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QUESTION PAPER CODE: X10666

B.E. / B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020
Fifth Semester
Computer Science and Engineering
MA8551 –ALGEBRA AND NUMBER THEORY
(Common to Computer and Communication Engineering and Information Technology)
(Regulations 2017)
Answer ALL Questions

Time: 3 Hours **Maximum Marks:100**

PART-A

(10×2=20 Marks)

1. Find the inverse of 3 under the binary operation $*$ defined in R by $a * b = \frac{ab}{3}$.
2. How many units and proper zero divisors are there in Z_{17}.
3. Given an example of a polynomial that is irreducible in $Q[x]$ and reducible in $C[x]$.
4. If $f(x) = 2x^4 + 5x^2 + 2$ and $g(x) = 6x^2 + 4$, then determine $f(x) \cdot g(x)$ in $Z_7[x]$.
5. State the pigeonhole principle.
6. Find six consecutive integers that are composite.
7. When does the linear congruence $ax \equiv b \pmod{m}$ has a unique soloution?
8. Find the remainder when 4^{117} is divided by 15.
9. State Wilson's theorem.
10. Find the value of $\tau(n)$ and $\sigma(n)$ for $n = 29$.

PART-B

(5×16=80 Marks)

11. (a) (i) Determine whether $(Z, \oplus, \odot)$ is a ring with the binary operation $x \oplus y = x + y - 7$, $x \odot y = x + y - 3xy$ for all $x, y \in Z$. (8)

(ii) For any group G, prove that G is abelian, if and only if, $(ab)^2 = a^2b^2$ for all $a, b \in G$. (8)

(OR)

(b) (i) Prove that Z_n is field, if and only if, n is a prime. (8)

(ii) Find$[777]^{-1}$ in Z_{1009}. (8)

12. (a) (i) State and prove the factor theorem and remainder theorem. (8)

(ii) Find the remainder, when $f(x) = x^{100} + x^{90} + x^{80} + x^{50} + 1$ is divided by $g(x) = x - 1$ in $Z_2[x]$. (8)

(OR)

(b) (i) If $(F, +, \cdot)$ is a field and $char(F) > 0$, then prove that $char(F)$ must be prime. (8)

(ii) Find the gcd of $x^4 + x^3 + x + 1$ and $x^3 + x^2 + x + 1$ in $Z_2[x]$. (8)

13. (a) (i) Find the number of positive integers ≤ 3000 and divisible by 3, 5 or 7. (8)

(ii) Apply Euclidean algorithm to express the gcd of 2076 and 1776 as a linear combination of themselves. (8)

(OR)

(b) (i) Prove that there are infinitely many primes. (8)

(ii) State and prove the fundamental theorem of arithmetic. (8)

14. (a) (i) Find the general solution of the linear Diophantine equation $6x + 8y + 12z = 10$. (8)

(ii) Prove that no prime of the form $4n + 3$ can be expressed as the sum of two squares. (8)

(OR)

(b) (i) Solve $x \equiv 2 (\mod 5)$, $x \equiv 3 (\mod 7)$ using Chinese remainder theorem. (8)

(ii) Solve the linear system $\begin{array}{l} 3x + 4y \equiv 5 (\mod 7) \\ 4x + 5y \equiv 6 (\mod 7) \end{array}$. (8)

15. (a) (i) State and prove Fermat's little theorem. (8)

(ii) Let n be a positive integer with canonical decomposition $n = p_1^{\theta_1} p_2^{\theta_2} \ldots p_k^{\theta_k}$. Derive the formula for evaluating Euler's phi function $\phi(n)$ and hence, evaluate the same for $n = 6125$. (8)

(OR)

(b) (i) Solve the linear congruence $25x \equiv 13 (\mod 18)$. (8)

(ii) Prove that tau and sigma functions are multiplicative. (8)

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