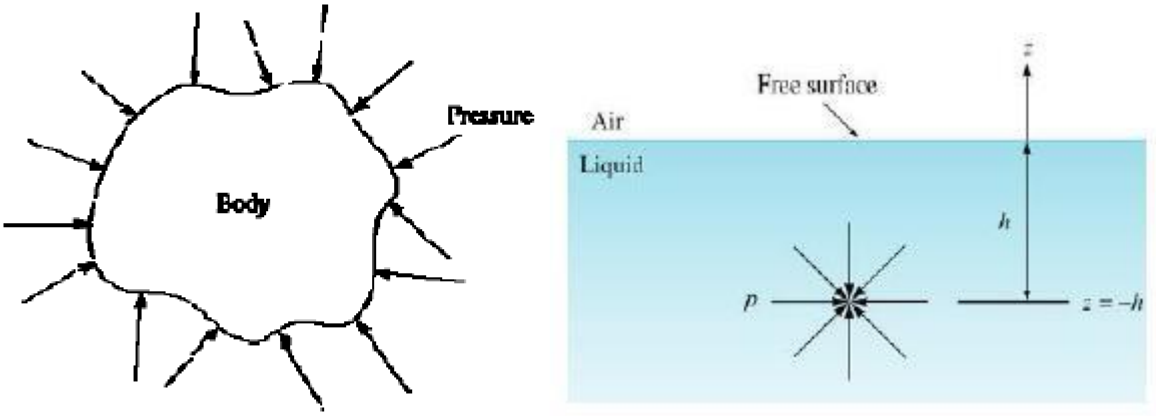


Figure 1.4.1 Definition of Pressure

[Source: "https://en.wikiversity.org/wiki/Fluid_Mechanics_for_Mechanical_Engineers/fluid Statics"]

In fluids, gases and liquids, we speak of pressure; in solids this is normal stress. For a fluid at rest, the pressure at a given point is the same in all directions. Differences or gradients in pressure drive a fluid flow, especially in ducts and pipes.

Definition of Pressure: Pressure is one of the basic properties of all fluids. Pressure (p) is the force (F) exerted on or by the fluid on a unit of surface area (A).

Mathematically expressed:

$$p = \frac{F}{A} \quad \left(\frac{N}{m^2}\right)$$

The basic unit of pressure is Pascal (Pa). When a fluid exerts a force of 1 N over an area of $1m^2$, the pressure equals one Pascal, i.e., 1 Pa = 1 N/m^2.Pascal is a very small unit, so that for typical power plant application, we use larger units:

Units: 1 kilopascal (kPa) = 10^3 Pa, and

1 megapascal (MPa) = 10^6 Pa = 10^3 kPa.

Pressure at a Point and Pascal's Law:

Pascal's Principle: Pressure extends uniformly in all directions in a fluid.

By considering the equilibrium of a small triangular wedge of fluid extracted from a static fluid body, one can show (Fig.1.4.2) that for *any* wedge angle θ, the pressures on the three faces of the wedge are equal in magnitude.

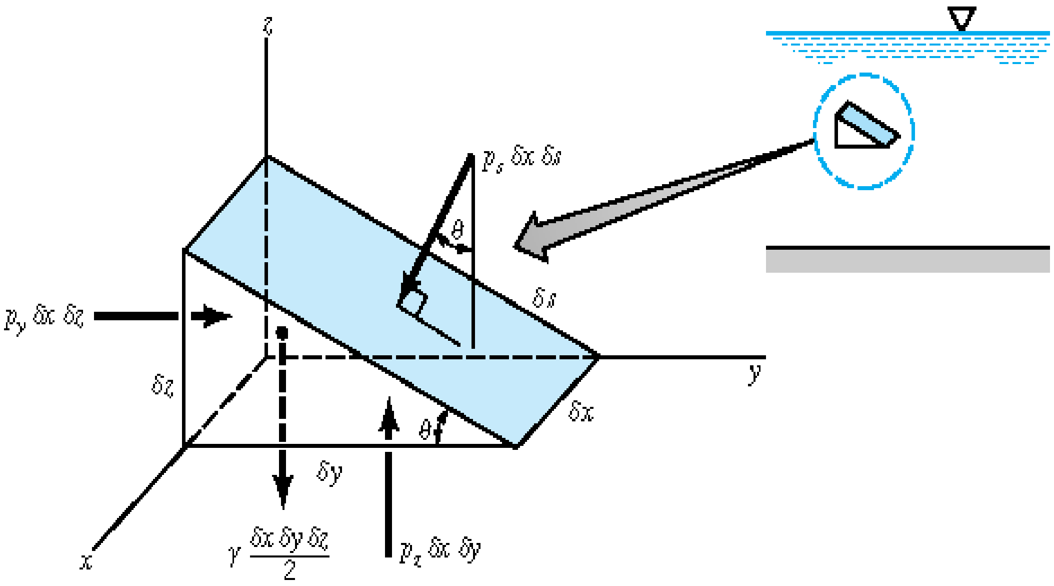


Figure 1.4.2 Pascal's Law

[Source: "https://en.wikiversity.org/wiki/Fluid_Mechanics_for_Mechanical_Engineers/fluid Statics"]

Independent of $p_x = p_y = p_z$ independent of 'θ'

Pressure at a point has the same magnitude in all directions, and is called **isotropic.** This result is known as **Pascal's law**.

Pascal's Law: In any closed, static fluid system, a pressure change at any one point is transmitted undiminished throughout the system.

Application of Pascal's Law:

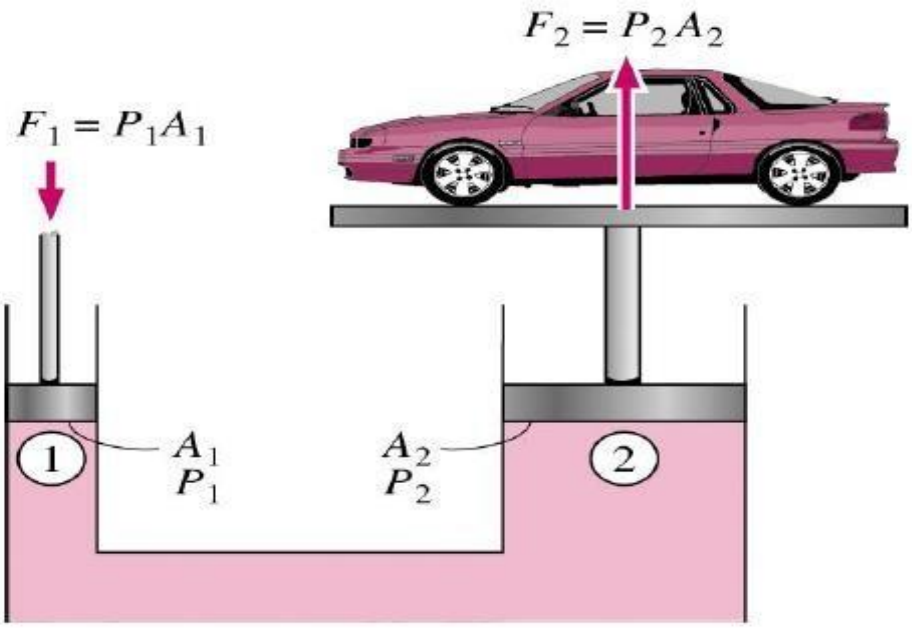


Figure 1.4.2 ***Application of Pascal's Law***

[Source: "https://en.wikiversity.org/wiki/Fluid_Mechanics_for_Mechanical_Engineers/fluid Statics"]

- Pressure applied to a confined fluid increases the pressure throughout by the same amount.
- In picture, pistons are at same height:

$$P_1 = P_2 \rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \rightarrow \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

- Ratio A_2/A_1 is called ideal mechanical advantage.

1.5 PRESSURE MEASUREMENTS BY MANOMETERS

MANOMETER

A manometer is an instrument that uses a column of liquid to measure pressure, although the term is currently often used to mean any pressure instrument.

Two types of manometer, such as

1. Simple manometer
2. Differential manometer

The U type manometer, which is considered as a primary pressure standard, derives pressure utilizing the following equation:

$$P = P2 - P1 = h\omega \, \rho g$$

Where:

P = Differential pressure

P1 = Pressure applied to the low pressure connection

P2 = Pressure applied to the high pressure connection

$h\omega$ = is the height differential of the liquid columns between the two legs of the manometer

ρ = mass density of the fluid within the columns

g = acceleration of gravity

SIMPLE MANOMETER

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are:

1.Piezometer

2.U tube manometer

3.Single Column manometer

PIEZOMETER

A piezometer is either a device used to measure liquid pressure in a system by measuring the height to which a column of the liquid rises against gravity, or a device which measures the pressure (more precisely, the piezometric head) of groundwater at a specific point. A piezometer is designed to measure static pressures, and thus differs from a pitot tube by not being pointed into the fluid flow.

It is the simplest form of manometer used for measuring gauge pressures. One end of this manometer is connected to the point where pressure is to be measured and other end is open to the atmosphere as shown in Fig. The rise of liquid gives the pressure head at that point. If at a point A, the height of liquid say water is h in piezometer tube, then pressure at A

$$= \rho \times g \times h \ \frac{\text{N}}{\text{m}^2}.$$

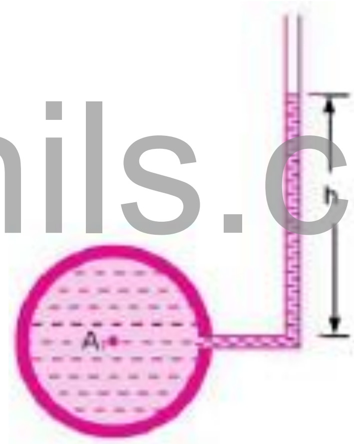


Figure 1.5.1 Piezometer

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 43]

U TUBE MANOMETER

Manometers are devices in which columns of a suitable liquid are used to measure the difference in pressure between two points or between a certain point and the atmosphere.

Manometer is needed for measuring large gauge pressures. It is basically the modified form of the piezometric tube.

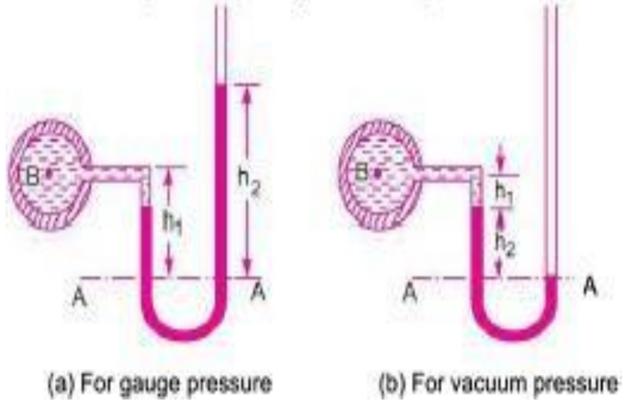


Figure 1.5.2 U Tube Manometer

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 43]

(*a*) **For Gauge Pressure.** Let B is the point at which pressure is to be measured, whose value is p. The datum line is A-A.

Let h_1 = Height of light liquid above the datum line

h_2 = Height of heavy liquid above the datum line

S_1 = Sp. gr. of light liquid

ρ_1 = Density of light liquid = $1000 \times S_1$

S_2 = Sp. gr. of heavy liquid

ρ_2 = Density of heavy liquid = $1000 \times S_2$

As the pressure is the same for the horizontal surface. Hence pressure above the horizontal datum line A-A in the left column and in the right column of U-tube manometer should be same.

Pressure above A-A in the left column $= p + \rho_1 \times g \times h_1$

Pressure above A-A in the right column $= \rho_2 \times g \times h_2$

Hence equating the two pressures $p + \rho_1 g h_1 = \rho_2 g h_2$

$\therefore \quad p = (\rho_2 g h_2 - \rho_1 \times g \times h_1).$

(*b*) **For Vacuum Pressure.** For measuring vacuum pressure, the level of the heavy liquid in the manometer will be as shown in Fig. 2.9 (*b*). Then

Pressure above A-A in the left column $= \rho_2 g h_2 + \rho_1 g h_1 + p$

Pressure head in the right column above A-A $= 0$

$\therefore \quad \rho_2 g h_2 + \rho_1 g h_1 + p = 0$

$\therefore \quad p = -(\rho_2 g h_2 + \rho_1 g h_1).$

Single Column Manometer

Single column manometer is a modified form of a U-tube manometer in which one side is a large reservoir and the other side is a small tube, open to the atmosphere.

There are two types of single column manometer:

1. Vertical single column manometer.
2. Inclined single column manometer.

1. Vertical single column Manometer

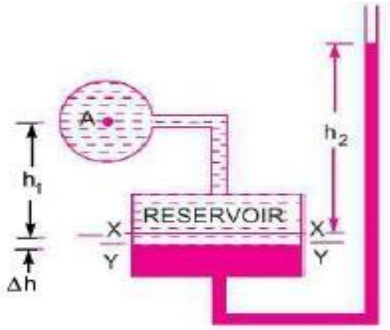


Figure 1.5.3 Vertical single column Manometer
[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 49]

Fall of heavy liquid in reservoir will cause a rise of heavy liquid level in the right limb.

$\therefore$ $$A \times \Delta h = a \times h_2$$

$\therefore$ $$\Delta h = \frac{a \times h_2}{A} \qquad \ldots(i)$$

$$p_A = \frac{a \times h_2}{A}[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

A>> a

Then: $p_A = h_2\rho_2 g - h_1\rho_1 g$

2. Inclined single column Manometer

This manometer is more sensitive. Due to the inclination the distance moved by the heavy liquid in the right limb will be more.

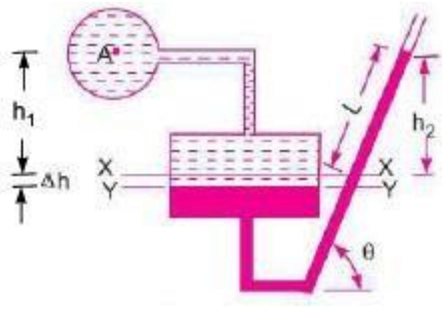


Figure 1.5.4 Inclined single column Manometer

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 49]

Let L = Length of heavy liquid moved in right limb from X-X

θ = Inclination of right limb with horizontal

h_2 = Vertical rise of heavy liquid in right limb from X-$X = L \times \sin\theta$

From the eq.

$$p_A = h_2\rho_2 g - h_1\rho_1 g$$

By substituting the value of h2, We get:

$$p_A = \sin\theta \times \rho_2 g - h_1\rho_1 g.$$

DIFFERENTIAL MANOMETER

Differential Manometers are devices used for measuring the difference of pressure between two points in a pipe or in two different pipes . A differential manometer consists of a U-tube, containing a heavy liquid, whose two ends are connected to the points, which difference of pressure is to be measure.

Most commonly types of differential manometers are:

1.U-tube differential manometer.

2.Inverted U-tube differential manometer

1.U-tube differential Manometer

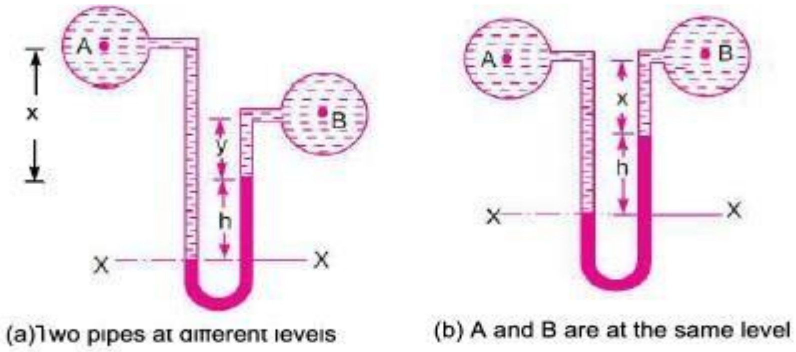


Figure 1.5.5 U-tube differential Manometer

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 51]

In Fig. (*b*), the two points *A* and *B* are at the same level and contains the same liquid of density ρ_1. Then

Pressure above *X-X* in right limb $= \rho_g \times g \times h + \rho_1 \times g \times x + p_B$

Pressure above *X-X* in left limb $= \rho_1 \times g \times (h + x) + p_A$

Equating the two pressure

$$\rho_g \times g \times h + \rho_1 gx + p_B = \rho_1 \times g \times (h + x) + p_A$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_1 gx - \rho_1 g(h + x)$$

$$= g \times h(\rho_g - \rho_1).$$

Taking datum line at *X-X*.

Pressure above *X-X* in the left limb $= \rho_1 g(h + x) + p_A$

where p_A = pressure at *A*.

Pressure above *X-X* in the right limb $= \rho_g \times g \times h + \rho_2 \times g \times y + p_B$

where p_B = Pressure at *B*.

Equating the two pressure, we have

$$\rho_1 g(h + x) + p_A = \rho_g \times g \times h + \rho_2 gy + p_B$$

$$\therefore \quad p_A - p_B = \rho_g \times g \times h + \rho_2 gy - \rho_1 g(h + x)$$

$$= h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$$

$\therefore$ Difference of pressure at *A* and $B = h \times g(\rho_g - \rho_1) + \rho_2 gy - \rho_1 gx$

2.Inverted U-tube differential Manometer

It consists of inverted U-tube, containing a light liquid. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measuring differences of low pressures.

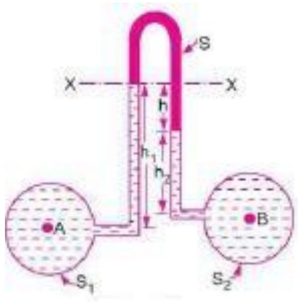


Figure 1.5.6 Inverted U-tube differential Manometer

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 53]

Let the pressure at A is more than the pressure at B.

Let h_1 = Height of liquid in left limb below the datum line

h_2 = Height of liquid in right limb

h = Difference of light liquid

ρ_1 = Density of liquid at A

ρ_2 = Density of liquid at B

ρ_s = Density of light liquid

p_A = Pressure at A

p_B = Pressure at B.

Taking X-X as datum line. Then pressure in the left limb below X-X

$$= p_A - \rho_1 \times g \times h_1.$$

Pressure in the right limb below X-X

$$= p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

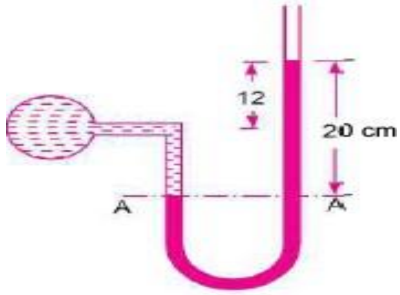
Equating the two pressure

$$p_A - \rho_1 \times g \times h_1 = p_B - \rho_2 \times g \times h_2 - \rho_s \times g \times h$$

or

$$p_A - p_B = \rho_1 \times g \times h_1 - \rho_2 \times g \times h_2 - \rho_s \times g \times h.$$

Problem1: The right limb of a simple U – tube manometer containing mercury is open to the atmosphere, while the left limb is connected to a pipe in which a fluid of sp.gr.0.9 is flowing. The centre of pipe is 12cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe, if the difference of mercury level in the two limbs is 20 cm.



Given, Sp.gr. of liquid S1= 0.9

Density of fluid $\rho 1$ = S1 × 1000 = 0.9 × 1000 =900 kg/ m 3

Sp.gr. of mercury S2 = 13.6

Density of mercury $\rho 2$ =13.6× 1000 = 13600 kg/m3

Difference of mercury level h2 = 20cm = 0.2m

Height of the fluid from A – A h1 = 20 – 12 = 8cm = 0.08 m

Let 'P' be the pressure of fluid in pipe

Equating pressure at A – A, we get p + ρ1gh1 = ρ2gh2

p + 900 × 9.81 × 0.08 = 13.6 × 1000 × 9.81 × 0.2

p = 13.6 × 1000 × 9.81 × 0.2 – 900 × 9.81 × 0.08

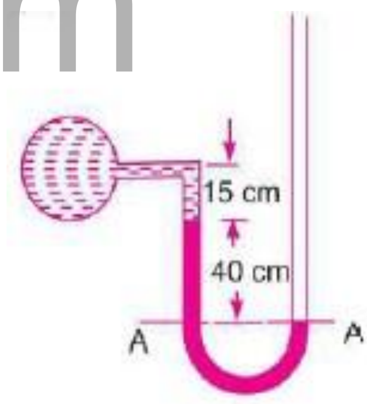
p = 26683 – 706

p = 25977 N/m^2

p = 2.597 N/cm^2

Pressure of fluid = 2.597 N/ cm2

Problem2: A simple U – tube manometer containing mercury is connected to a pipe in which a fluid of sp.gr. 0.8 And having vacuum pressure is flowing. The other end of the manometer is open to atmosphere. Find the vacuum pressure in pipe, if the difference of mercury level in the two limbs is 40cm. and the height of the fluid in the left tube from the centre of pipe is 15cm below.



Given,

Sp.gr of fluid S1 = 0.8

Sp.gr. of mercury S2 = 13.6

Density of the fluid = S1 × 1000 = 0.8 × 1000 = 800

Density of mercury = 13.6 × 1000

Difference of mercury level h2 = 40cm = 0.4m

Height of the liquid in the left limb = 15cm =0.15m

Let the pressure in the pipe = p

Equating pressures above datum line A—A

ρ2gh2 + ρ1gh1 + P = 0

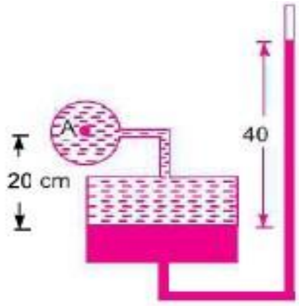
P = - [ρ2gh2 + ρ1gh1] = - [13.6 ×1000 × 9.81 × 0.4 + 800 × 9.81 × 0.15]

= 53366.4 + 1177.2 = -54543.6 N/m^2

P = - 5.454 N/cm^2

Problem 3: A single column manometer is connected to the pipe containing liquid of sp.gr.0.9. Find the pressure in the pipe if the area of the reservoir is 100 times the area of the tube of manometer. sp.gr. of mercury is 13.6. Height of the liquid from the centre of pipe is 20cm and difference in level of mercury is 40cm.

Given,



Sp.gr. of liquid in pipe S1 = 0.9

Density $\rho 1$= 900 kg/ m 3

Sp.gr. of heavy liquid S2 = 13.6

Density $\rho 2$= 13600

$$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$$

Height of the liquid h1 = 20cm = 0.2m

Rise of mercury in the right limb h2 = 40cm = 0.4m

$$p_A = \frac{a}{A} h_2[\rho_2 g - \rho_1 g] + h_2\rho_2 g - h_1\rho_1 g$$

$$= \frac{1}{100} \times 0.4[13.6 \times 1000 \times 9.81 - 900 \times 9.81] + 0.4 \times 13.6 \times 1000 \times 9.81 - 0.2 \times 900 \times 9.81$$

$$= \frac{0.4}{100} [133416 - 8829] + 53366.4 - 1765.8$$

$$= 533.664 + 53366.4 - 1765.8 \text{ N/m}^2 = 52134 \text{ N/m}^2 = \mathbf{5.21 \text{ N/cm}^2. \text{ Ans.}}$$

Pressure in pipe A= 5.21 N/ cm^2

Problem 4: A pipe contains an oil of sp.gr.0.9. A differential manometer is connected at the two points A and B shows a difference in mercury level at 15cm. find the difference of pressure at the two points.

Given:

Sp.gr. of oil S1 = 0.9: density $\rho 1$ = 0.9 x 1000 = 900 kg/ m^3

Difference of level in the mercury h = 15cm = 0.15 m

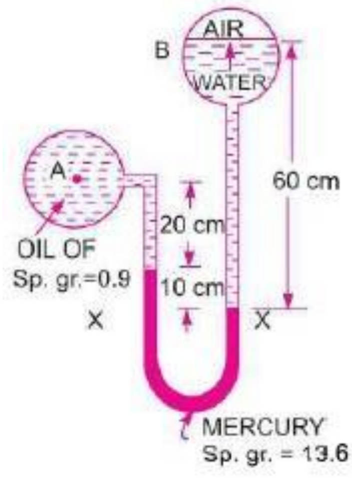
Sp.gr. of mercury = 13.6, Density = 13.6 × 1000 = 13600 kg/m3

The difference of pressure pA – pB = g × h × (ρg - $\rho 1$)

= 9.81 x 0.15 (13600 – 900)

pA – pB = 18688 N/ m^2

Problem 5: A differential manometer is connected at two points A and B .At B air pressure is 9.81 N/cm2 . Find absolute pressure at A.



Given:

Density of air = 0.9 × 1000 = 900 kg/m^3

Density of mercury = 13.6 × 103 kg/ m^3.

Let pressure at A is pA

Taking datum as X – X

Pressure above X – X in the right limb

= 1000 × 9.81 × 0.6 + pB = 5886 + 98100 = 103986

Pressure above X – X in the left limb

= 13.6 × 103 × 9.81 × 0.1 +0900 × 9.81 × 0.2 + pA

= 13341.6 +1765.8 +pA

Equating the two pressures heads

103986 = 13341.6 + 1765.8 + pA

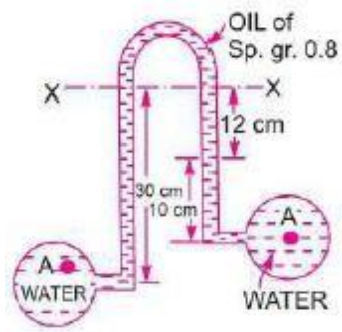
= 15107.4 + pA

pA = 103986 – 15107.4

= 88878.6 N/m2

pA = 8.887 N/cm

Problem 6: Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp.gr. 0.8 is connected. The pressure head in the pipe A is 2m of water. Find the pressure in the pipe B for the manometer readings shown in fig.



Given:

Pressure head at $A = \frac{p_A}{\rho g} = 2$ m of water

$\therefore \quad p_A = \rho \times g \times 2 = 1000 \times 9.81 \times 2 = 19620 \text{ N/m}^2$

Pressure below X – X in the left limb

$= pA - \rho 1gh1$

$= 19620 - 1000 \times 9.81 \times 0.3$

$= 16677 \text{ N/m}^2$

Pressure below X – X in the right limb

$= pB - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$

$= pB - 981 - 941.76 = pB - 1922.76$

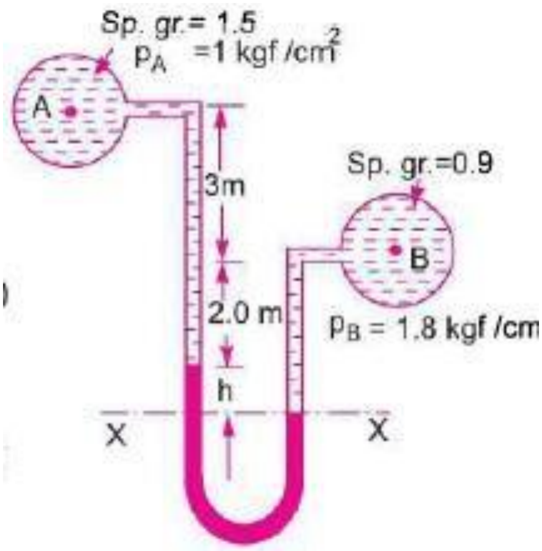
Equating the two pressures, we get,

$16677 = pB - 1922.76$

$pB = 16677 + 1922.76$

$pB = 18599.76 \text{ N/m}^2$

Problem 7: A different manometer is connected at two points A and B of two pipes. The pipe A contains liquid of sp.gr. = 1.5 while pipe B contains liquid of sp.gr. = 0.9. The pressures at A and B are 1 kgf/cm^2 and 1.80 Kg f/cm^2 respectively. Find the difference in mercury level in the differential manometer.



Sp.gr. of liquid at A S1 = 1.5

Sp.gr. of liquid at B S2 == 0.9

Pressure at A pA= 1 kgf/c m^2 = 1 × 104 × kg/m2 = 1 × 10^4 × 9.81N/m^2

Pressure at B pB = 1.8 kgf/cm^2 = 1.8 × 104 × 9.81 N/m^2 [1kgf = 9.81 N]

Density of mercury = 13.6 × 1000 kg/m^3

Taking X – X as datum line

Pressure above X – X in left limb

= 13.6 × 1000 × 9.81 × h + 1500 × 9.81(2+3) + (9.81 x 10^4)

Pressure above X – X in the right limb = 900 × 9.81(h + 2) + 1.8 × 9.81 × 10^4

Equating the two pressures, we get

13.6 × 1000 × 9.81h + 1500 × 9.81 × 5 + 9.81 × 10^4 = 900 × 9.81(h + 2) + 1.8 × 9.81 × 10^4

Dividing both sides by 1000 × 9.81

13.6 h + 7.5 +10 = 0.9(h+2) + 18

(13.6 – 0.9) h = 1.8 + 18 – 17.5 = 19.8 – 17.5 =2.3

h = 2.3 / 12.7 = 0.181m

h = 18.1 cm

1.6 FORCES ON PLANES

Total Pressure and Centre of Pressure

Total Pressure: It is defined as the force exerted by static fluid on a surface (either plane or curved) when the fluid comes in contact with the surface. This force is always at right angle (or normal) to the surface.

Centre of Pressure: It is defined as the point of application of the total pressure on the surface. Now we shall discuss the total pressure exerted by a liquid on the immersed surface. The immersed surfaces may be:

1. Horizontal plane surface
2. Vertical plane surface
3. Inclined plane surface
4. Curved surface

Derivation of total pressure

In order to determine the total pressure, we will consider the object in terms of small strips as displayed here in following figure. We will determine the force acting on small strip and then we will integrate the forces on small strips for calculating the total pressure or hydrostatic force on object.

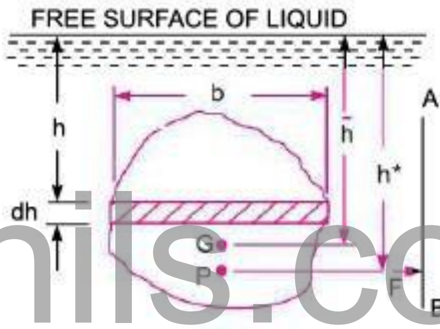


Figure 1.6.1 Vertical plane Immersed surface

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 70]

Let us consider the small strip of thickness dh, width b and at a depth of h from free surface of liquid as displayed here in above figure.

Intensity of pressure on small strip, dp = ρgh

Area of strip, dA = b x dh

Total pressure force on small strip, dF = dP x dA

Total pressure force on small strip, dF = ρgh x b x dh

Total pressure force on whole surface, F = Integration of dF

$$F = \int dF = \int \rho g h \times b \times dh = \rho g \int b \times h \times dh$$

But $\int b \times h \times dh = \int h \times dA$

= Moment of surface area about the free surface of liquid

= Area of surface × Distance of C.G. from free surface

$= A \times \bar{h}$

$\therefore \quad F = \rho g A \bar{h}$

Where,

ρ = Density of liquid (Kg/m^3)

g = Acceleration due to gravity (m/s^2)
A = Area of surface (m^2)
ħ = Height of C.G from free surface of liquid (m)

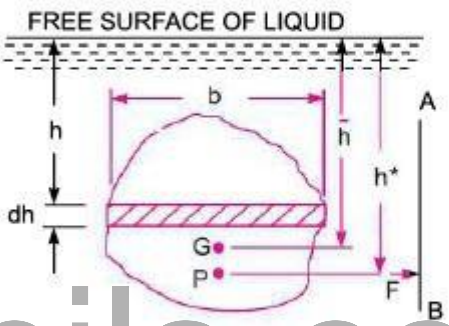
Unit of total pressure

As total pressure is basically a hydrostatic force and therefore total pressure will be measured in terms of N or KN.

Centre of pressure

Centre of pressure is basically defined as a single point through which or at which total pressure or total hydrostatic force will act.

Let us consider that we have one tank filled with liquid e.g. water. Let us consider that there is one object of arbitrary shape immersed inside the water as displayed here in following figure.



Let us consider G is the centre of gravity and P is the centre of pressure. ħ is the height of C.G from free surface of liquid and h* is the height of centre of pressure from free surface of liquid.

Derivation of Centre of Pressure

In order to determine the centre of pressure, we will consider the object in terms of small strips as displayed here in above figure.We will use the concept of "principle of moments" to determine the centre of pressure.

According to the principle of moments, moment of the resultant force about an axis will be equal to the sum of the moments of components about the same axis.

As we have shown above in figure, total hydrostatic force F is applied at centre of pressure P which is at height of h* from the free surface of liquid.

Therefore, let us determine the moment of resultant force F about the free surface of liquid and it will be determined as F x h*.

As we have considered here the object in terms of small strips as displayed here in above figure and hence we will determine the moment of force dF acting on small strip about the free surface of liquid.

$$\text{Moment of force } dF = dF \times h$$

$$\text{Moment of force } dF = \rho\, g\, h \times b\, dh \times h$$

Let us sum of all moments of such small forces about the free surface of liquid and it will be written as mentioned here.

$$= \int \rho gh \times b \times dh \times h = \rho g \int b \times h \times hdh$$

$$= \rho g \int bh^2 \, dh = \rho g \int h^2 \, dA \qquad (\because \quad bdh = dA)$$

But $\int h^2 dA = \int bh^2 dh$

= Moment of Inertia of the surface about free surface of liquid

$= I_0$

$\therefore$ Sum of moments about free surface

$= \rho g I_0$

$$F \times h^* = \rho g I_0$$

But $F = \rho g A \bar{h}$

$$\therefore \quad \rho g A \bar{h} \times h^* = \rho g I_0$$

or $$h^* = \frac{\rho g I_0}{\rho g A \bar{h}} = \frac{I_0}{A \bar{h}}$$

By the theorem of parallel axis, we have

$$I_0 = I_G + A \times \bar{h}^2$$

where I_G = Moment of Inertia of area about an axis passing through the C.G. of the area and parallel to the free surface of liquid.

Substituting I_0 in equation (3.4), we get

$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}} = \frac{I_G}{A\bar{h}} + \bar{h}$$

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$$

Total Pressure and Centre of Pressure for Inclined Plane Surface Immersed in a Liquid

Centre of pressure for inclined plane surface submerged in liquid will be given by

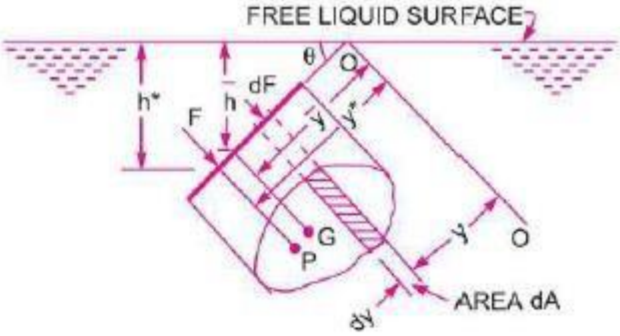


Figure 1.6.2 Inclined Immersed surface

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 86]

Let us consider that we have following data from above figure.
A = Total area of inclined surface
ħ = Height of centre of gravity of inclined area from free surface
h* = Distance of centre of pressure from free surface of the liquid
θ = Angle made by the surface of inclined plane with free surface of the liquid

Total pressure which is basically defined as the hydrostatic force applied by a static fluid on a plane or curved surface when fluid will come in contact with the surfaces.

Total pressure for inclined plane surface submerged in liquid will be given by following formula as mentioned here.

Total pressure = ρ g A ħ

Centre of pressure is basically defined as a single point through which or at which total pressure or total hydrostatic force will act.

Centre of pressure for inclined plane surface submerged in liquid will be given by following formula as mentioned here.

Let us consider a curved surface AB sub-merged in a static liquid as displayed here in following figure.

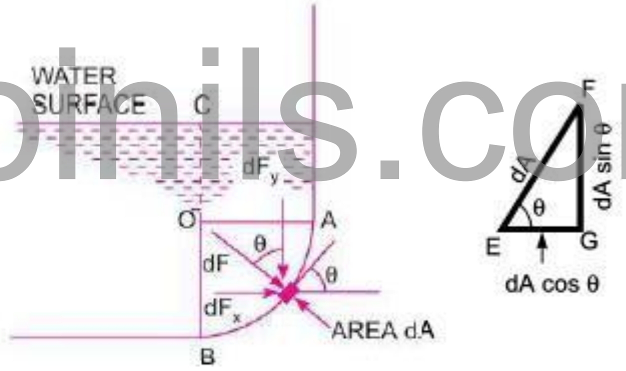


Figure 1.6.3 Curved surface sub-merged in a static liquid

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 98]

Let us consider one small strip area dA at a depth of h from free surface of liquid. We have following data from above figure.

A = Total area of curved surface
ρ = Density of the liquid
g = Acceleration due to gravity

Pressure intensity on small area dA = ρ g h

Hydrostatic force on small area dA will be given by following formula as mentioned here.

$$dF = \rho\ g\ h \times dA$$

Direction of this hydrostatic force will be normal to the curved surface and will vary from point to point. Therefore, in order to secure the value of total hydrostatic force we will not integrate the above equation.

We will secure the value or expression for total hydrostatic force on curved surface by resolving the force dF in its two components or we can say that dF force will be resolved in X direction i.e. dFx and in Y direction i.e. dFy.

$$dFx = dF \sin\theta = \rho\ g\ h \times dA \sin\theta$$
$$dFy = dF \cos\theta = \rho\ g\ h \times dA \cos\theta$$

Total force in X- direction and in Y- direction will be given as mentioned here.

$$F_x = \int dF_x = \int \rho g h dA \sin\theta = \rho g \int h dA \sin\theta$$

$$F_y = \int dF_y = \int \rho g h dA \cos\theta = \rho g \int h dA \cos\theta$$

Let us analyze the above equation

FG will be dA Sin θ or vertical projection of area dA. Therefore, the expression for Fx will be total pressure force on the projected area of the curved surface on the vertical plane.

Fx = Total pressure force on the projected area of the curved surface on the vertical plane

EG will be dA Cos θ or horizontal projection of dA. Therefore, the expression for Fy will be the weight of the liquid contained between the curved surface extended up to free surface of liquid.

Fy = Weight of the liquid contained between the curved surface extended up to free surface of liquid

Then total force on the curved surface is

$$F = \sqrt{F_x^2 + F_y^2}$$

where F_x = Horizontal force on curved surface and is equal to total pressure force on the projected area of the curved surface on the vertical plane,

$= \rho g A \bar{h}$

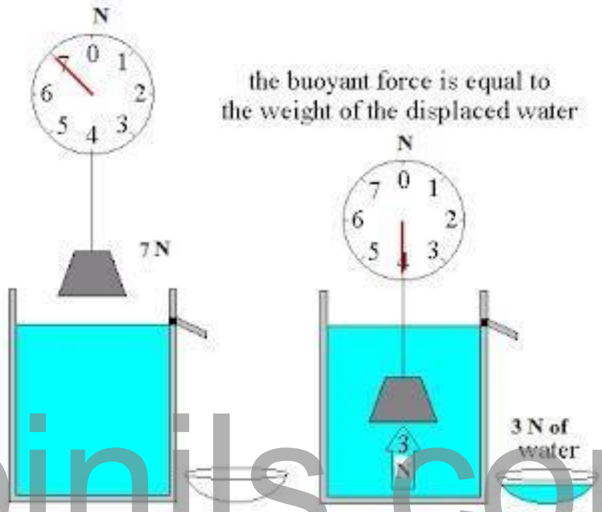
and F_y = Vertical force on submerged curved surface and is equal to the weight of liquid actually or imaginary supported by the curved surface.

1.7 BUOYANCY AND FLOATATION

Buoyancy or buoyancy force

When a body is immersed in fluid, an upward force is exerted by the fluid on the body. This force will be equal to the weight of the fluid displaced by the body and this force will be termed as force of buoyancy or buoyancy.

Let us consider we have one container filled with water as displayed here in following figure. We have one object of weight 7 N. Let us think that we are now immersing the object in to the liquid i.e. water.



Once object will be immersed in the water, some amount of water will be displaced by the object and one upward force will be applied over the object by the water.

Weight of the displaced water will be equal to this upward force which will be exerted by the water on the object. As we can see from above figure that, water of weight 3N is displaced here and one upward force of 3N is exerted by the water over the object.

Conclusion for buoyancy force

Buoyancy force is the force which will be exerted on the object by the surrounding fluid. When one object will be immersed in the water, object will push the water and water will push back the object with as much force as it can.

Force of buoyancy = Weight of the displaced fluid

Force of buoyancy = Weight of the object in air – Weight of the object in given water

Positive buoyancy

Force of buoyancy will be greater than the weight of the object. Hence, object will float and this case will be termed as positive buoyancy.

Neutral buoyancy

Force of buoyancy will be equal to the weight of the object. Hence, object will be suspended in the fluid and this case will be termed as neutral buoyancy.

Negative buoyancy

Force of buoyancy will be less than the weight of the object. Hence, object will be sunk and this case will be termed as negative buoyancy.

Centre of buoyancy

As we know that when a body is immersed in fluid, an upward force is exerted by the fluid on the body. This force will be equal to the weight of the fluid displaced by the body and this force will be termed as force of buoyancy or buoyancy.

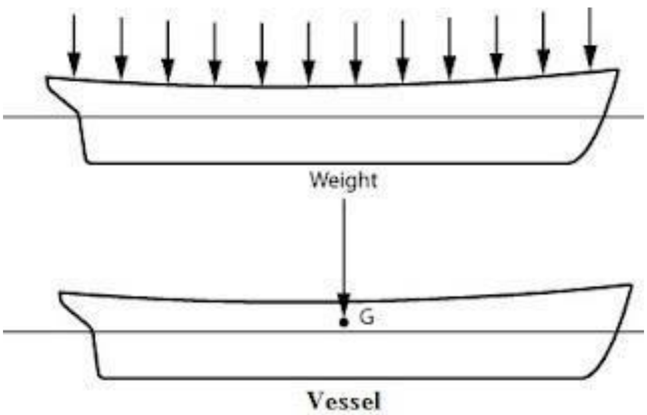
Buoyancy force will act through the centre of gravity of the displaced fluid and that point i.e. centre of gravity of the displaced fluid will be termed as centre of buoyancy.

Therefore we can define the term centre of buoyancy as the point through which the force of buoyancy is supposed to act.

Centre of buoyancy = Centre of gravity of the displaced fluid = Centre of gravity of the portion of the body immersed in the liquid

Let us explain the term centre of buoyancy

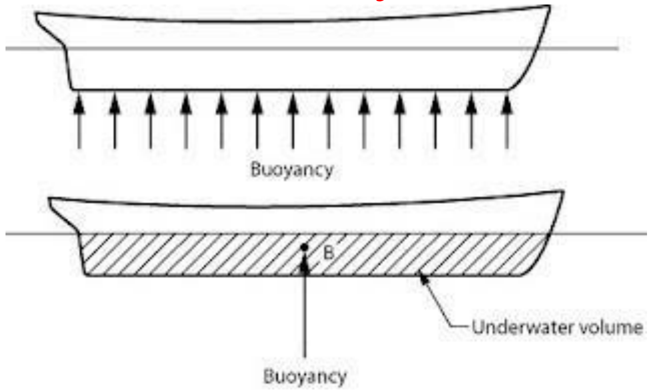
Let us consider one vessel as displayed here in following figure. Weight of vessel will be distributed throughout the length of vessel and will act downward over the entire structure of vessel.



But, what do we consider?

We consider that complete weight of the vessel will act downward vertically through one point and that point will be termed as the centre of gravity of that vessel.

In similar way, buoyancy force will be supposed to act vertically in upward direction through a single point and that point will be termed as centre of buoyancy.



Meta-centre

Meta-centre is basically defined as the point about which a body in stable equilibrium will start to oscillate when body will be displaced by an angular displacement.

We can also define the meta-centre as the point of intersection of the axis of body passing through the centre of gravity and original centre of buoyancy and a vertical line passing through the centre of buoyancy of the body in tilted position.

Let us consider a body which is floating in the liquid. Let us assume that body is in equilibrium condition. Let us think that G is the centre of gravity of the body and B is the centre of buoyancy of the body when body is in equilibrium condition.

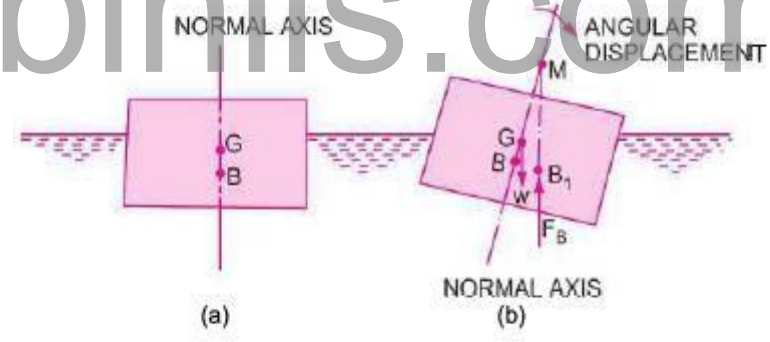


Figure 1.7.1 Meta-centre

[Source: "Fluid Mechanics and Hydraulics Machines" by Dr.R.K.Bansal, Page: 136]

In equilibrium situation, centre of gravity G and centre of buoyancy B will lie on same axis which is displayed here in above figure with a vertical line.

Let us assume that we have given an angular displacement to the body in clockwise direction as displayed here in above figure.

Centre of buoyancy will be shifted now towards right side from neutral axis and let us assume that it is now B1.

Line of action of buoyancy force passing through this new position will intersect the normal axis passing through the centre of gravity and centre of buoyancy in original position of the body at a point M as displayed here in above figure. Where, M is the meta-centre.

Meta-centric height

Meta-centric height is basically defined as the distance between the meta-centre of the floating body and the centre of gravity of the body.

Therefore, MG in above figure will be termed as meta-centric height.