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QUESTION PAPER CODE: X10663

B.E. / B.Tech. DEGREE EXAMINATIONS, NOV/DEC 2020 AND APRIL /MAY 2021

Fourth Semester

Electronics and Communication Engineering

MA8451–PROBABILITY AND RANDOM PROCESSES

(Common to Computer and Communication Engineering and Electronics and Telecommunication Engineering)

(Regulations 2017)

Answer ALL Questions

Time: 3 Hours **Maximum Marks:100**

PART-A

(10×2=20 Marks)

1. In a community, 32% of the population are male smokers, 27% are female smokers. What percentage of the population of this community smoke?

2. Suppose that five good fuses and two defective ones have been mixed up. To find the defective fuses, we test them one-by-one, at random and without replacement. What is the probability that we are lucky and find both of the defective fuses in the first two tests?

3. Given $F_{XY}(x,y) = \begin{cases} 1-5e^{-5(x+y)}, & x>0, y>0 \\ 0, & \text{otherwise} \end{cases}$. Determine $f(x,y)$, if F is the joint probability distribution function of ywo randon variables X and Y.

4. Given $F_{XY}(x,y) = \begin{cases} \frac{6}{5}(x+y^2), & 0 \le x \le 1, 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$. Find the marginal probability density function of X.

5. Consider the random process $X(t) = \cos(t+\phi)$, ϕ is a uniform random variable in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Check whether the process is stationary.

6. A hospital receives on an average of 3 emergency calls in a 10-minute interval. What is the probability that there are at most 3 emergency calls in a 10-minute interval?

7. State any two properties of a cross correlation function.

8. An autocorrelation function of a random process $\{X(t)\}$ is given by
$$R_{XX}(\tau) = Ce^{-\alpha|\tau|},\ C>0,\ \alpha>0.$$
Obtain the spectral density of $X(t)$.

9. What is meant by a linear time invariant system?

10. Prove that the mean of the output $Y(t)$ of a linear system is given by $\mu_Y = H(0)\mu_X$, where the input $X(t)$ is wide sense stationary and $H(\cdot)$ is the transfer function.

PART-B (5×16=80 Marks)

11. (a) (i) For a Gamma random variate with parameters (n, λ), derive the moment generating function and hence, obtain its mean and variance. (8)

(ii) The radius of a sphere is a random number between 2 and 4. What is the expected value of its volume? What is the probability that its volume is at most 36π? (8)

(OR)

(b) (i) For a Poisson random variable with parameter λ, derive the moment generating function and hence, obtain its mean and variance. (8)

(ii) Only 60% of certain kinds of seeds germinate when planted under normal conditions. Suppose that 4 such seeds are planted and X denotes the number of those that will germinate. Find the probability functions of X and $Y = 2X + 1$. (8)

12. (a) Let X and Y be continuous random variables with joint probability density function $f(x, y) = \frac{1}{3}(x + y)$, $0 \leq x \leq 1$, $0 \leq y \leq 2$. Determine the correlation coefficient of the random variables X and Y. (16)

(OR)

(b) (i) Let the joint probability density function of the random variables X and Y be given by $f(x, y) = x + y$, $0 \leq x \leq 1$, $0 \leq y \leq 1$. Determine the probability density function of the random variable $U = XY$. (8)

(ii) What is the probability that the average of 150 random points from the interval $(0, 1)$ is within 0.02 of the midpoint of the interval? (8)

13. (a) (i) State the postulates for a Poisson process $\{N(t), t \geq 0\}$ with parameter λ and hence, derive

$$P_n(t) = P\{N(t) = n\} = \frac{e^{-\lambda t}(\lambda t)^n}{n!}, \; n = 0, 1, 2, \ldots$$

(10)

(ii) Given $\{N(t), t \geq 0\}$ to be a Poisson process with parameter λ, for $s < t$, find $P\{N(s) = k | N(t) = n\}$. (6)

(OR)

(b) Given the TPM

$$P = \begin{bmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

a Markov chain state space $S = \{1, 2, 3\}$. Classify the states and obtain the stationary distribution, if it exists. (16)

14. (a) (i) If $X(t)$ and $Y(t)$ are uncorrelated random processes then find the power density of Z, if $Z(t) = X(t) + Y(t)$. Also find the cross spectral density $S_{XZ}(\omega)$ and $S_{YZ}(\omega)$. (8)

(ii) Given the power spectral density of a continuous random process as

$$S_{XX}(\omega) = \frac{\omega^2 + 9}{\omega^4 + 5\omega^2 + 4},$$

find the mean square value of the prcoess. (8)

(OR)

(b) (i) If $X(t)$ and $Y(t)$ be both zero-mean and wide sense stationary random processes with $Z(t) = X(t) + Y(t)$. Find (A) the autocorrelation function and the power density, if $X(t)$ and $Y(t)$ are jointly wide sense stationary, (B) the power spectral density of $Z(t)$, if $X(t)$ and $Y(t)$ are orthogonal, and (C) the mean square of $Z(t)$, if $X(t)$ and $Y(t)$ are orthogonal. (8)

(ii) The power spectral density function of zero-mean wide sense stationary process $\{X(t)\}$ is given by $S_{XX}(\omega) = 1,\ |\omega| < \omega_0$. Find the autocorrelation function of $X(t)$ and also show that $X(t)$ and $X\left(t + \frac{\pi}{\omega_0}\right)$ are uncorrelated. (8)

15. (a) (i) Let $X(t)$ be the input voltage to a system and $Y(t)$ is the output voltage. $X(t)$ is a stationary random process with $\mu_X = 0$ and $R_{XX}(\tau) = e^{-\alpha|\tau|}$. Find μ_Y, $S_{YY}(\omega)$ and $R_{YY}(\tau)$, with the power transfer function $H(\omega) = \frac{R}{R + iL\omega}$. (8)

(ii) A linear time invariant system has an impulse response $h(t) = e^{-\beta t}u(t)$. Find the output autocorrelation function $R_{YY}(\tau)$ corresponding to an input $X(t)$. (8)

(OR)

(b) (i) A system has unit impulse response given by

$$h(t) = \begin{cases} \frac{1}{T}, & 0 \le t \le T \\ 0, & \text{elsewhere} \end{cases}$$

Evaluate $S_{YY}(\omega)$ in terms of $S_{XX}(\omega)$. (8)

(ii) If $Y(t) = A\cos(\omega_0 t + \theta) + N(t)$, where A is a constant, θ is uniform random variable in $(-\pi, \pi)$ and $\{N(t)\}$ is a band-limited Gaussian white noise with power spectral density

$$S_{NN}(\omega) = \frac{N_0}{2},\ |\omega - \omega_0| < \omega_B.$$

Find the power spectral density of $\{Y(t)\}$. Assume that $N(t)$ and θ are independent.

(8)

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