ST5103- Theory of Elasticity and Plasticity

13 Marks Question Bank

Part-B

Unit- I

1. The three stress components at a point are given by $\begin{bmatrix} 10 & 5 & 6 \\ 5 & 8 & 10 \\ 6 & 10 & 6 \end{bmatrix}$ MPa calculate the principal stresses and principal planes?
2. Derive the compatibility equations for plane stress problem and plane strain problem in cartesian coordinate considering the body forces. Also show that when the body forces are constant or zero, the compatibility equations for plane stress problem and plane strain problem reduces to the same?
3. Using Fourier integral method, determine the solution of biharmonic equation in Cartesian coordinates?
4. Following unit elongations were measured with a rectangular strain rosette $\epsilon_0 = 3 \times 10^{-4}, \epsilon_{45} = -4 \times 10^{-4}\ and\ \epsilon_{90} = 5 \times 10^{-4}$ Determine the principal strains and their directions?

5. For the stress tensor given below, determine the principal stresses and the direction cosines associated with the normal to the surfaces of each principal stress. $\begin{bmatrix} 15 & 10 & -10 \\ 10 & 10 & 0 \\ -10 & 0 & 40 \end{bmatrix}$
6. Derive the Elastic Stress – Strain relationship by the understanding of Hook's law for isotropic and homogeneous materials.
7. Explain about the reduction of elastic constants for homogeneous and isotropic materials.
8. Discuss the state of stress at a point. Explain the following basic equations in cartesian and polar coordinates. i)equation of equilibrium and ii) strain displacement relations.
9. Explain the identification of soil medium in beams on elastic foundation by using wrinkler's elastic model.
10. The state of strain at a point ∑x=0.001, ∑y=-0.003, ∑z=0.002, γ_{xy}=0.001, γ_{yz}=0.0005, γ_{xz}=0.002. Assess the strain invariants and the principal strains.
11. Investigate the equation for Stress transformation law in 3-D Cartesian co-ordinates.
12. Illustrate the differential equation of equilibrium in 3-D rectangular co-ordinates.

13. Illustrate the Navier's equations of equilibrium in terms of displacement.
14. The state of stress at a point with respect to the xyz coordinate system is given by stress matrix $\begin{bmatrix} 15 & 10 & -10 \\ 10 & 10 & 0 \\ -10 & 0 & 40 \end{bmatrix}$kN/m2. Determine the normal stress and the magnitude and direction of the shear stress on a surface intersecting the point is parallel to the plane given by the equations 4x-y+3z=11.
15. The state of stress at a point is given by σ_x = 100, σ_y = 200, σ_z = - 100, τ_{xy} = 200, τ_{yz} = 100, τ_{xz} = 300 kpa. Compose a) The stress in variants b) The principal stresses c). The direction cosines of the principal planes.
16. The displacement field in a homogeneous isotropic elastic body is given by u=k{($3x^2z$+60x)i+($5z^2$+10xy);+($6z^2$+2xyz)k}, where k=1x10^{-6}mm, if E=2x10^5N/mm^2, r=0.25. Evaluate the stress components at a point P(5,10,-15) mm.
17. The state of strain at a point is given by ∑x=0.001, ∑y=-0.003, ∑z=0.002, γxy=0.001, γyz=0.0005, γxz=0.002 Determine the strain invariants and the principal strains.
18. Compose the compatibility equation in 3-D Cartesian co-ordinates.
19. Consider an isotropic material that is subjected to uniform stress. Show that the elastic constants are only two by generalized Hooke‟s law.
20. The principal strains at a point are given by ε1= 2x10-3, ε2 = -3x10-3, ε3= -4x10-3. Calculate the octahedral normal and shearing strains.

Unit- II

1. Discuss the use of polynomials in the solution of structural problems?
2. Show that the Airy's stress function $\emptyset = A\left(xy^3 - \left(\frac{3}{4}\right)xyh^2\right)$ represents the stress distribution in a cantilever beam loaded at the free end with load P. find the value of A, if $r_{xy} = 0 \; at \; y = \pm h/2$ where b and h are width and depth respectively of the cantilever beam cross section?
3. The following stress function is proposed for a long cantilever carrying a point load at the free end. Determine the stress components and verify the same.
4. Show that *Ψ* = C (Constant) solves the torsion problem of a solid circular shaft using warping function approach. Evaluate the maximum shear stress and torsional moment, in terms of torsional rigidity, and verify the results are in agreement with those given by the strength of materials approach.
5. State the plain stress and plain strain. Discuss the plain stress and plain strain for two dimensional problems with illustration.

6. Explain the stress concentration in stressed plate with circular hole and elliptical hole.
7. Describe the deflection equation for the bending of a cantilever loaded (point load) at the end in terms of Cartesian coordinates.
8. Describe the deflection equation for the bending of a cantilever loaded (UDL) at the end in terms of Cartesian coordinates
9. Describe the deflection equation for bending a simply supported beam uniformly loaded over the entire span in terms of Cartesian coordinates.
10. Show that the following Airy's stress functions and examine the stress distribution represented by them: a) $\phi=Ax^2+By^2$, b) $\phi=Ax^3$, c)³ $\phi=A(x^4-3x^2y^2)$
11. Show that the Airy‟s stress function $\phi=A(xy^3-3/4\ xyh^2)$ represents stress distribution in a cantilever beam loaded at the free end with load P, examine the value of A if $\tau xy = 0$ at $y = \pm h/2$ where b and h are width and depth respectively of the cantilever. Show that the following function is a stress function and illustrate what problem it solves when applied to the region bounded in $y = \pm c$, $x = 0$ on positive x-direction $\Phi = [q/8c^3 \{x^2 (y^3 -c^2 y-x^3)-1/5(y^3 (y^2 -2c))\}]$
12. Predict the stress function for a cantilever beam having narrow c/s of unit width and depth 2C, subjected to a concentrated load applied at the free end. Discuss the stress function & compare these stresses with those derived by SOM approach.

13. Investigate what problem of plane stress solved by given stress function, $\Phi=-F/d^3 \{xy^2 (3d-2y)\}$ apply at the bounded region of $y = d$; $y = 0$ towards +ve direction
14. Using an equiangular strain rosette, the following strains were measured at a point in a material. 600 e 0 = micrometers/ms, 200 e 60 = - micrometers/m, 300 e 120 = micrometers/m Calculate the magnitudes and directions of principal strain.
15. Determine the stress fields that arise from the following stress functions (a) $\varphi = cy^2$ (b) $\varphi = Ax^2 + Bxy + Cy^2$ (c) $\varphi = Ax^2 + Bx^2 y + Cxy^2 + Dy^3$
16. A steel turbine rotor of 750mm outer diameter, 150mm inner diameter and 50mm thickness has 100 blades 150mm long, each weighing 4N. It is shrink-fitted on a rigid shaft. Compose the initial shrinkage allowance on the inner diameter of the rotor so that it just loosens on the shaft at 3000 rev/min. Take E = 200 GPa, v = 0.3. The density of shaft and rotor is 7500 kg/m³.
17. Assess the two-dimensional biharmonic equation in terms of Cartesian Coordinates.
18. Predict the two-dimensional biharmonic equation in terms of polar coordinates.
19. A cantilever beam of rectangular cross section 5cm wide and 6cm thick is 1m in length. It carries a load of 5kN at the free end. Determine the stresses in the cantilever at mid length.

20. A thin square plate whose sides are parallel to x and y axes has the following stress distribution σx = Ay, σy = Ax and some shear stress where A is a constant. Find the suitable stress function and the nature of shear stress which can be associated with the given normal stresses.

Unit- III

1. A 300mm steel I beam with flanges and web 12.5mm thick is subjected to a torque of 4kNm. Find the maximum shear stress and angle of twist per unit length. Assume G = 100GPa.
2. An I section with flanges 50mm × 5mm and web 140mm × 3mm is subjected to a twisting moment of 200Nm. Find the maximum shearing stress and twist per unit length. Assume G=80Gpa. If the I section is stiffened by welding two steel plates of size 140mm × 5mm, find the stress due to the same torque.
3. A shaft is of elliptical cross section having semi major axis 50mm and semi minor axis 25mm. It is subjected to a torque of 1000Nm. Determine the maximum shear stress developed in the shaft.
4. Explain in detail membrane analogy used in solving torsion problems.
5. A thin-walled closed tube of non-circular section is subjected to a torque T. Derive the expression for shear stress and angle of twist.
6. Discuss the design method analysis of torsion on thin walled open and closed section.
7. Write short notes on the following i) Analogy by St. Venant's Approach ii) Analogy by Prandit's Approach.
8. Describe the torsion equation of a hollow cylinder.
9. Describe the torsion equation of thin-walled hollow rectangular section.
10. Predict the torsion equation of thin-walled closed rectangular section.
11. Derive the equations for torsion of an elliptical cross section bar section by analogous methods.
12. Solve the expression for torsion and angle of twist of an equilateral triangular section by analogous methods.
13. Investigate the torque equation of a prismatic bar subjected to twist T, according to st.Venant's theory.
14. Investigate that in the torsion effect of thin-walled tubes, the ratio of rate of twists approaches to unity.
15. Assess the torsion equation for hollow bars.

16. Outline the conformal mapping to an epitrochoid section and derive the torsional rigidity.
17. A square shaft rotating at 250 rpm transmits torque to a crane which is designed to lift maximum load of 150 kN at a speed of 10m/min. If the efficiency of crane gearing is 65%, predict the size of the shaft for the maximum permissible shear stress of 35MPa. Also predict the angle of twist of the shaft for a length of 3m. Take G = 100 GPa
18. Derive the expression for shear stresses of a bar with elliptical cross section subjected to a torque of "T" and compare the same with hexagon of side 'a'.
19. Illustrate the concept of membrane analogy with case study
20. Investigate the equation for torque by prandlt's stress function approach.

Unit- IV

1. What are the two types of elastic foundations? Explain them briefly.
2. Derive the governing differential equation for the elastic line of a beam resting on an elastic foundation.
3. The foundation for a machine base comprises of standard I beams of overall depth 75mm and 8m long supported on coil springs spaced at 120N/mm. The machine transmits a concentrated load of 10KN acting at the mid-point of the beam. Estimate the maximum deflection and the bending stress in the beam, assuming the modulus of elasticity of the materials of the beam is $70 kN/mm^2$ and second moment of area of the beam is $1\times10^6 mm^4$.
4. Illustrate the Rayleigh Ritz method of analysis in the application to beams and columns.
5. State and prove energy theorems and also explain the applications of theorem to beams and columns.
6. Describe the differential equation for the elastic line of beam resting on an elastic foundation.
7. Describe the expression for deflection, shear force and bending moment equation for an infinite beam loaded with concentrated load.
8. Describe the expression for deflection, shear force and bending moment equation for an infinite beam loaded with UDL.
9. Predict the expression for deflection, shear force and bending moment equation for an Semi-infinite beam loaded with concentrated load.
10. Predict the expression for deflection, shear force and bending moment equation for an Semi-infinite beam loaded with UDL.
11. Solve the deflection of an sinusoidally loaded infinite beam.

12. Investigate the expression for deflection, slope, shear force and bending moment for an semi-infinite beam loaded with free ends.
13. Investigate the expression for deflection, slope, shear force and bending moment for an semi-infinite beam loaded with hinged ends.
14. Assess the expression for deflection, slope, shear force and bending moment for an infinite beam loaded with both the ends fixed.
15. An aluminum alloy I-beam of depth 100mm, Ix=2.45x10^6 mm^4 , E= 72Gpa has a length = 7m, and is supported by 8 springs (k=100N/mm) spaced at a distance l-1m c/c along the beam. A load P=15kN is applied at the centre of the beam over on the springs. Compose the deflection of the beam under the load, the maximum bending moment and maximum bending stress in the beam.
16. Predict the expression for deflection, shear force and bending moment equation for an Semi-infinite beam loaded with combination of loadings.
17. Solve the differential equation for the elastic line of beam of finite length (free ends) resting on an elastic foundation.
18. Derive the expression for deflection of a simply supported beam with μdl over the entire span by Rayleigh-Ritz Method.

19. A semi-infinite beam with free ends is resting on an elastic foundation. The beam is 6cm wide and 8cm thick. It carries a uniformly distributed load of 6kN/m over a length of 50cm at one end. Determine the maximum deflection and the stresses in the beam. Assume E =2 X 10^5 N/mm2, μ = 0.30 and modulus of elastic foundation as 63 N/mm^2.
20. Show that a longitudinal element of a thin cylindrical shell subjected to radial forces uniformly distributed along the circumference can be considered as a beam resisting on an elastic foundation.

Unit- V

1. Discuss the various yield criteria and its use to predict the onset of yielding in metals.
2. The state of stress in a material is given by σ_x=75 *MPa*, σ_y=95 *MPa* and τ_{xy} = 55 *MPa*. If the yield strength of the material is 120 MPa, determine whether yielding will occur or not.
3. Write a detailed note on Plastic bending of beams.
4. Explain the sand heap analogy for solving plastic torsion problems. What are the limitations of this analogy?
5. Briefly explain about elastic plastic problems in bending and torsion with a typical idealized stress-strain diagram.

6. Explain the various failure Theories adopted in elastic-plastic analysis with necessary sketches.
7. Compose various types of materials and their mathematical models.
8. Derive the expression showing stress strain relationship.
9. Explain the theories of plastic flow.
10. A hollow circular shaft of inner radius 5cm and outer radius 10cm is subjected to a twisting couple of 5000Nm. If the shear stress-strain diagram for the shaft material is given by $\tau=350\ \gamma^{0.3}$, investigate the maximum shear stress induced in the shaft and the angle of twist per unit length.
11. Discuss in detail about the various failure theories of plasticity with its limitations.
12. Discuss in detail the various theories of failure normally adopted to find the yield criteria.
13. A rectangular beam having linear stress-strain behavior is 6cm wide and 8cm deep.It is 3m long, simply supported at the ends and carries a uniformly distributed load over the whole span. The load is increased so that the outer 2cm depth of the beam yields plastically. If the yield stress for the beam material is 240MPa, illustrate the residual stress distribution in the beam.
14. A circular shaft of inner radius 4cm and outer radius 10cm is subjected to a twisting couple so that the outer 2cm deep shell yields plastically. Find the twisting couple applied to the shaft yield stress in shear for the shaft material is 425N/mm². Also find the couple for full yielding.
15. A solid circular shaft of radius 12cm is subjected to transmit 600 kW at 540rpm.The maximum torque is 30 percent greater than the mean torque. If the shear stress strain curve for the shaft materials is given by $\tau=280\gamma^{0.25}$, assess the maximum stress induced in the shaft and the corresponding angle of twist, prioritize these values if the shear stress-strain curve is a linear one? $G=0.84\times10^5$ N/mm².
16. A hollow circular shaft of inner radius 2cm and outer radius 5cm is subjected to a twisting moment so that the outer 1cm deep shall yields plastically. The yield stress in shear for the shaft material is 175 MPa and it is made of a non-linear material whose shear stress-shear strain curve is given by $\tau=280\gamma^{0.25}$. If this twisting moment is now released, compose the residual stress distribution in the shaft and the associated residual angle of twist, $G=0.84\times10^5$ N/mm².
17. A thick cylinder of internal radius 15cm and external radius 25cm is subjected to an internal pressure P MPa. If the yield stress for the cylinder material is 220N/mm², find

a) the pressure at which the cylinder will start yielding just at the inner radius b) the stresses when the cylinder has a plastic front radius of 20cm and c) the stresses when whole of the cylinder has yielded. Assume Von-Mises yield condition is a state of plane strain.

18. A solid circular shaft of 8cm radius is subjected to a twisting couple so that the outer 3cm deep shell of the shaft yields plastically. If the yield stress in shear for the shaft material is 150MPa, investigate the value of twisting couple applied and the associated angle of twist, $G=0.84\times10^5$ N/mm^2.
19. A hollow circular shaft of inner radius 5cm and outer radius 10cm is subjected to a twisting couple of 5000Nm. If the shear stress-strain diagram for the shaft material is given by $\tau=350\ \gamma^{0.3}$, investigate the maximum shear stress induced in the shaft and the angle of twist per unit length.
20. With the help of case study justify any two theories of failure.