# MA 5151 Advanced Mathematical Methods

## Important 13 Mark Questions

## Unit I

1. Apply the convolution theorem to evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^2}t\right]$.
2. Solve the initial boundary value problem using the Laplace transform technique:
   PDE: $u_t = u_{xx,} 0 < x < 1, t > 0$
   BCs: $u(0,t) = 1, u(1,t) = 1, t > 0$
   IC: $u(x,0) = 1 + \sin \pi x, 0 < x < 1$.
3. State and prove convolution theorem of Laplace Transforms.
4. Using complex inversion formula, find the inverse Laplace transform of $\frac{1}{(s+1)(s-2)^2}$.
5. A string is stretched between two fixed points $(0,0)$ and $(c,0)$. If it is displaced into the curve $y = b\, sin\left(\frac{\pi x}{a}\right)$ and released from rest in that position at time $t = 0$, find its displacement at any time $t > 0$ and at any point $0 < x < c$. [use Laplace transform to solve]

## Unit II

1. Using the Fourier cosine transformation of $e^{-ax}$ and $e^{-bx}$, show that $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{2ab(a+b)}$.
2. Using the Fourier transform, determine the temperature distribution by solving the heat equation in the semi-infinite medium $x \geq 0$, when the end $x = 0$ is maintained at zero temperature and the initial temperature distribution is given by $f(x)$.
3. Find the Fourier transform of $f(x)$ defined by $f(x) = \begin{cases} a - |x|, & |x| < a \\ 0, & |x| > a \end{cases}$, and hence show that
   (i) $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$
   (ii) $\int_0^{\infty} \left(\frac{\sin t}{t}\right)^4 dt = \frac{\pi}{3}$
4. Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ and hence evaluate $\int_0^{\infty} \frac{x\cos x - sinx}{x^3} cos\left(\frac{x}{2}\right) dx$.
5. If the initial temperature of an infinite bar is given by $u(x,0) = \begin{cases} 1, & for\ -c < x < c \\ 0, & otherwise \end{cases}$, determine the temperature at any point x and at any time $t(> 0)$.

## Unit III

1. Derive the Euler's equation and use it, find the extremals of the functional $V[y(x)] = \int_{x_0}^{x_1} \left(y^2 + y'^2 - 2y sinx\right) dx$.

2. Find the approximate solution to the problem of the minimum, of the functional $J(y) = \int_0^1 (y'^2 - y^2 + 2xy)dx,\ y(0) = 0 = y(1)$ by Ritz method and compare it with the exact solution.
3. Find the extremals of the functional $\int_0^{\pi/2} (y''^2 - y^2 + x^2)\,dx$ that satisfies the conditions $y(0) = 1, y'(0) = 0, y\left(\frac{\pi}{2}\right) = 0, y'\left(\frac{\pi}{2}\right) = -1$.
4. Find the shortest distance between the point (1, -4) and the parabola $y^2 = 4x$.
5. Find the solid of maximum volume formed by the revolution of a given surface area.

**<u>Unit IV</u>**

1. Find the bilinear transformation that maps $z_1 = -1, z_2 = 0, z_3 = 1$ onto $w_1 = -1, w_2 = -i, w_3 = 1$ respectively.
2. Find the complex potential due to a source at $z = -a$ and a sink at $z = a$ of equal strengths k. Determine the equipotential lines and streamlines and represent graphically. Also find the speed of the fluid at any point.
3. Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. Hence find
   (i) The image of $|z| < 1$
   (ii) The invariant points of this transformation.
4. Establish the validity of Schwarz-Christoffel transformation and hence prove that this transformation also maps the upper half plane onto the interior of the polygon.
5. Find the transformation which maps the semi-infinite strip bounded by $v = 0, u = 0$ and $v = b$ into the upper half of the z-plane.

**<u>Unit V</u>**

1. Find the components of the metric tensor and the conjugate tensor in cylindrical coordinates.
2. Given the covariant components in rectangular co-ordinates $2x - z, x^2y, yz$. Find the covariant components in
   (i) Spherical polar coordinates $(r, \theta, \varphi)$
   (ii) Cylindrical co-ordinates $(\rho, \varphi, z)$
3. Derive the law of transformation of Christoffel symbol of first and second kind.
4. Prove that the covariant derivative of $g^{ij}$ is zero.
5. If the metric is given by $ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3$, evaluate $g$ and $g^{ij}$.