MA5165 Statistical Methods for Engineers

Important 13 Mark Questions

Unit I

1. State and Prove Cramer-Rao Inequality.
2. For random sampling from a normal population, find the maximum likelihood estimators for
   (i) The population mean, when the population variance is known.
   (ii) Population variance, when the population mean in known.
   (iii) The simulations estimation of both the population mean and variance.
3. Define unbiasedness in estimators. Show that in sampling from a population with mean μ and variance σ^2, sample mean $\bar{x}$ is an unbiased estimator of μ, but the sample variance s^2 is not unbiased to σ^2.
4. In a random sampling from normal population $N(\mu, \sigma^2)$, find the maximum likelihood estimator for μ when σ^2 is known.
5. Find the maximum likelihood estimator for the parameter λ of a Poisson distribution on the basis of a sample of size n. Also, find its variance.

Unit II

1. To determine whether there really is a relationship between an employee's performance in the company's training program and his or her ultimate success in the job, the company takes a sample of 400 cases from its very extensive files and obtain the results shown in the following contingency table:

		Performance in training program :		
		Below Average	Average	Above average
	Poor	23	60	29
Success in Job :	Average	28	79	60
	Very good	9	49	63

   Use the $\propto = 0.01$ level of significance to test the null hypothesis that performance in the training program and success in the job are independent.
2. The following table gives the number of accidents that occurred during the various days of a week. Test whether the accidents are uniformly distributed over the week.

Days	Mon	Tue	Wed	Thu	Fri	Sat
No. of Accidents	15	19	13	12	16	15

3. Survey of 320 families with 5 children each revealed the following information:

No. of boys	5	4	3	2	1	0
No. of girls	0	1	2	3	4	5
No. of families	14	56	110	88	40	12

   Is this result consistent with Hypothesis that male and female birth are equally probable?

4. Explain the test procedure for testing the equality of two means in large samples with an example in industrial applications.
5. A spare part manufacturer is making spare parts with axle diameter of 0.700 inch. A random sample of 10 parts shows a mean diameter of 0.742 inch with a S.D. of 0.04 inch. Verify whether the work satisfies the specifications.

Unit III

1. Given $r_{12} = 0.28;\ r_{23} = 0.49, r_{31} = 0.51, \sigma_1 = 2.7,\ \sigma_2 = 2.4, \sigma_3 = 2.7.$ Find the regression equation of $x_{3,}$ and x_1 and x_2.
2. For a trivariate distribution:

   $\overline{X_1} = 28.02\ \overline{X_2} = 4.91\ \overline{X_3} = 594$

   $\sigma_1 = 4.42\ \sigma_2 = 1.10\ \sigma_3 = 85$

   $r_{12} = 0.8\ r_{23} = -0.56\ r_{13} = -0.4$

   Find:

   (i) $R_{1.23}$
   (ii) $r_{23.1}$
   (iii) The regression equation of X_1 on X_2 and X_3.
3. Ten students got the following percentage of marks in economics and statistics. Calculate the correlation coefficient.

Marks in Economics :	78	36	98	25	75	82	90	62	65	39
Marks in mathematics:	84	51	91	60	68	62	86	58	53	47

4. The following bivariate data shows the length, X and weight, of a sample of 10 iron rods.

X:	12	18	24	20	17	13	20	24	32	16
Y:	4	7	12	10	13	15	13	14	20	12

   (i) Obtain the Pearson's correlation coefficient of X and interpret the result
   (ii) Estimate the weight (Y) of rod when the length is 25 inches.
   (iii) Predict the length (X) of rod when the weight is 18 kgs.
5. Prove that the sum of the product of any residual of order zero with any other residual of higher order is zero, provided the subscript of the former occurs among the secondary subscripts of the latter.

Unit IV

1. A farmer wishes to test the effects of four different fertilizers A, B, C, D on the yield of wheat. In order to eliminate sources of error due to variability in soil fertility, he uses the fertilizers in a Latin square arrangement as indicated in the following table, where the numbers indicate yields in bushels per unit area.

A 18	C 21	D 25	B 11
D 22	B 12	A 15	C 19
B 15	A 20	C 23	D 24
C 22	D 21	B 10	A 17

Perform an analysis of variance to determine if there is a significant difference between the fertilizers at the $\propto = 0.05$ significance levels.

2. The following data resulted from an experiment to compare three burners B_1, B_2, B_3. A Latin square design was used as the tests were made on three engines and were spread over three days.

		Engine 1	Engine 2	Engine 3
Day	1	$B_1 - 16$	$B_2 - 17$	$B_3 - 20$
	2	$B_2 - 16$	$B_3 - 21$	$B_1 - 15$
	3	$B_3 - 15$	$B_1 - 12$	$B_2 - 13$

Test the hypothesis that there is no significant difference between the burners.

3. To study the performance of three detergents and three different temperatures, the following 'whiteness' reading were obtained with specially designed equipment:

Water temp.	Detergent A	Detergent B	Detergent C
Cold water	57	55	67
Warm water	49	52	68
Hot water	54	46	58

Perform a two-way analysis of variance at 5 % level

4. A set of data involving four "tropical feed stuffs A, B, C, D" tried on 20 chicks is given below. All the 20 chicks are treated alike in all respects expect the feeding treatments and each feeding treatment is given to 5 chicks. Analyse the data using CRD layout and interpret the result.

Feed	Gain in weight				
A	55	49	42	21	52
B	61	112	30	89	63
C	42	97	81	95	92
D	169	137	169	85	154

5. Analyse the variance in the following Latin square of yields (in kgs) of paddy where A, B, C, D denote the different methods of cultivation.

D122	A121	C123	B122
B124	C123	A122	D125
A120	B119	D120	C121
C122	D123	B121	A122

Examine whether the different methods of cultivation have given significantly different yields.

Unit V

1. Let X have covariance matrix $\Sigma = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{bmatrix}$. Obtain the standard deviation matrix $V^{1/2}$ and the correlation matrix ρ.
2. Calculate the population principal component of the random variables $X_{1,}\ X_{2,}$ and X_3 which has the covariance matrix $\Sigma = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
3. Let $X = (X_{1,}\ X_{2,\dots}\ X_n)$ be distributed as $N(\mu, \Sigma)$, then $Z = CX$ is distributed as $N(C\mu, C\Sigma C)$ where C is non-singular.
4. A random sample of 10 observations be drawn from a trivariate normal population whose data is as follows:

X_1 :	12	15	17	18	20	16	17	14	13	10
X_2 :	20	23	21	24	28	23	26	27	20	19
X_3 :	31	28	32	34	28	27	24	20	32	30

   (i) Obtain the sample mean vector
   (ii) Calculate the covariance matrix
   (iii) Obtain the sample correlation matrix.
5. The covariance matrix of two random variables X_1 and $X_{2,}$ is given by $\Sigma = \begin{bmatrix} 1 & 4 \\ 4 & 100 \end{bmatrix}$ find the principal components of covariance matrix and correlation matrix.