Reg. No. :

Question Paper Code : X10665

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020 AND APRIL/MAY 2021

Fourth/Fifth/Sixth Semester

Civil Engineering

MA 8491 – NUMERICAL METHODS

(Common to Agriculture Engineering/Aeronautical Engineering/Aerospace Engineering/Electrical and Electronics Engineering/Electronics and Instrumentation Engineering/Instrumentation and Control Engineering/ Manufacturing Engineering/Mechanical Engineering (Sandwich)/Mechanical and Automation Engineering/Chemical Engineering/Chemical and Electrochemical Engineering/Plastic Technology/Polymer Technology/Textile Technology)

(Regulations 2017)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. State the order and criterion of convergence of Newton-Raphson method for f(x) = 0.
2. Find all the Eigen values and Eigen Vectors of $A = \begin{bmatrix} 2 & 2 \\ 2 & -1 \end{bmatrix}$ by Jacobi method.
3. Establish the relation $1 + \mu^2\delta^2 = (1 + \delta^2)^2$, where μ is the averaging operator and δ is the central difference operator.
4. Form the divided difference table for x = 1, 3, 6, 11 and $f(x) = x^2 + x + 2$.
5. Apply Simpson's 1/3 rule to evaluate $I = \int_0^2 \frac{1}{x^2 + x + 1}$ dx, taking h = 0.25.
6. Derive the formula for finding Integral value I by Romberg's method given I_1 and I_2 the two values of I got from two different values of $h_1 = h$ and $h_2 = h/2$.
7. Use Euler's modified formula to find y(0.1) given $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$.
8. Write down the Adam-Bashforth predictor and corrector formulae to solve $\frac{dy}{dx} = f(x,y)$.
9. Derive the difference equation for $y''(x) + a(x)y'(x) + b(x)\, y(x) = f(x)$ with $y(x_0) = \alpha$, $y(x_1) = \beta$ by using the difference approximation formula for first and second derivatives of y.
10. Write down the Leibmann's iteration formula for solving Laplace equation.

PART – B (5×16=80 Marks)

11. a) i) Find a positive root of $f(x) = 3x - \sqrt{1 + \sin x} = 0$, using fixed point iteration.

ii) Find the dominant Eigen value and the corresponding Eigen vector by power method for the matrix $A = \begin{bmatrix} 1 & 3 & -1 \\ 3 & 2 & 4 \\ -1 & 4 & 10 \end{bmatrix}$.

(OR)

b) i) Solve the following linear system of equations by Gauss elimination method $2x + 3y + z = -1$; $5x + y + z = 9$; $3x + 2y + 4z = 11$.

ii) Use Gauss-Seidal iterative method to obtain the solution of the equations $30x - 2y + 3z = 75$; $2x + 2y + 18z = 30$; $x + 17y - 2z = 48$.

12. a) i) If f(1) = 1, f(2) = 5, f(7) = 5 and f(8) = 4, find a polynomial that satisfies this data using Newton's divided difference formula. Hence, find f(6).

ii) Find the cubic spline in [0, 2] and [2, 4] given $M_0 = 0$, $M_3 = -12$, for the data below :

X	0	2	4	6
y = f(x)	1	9	41	41

(OR)

b) i) Find the number of students who scored marks not more than 45, from the following data.

x	30-40	40-50	50-60	60-70	70-80
y	35	48	70	40	22

ii) Find y(10) given y(5) = 12, y(6) = 13, y(9) = 14 and y(11) = 16 by Lagrange's formula.

13. a) i) Given the data below, find y'(6) and the maximum value of y.

x	0	2	3	4	7	9
y	4	26	58	112	466	922

ii) Evaluate $\int_1^2 \int_3^4 \frac{dxdy}{(x+y)^2}$ by Trapezoidal and Simpson's formula by taking $h = k = 0.5$.

(OR)

b) i) Use Gaussian three-point formula to evaluate $\int_1^5 \frac{dz}{z}$ and compare with exact value.

ii) Use Romberg's method to evaluate $I = \int_0^1 \frac{1}{(1+x)} dx$ correct to four decimal places by taking h = 0.5, 0.25 and 0.125

14. a) i) Solve $\frac{dy}{dx} = y - \frac{2x}{y}$, given y(0) = 1 and find values of y(0.1) and y(0.2) using improved Euler's method, correct to four decimal places.

ii) Compute y(0.1) given $\frac{dy}{dx} + y + xy^2 = 0$, y(0) = 1, by taking h = 0.1 using Runge-Kutta method of fourth order, correct to 4 decimal accuracy.

(OR)

b) i) Use Milne's predictor-corrector formula to find y(0.4), given $\frac{dy}{dx} = xy + y^2$, y(0) = 1, y(0.1) = 1.1167, y(0.2) = 1.2767 and y(0.3) = 1.5023.

ii) Solve by finite difference method $y'' - y = x$, y(0) = 0, y(1) = 0, by taking mesh length h = 1/4.

15. a) i) Solve : $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides x = 0, x = 3, y = 3 with u = 0 on the boundary and mesh length 1 unit.

ii) Derive the Bender-Schmitt formula for one dimensional heat equation. Hence, slove , $\frac{\partial^2 u}{\partial x^2} = 2\frac{\partial u}{\partial t}$ given u(0, t) = 0, u(4, t) = 0, u(x, 0) = x(4 – x) taking h = 1. Find u(x, t) upto t = 5.

(OR)

b) Solve by explicit difference method :

$$25\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, \frac{\partial u}{\partial t}(x,0) = 0, u(0,t) = 0, u(5,t) = 0 \text{ and } u(x,0) = \begin{cases} 2x, \text{when } 0 \le x \le 2.5 \\ 10 - 2x, \text{when } 2.5 \le x \le 5 \end{cases}$$

Take h = 1 and compute u(x, t) upto t = 2.