MA 5151 Advanced Mathematical Methods

Important 2 Mark Questions

Unit I

1. Evaluate $\int_0^\infty \frac{e^{-t}\sin t}{t}dt$.
2. Find $L^{-1}\left[\frac{2s-5}{9s^2-25}\right]$.
3. Find the Laplace transform of unit step function.
4. Find the inverse Laplace transform of $log\left(\frac{s+1}{s-1}\right)$.
5. Find the Laplace transform of $(1 - cost)$.
6. Find $L(e^{at})$.
7. Find the inverse Laplace transform of the function $\frac{2p+1}{p(p+1)}$.
8. Show that $\int_0^t J_0(u)J_0\,(t-u)du = sint$.
9. Find $L\{t^2e^{2t}\}$.
10. Find $L(erf\sqrt{t})$.

Unit II

1. If $U(a,t)$ is the Fourier transform of $u(x,t)$ and if u and $\frac{\partial u}{\partial x}$ vanish as $x \to \pm\infty$, then find the Fourier transform of $\frac{\partial^2 u}{\partial x^2}$.
2. Find the Fourier cosine transform of $f(x) = \begin{cases} x, & 0 < x < a \\ 0, & x > a \end{cases}$.
3. Define Fourier transforms pair.
4. Find Fourier transform of Dirac Delta function.
5. Define Fourier cosine transform.
6. State convolution theorem for the Fourier transforms.
7. If the Fourier transform of $f(x)$ is $f(a)$, the find the Fourier transform of $f(x)\cos ax$.
8. State the Parseval's identity for Fourier transforms.
9. If $F(s)$ is the Fourier transform of $f(x)$, then show that the Fourier transform of $f(ax)$ is $\frac{1}{a}F\left(\frac{s}{a}\right)$.
10. Write the existence conditions for the Fourier transform of the function $f(x)$.

Unit III

1. Show that the symbols $\frac{d}{dx}$ and δ are commutative.
2. Find the transversality condition for the functional
$$U = \int_{x_0}^{x_1} A(x,y)\sqrt{\left[1+\left(\frac{dy}{dx}\right)^2\right]}\,dx.$$
3. Find the curves on which the functional $\int_{x_1}^{x_2}\left(y\sqrt{1+y^2}\right)dx$ with $y(x_1) = y_1$ and $y(x_2) = y_2$ can be extremized.

4. Write the necessary condition for the functional $I = \int_{x_0}^{x_1} f(x,\ y,\ y',y'',y''')dx$ to be stationary.
5. Define maxima and minima of $I[y(x)]$.
6. Find the Euler equation of the functional: $I[y(x)] = \int_{x_0}^{x_1}(xy + y^2 - 2y^2y)dx$.
7. Write the Euler-Ostrogradsky equation.
8. Find the partial differential equation of the extremal of the functional $V[z(x,y)] = \iint (p^2 - q^2)\, dxdy$.
9. State the fundamental lemma of Calculus of variations.
10. Find the tranversality condition for the functional of the form $v[y(x)] = \int_{x_0}^{x_1} A(x,y)\sqrt{1+y'^2}dx$ with the right boundary moving along $y_1 = \varphi(x_1)$.

Unit IV

1. Define conformal mapping and give an example.
2. Find the fixed points of the transform $\omega = \frac{2z-5}{z+4}$.
3. Define equipotential lines.
4. Find the fixed points of $f(z) = \frac{z-1}{z+2}$.
5. Define velocity potential.
6. Write the cross-ratio of four points z_1, z_2, z_3, z_4.
7. Show that the transformation $z = F(w) + iG(w)$ maps the curve C in the z-plane given by $x = F(t), y = G(t)$ onto the real axis of the w-plane.
8. Find the stagnation points of the flow represented by the complex potential $\Omega(z) = z^2$.
9. State any two properties of conformal mapping.
10. Find all the points at which the mapping $z(z^2 - 5)$ is not conformal.

Unit V

1. If φ is a function of the n quantities x^i, write the differential of φ using the summation convention.
2. Define divergence of a contravariant vector.
3. Define reciprocal tensor.
4. What is symmetric tensor?
5. Define divergence of a contra variant vector.
6. State the Quotient law of tensors.
7. Define inner product of two tensors.
8. If A_i is a covariant tensor, then prove that $\frac{\partial A_i}{\partial x^j}$ does not form a tensor.
9. Prove that $\frac{\partial g_{ij}}{\partial x^k} = [ik,j] + [jk,i]$.
10. Write the terms contained in $S = a_{ij}x^i x^j$, taking $n = 3$.