MA 5160 APPLIED PROBABILITY AND STATISTICS

Important 13 Marks Questions

Unit I

1. State and prove the memory less property of an exponential distribution.
2. A continuous random variable X has probability density functions $f(x) = kx^2e^{-x}, x \geq 0$. Find k, r^{th} moment, mean and variance.
3. Derive the recurrence relation for the moments of the Poisson distribution. Also obtain the first four moments.
4. A random variable X has the following problem values of

X:	0	1	2	3	4	5	6	7
P(X):	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

   (i) Find K
   (ii) Evaluate P(X<6), P(X>6) and P(0<x<5)
   (iii) Find minimum value of 'a' such that P(X$\leq a) > \frac{1}{2}$
5. Let X be a random variable such that $P[X = -2] = P[X = -1] = P[X = 1] = P[X = 2]$ and $P[X < 0] = P[X = 0] = P[X > 0]$. Find the probability mass function and the distribution of X.

Unit II

1. The joint probability function of (X, Y) is given by $P(x, y) = K(2x + 3y), x = 0, 1, 2; y = 1, 2, 3.$ Find the marginal distributions and conditional distributions.
2. Find the correlation co-efficient for the following data:

X	51	63	63	49	50	60	65	63	46	50
Y	49	72	75	50	48	60	70	48	60	56

3. The joint distribution function of X and Y is given by

   $F(x, y) = (1 - e^{-x})(1 - e^{-y}), x > 0, y > 0.$

   (i) Find the probability density function of X and Y.
   (ii) Find the marginal density function of X and Y.
   (iii) Are X and Y independent?
   (iv) Find $P(1 < X < 3, 1 < Y < 2)$.
4. Two random variables X and Y have the following joint probability density function
   $f(x, y) = \{2 - x - y, 0 \leq x, y \leq 1$
   0, otherwise
   (i) Find the Marginal density functions of X and Y.
   (ii) Find the conditional density functions of X and Y.
   (iii) Find the Var X, Var Y and $cov\ (X, Y)$.
5. If X, Y and Z are uncorrelated random variables with zero means and standard deviations 5, 12, 9 respectively and if $U = X + Y, V = Y + Z,$ find the coefficient of correlation between U and V.

Unit III

1. State and prove Cramer-Rao inequality. Given the p.d.f $f\left(x_{i,}\theta\right)=\frac{1}{\pi}\frac{1}{1+(x-\theta)^2}-\infty<x<\infty,-\infty<\theta<\infty$; Show that the Cramer-Rao bound of variance of an unbiased estimators of θ is $\frac{2}{n}$, where n is the size of the random sample from this distribution.
2. The following data relate to the marks of 10 students in the internal test and the university examination for the maximum of 50 in each.
   Internal Marks: 25 28 30 32 35 36 38 39 42 45
   University Marks: 20 26 29 30 25 18 26 35 35 46
   (i) Obtain the equations of the lines of regression.
   (ii) The most likely internal mark for the university mark of 25,
   (iii) The most likely university mark for the internal mark of 30.
3. Let X_k for k=1, 2, 3, …, n constitute a random sample of a normal random variable X with known mean μ and unknown variance σ^2. Find the maximum likelihood estimator of σ^2.
4. Obtain the equation of regression line $y=ax+b$ from the following data, using the method of least squares.

X:	22	26	29	30	31	31	34	35
Y:	20	20	21	29	27	24	27	31

5. Find the maximum likelihood estimate for the parameter λ of a Poisson distribution on the basis of a sample of size it.

Unit IV

1. The table below gives the number of aircraft accidents that occurred during the various days of the week. Test whether the accidents are uniformly distributed over the work.

Days:	Mon	Tues	Wed	Thurs	Fri	Sat
No of accidents:	14	18	12	11	15	14

2. The mean breaking strength of the cables supplied by a manufacturer is 1800, with a standard deviation of 100. By a new technique in the manufacturing process, it is claimed that the breaking strength of the cable has increased. To test this claim, a sample of 50 cables is tested and it is found that the mean breaking strength is 1850 at 0.01 level of significance?
3. The following data shows defective articles produced by 4 machines.

Machine	A	B	C	D
Production time	1 hr	1 hr	2 hrs	3 hrs
Number of defectives	12	30	63	98

   Do the data indicate a significant difference in the performance of the machine?
4. A survey of 320 families with 5 children each revealed the following information:

No of boys	5	4	3	2	1	0
No of girls	0	1	2	3	4	5
No of families	14	56	110	88	40	12

   Is the result consistent with Hypothesis that male and female birth are equally probable?

5. Samples of two types of electric bulbs were tested for length of life and the following data were obtained.

	Size	**Mean**	**S. D**
Sample I	8	1234 hours	36 hours
Sample I	7	1036 hours	40 hours

Is the difference in means sufficient to warrant that type I bulbs are superior to type II bulbs at 5% level?

<u>Unit V</u>

1. Computing the correlation matrix from the Covariance matrix $\begin{vmatrix} 4 & 1 & 2 \\ 1 & 9 & -3 \\ 2 & -3 & 25 \end{vmatrix}$.
2. Find the mean matrix, covariance matrix, standard, standard deviation matrix and correlation coefficient matrix for two random variables X_1 and X_2 whose joint mass function is given by.

x_1/x_2	0	1
-1	0.24	0.06
0	0.16	0.14
1	0.40	0.0

3. Suppose the random variables $X_{1,}\ X_{2,}\ X_3$ have the covariance matrix $\mathbf{\Sigma} = \begin{vmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{vmatrix}$. Calculate the population principle components and hence find variances and covariance of principal components.
4. Find the population principal components Y_k for k = 1, 2, 3 for $\mathbf{\Sigma}$.
5. Calculating the population principal components of random variables $X_1,\ X_{2,}\ X_3$ have the covariance matrix $\begin{vmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{vmatrix}$ and also find $Var(Y_1)$, $Cov(Y_{1,}Y_2)$ and total population variance.