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QUESTION PAPER CODE: X10662

B.E./B.Tech. DEGREE EXAMINATIONS, NOV/DEC 2020 & APRIL/MAY 2021

Fourth Semester

Computer Science and Engineering

MA8402 –PROBABILITY AND QUEUEING THEORY

(Regulations 2017)

Answer ALL Questions

Time: 3 Hours **Maximum Marks:100**

PART-A

(10×2=20 Marks)

1. Let A and B be two events such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{6}$. Compute $P(B/A)$ and $P(\bar{A} \cap B)$.

2. The p.d.f. of a random variable X is $f(x) = \frac{1}{2}e^{-|x|}$, $-\infty < x < \infty$. Find $E(X)$.

3. The joint p.d.f. of the random variable (X, Y) is given as

$$f(x,y) = \begin{cases} \frac{1}{2}xe^{-y}, & y > 0,\ 0 < x < 2 \\ 0, & \text{otherwise} \end{cases}.$$

   Calculate the marginal p.d.f. of X.

4. Show that the correlation coefficient , ρ_{XY}, of the random variables X and Y lies between -1 and 1.

5. Define (i) Markov Chain and (ii) Wide-sense stationary process.

6. Let $\{X_n; n \geq 0\}$ be a Markov chain having state space $S = \{1, 2\}$ and one-step TPM $P = \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$. Find the stationary probabilities of the Markov chain.

7. In an $M/M/1/\infty/FCFS$ queueing system, the arrival rate $\lambda = 3$ customers/minute and utilization ratio $\rho = 0.5$. Obtain L_s and W_s.

8. In an $M/M/c/N/FCFS$ queueing system, write the expressions for P_0 and P_N.

9. An $M/D/1$ queue has an arrival rate of 10 customers per second and a service rate of 15 customers per second. Calculate the mean number of customers in the system.

10. Consider a two-system random Markovian queueing network with customer arrival rate $\lambda = 2/minute$ and service rate $\mu_1 = 4/minute$ at station 1 and service rate $\mu_2 = 5/minute$ at station 2. Compute the probability that both the servers are idle.

PART-B (5×16=80 Marks)

11. (a) (i) Of three types of spark plugs, 6% of Type A spark plugs are defective, 4% of Type B spark plugs are defective, and 2% of Type C spark plugs are defective. A spark plug is selected at random from a batch of spark plugs containing 50 Type A plugs, 30 Type B plugs, and 20 Type C plugs. The selected plug is found to be defective. What is the probability that the selected plug was of Type A? (8)

(ii) Let $P(X=x) = \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{x-1}$, $x = 1, 2, 3, \ldots$, be the probability mass function of a random variable X. Obtain (A) $P(X > 5)$, (B) the moment generating function, $M_X(t)$, of the random variable X and (C) $E(X)$ and $Var(X)$. (8)

(OR)

(b) (i) The p.d.f. of a continuous random variable X is given as

$$f(x) = \begin{cases} \frac{1}{6}, & -3 \le x \le 3 \\ 0, & \text{otherwise} \end{cases}.$$

Find (A) $P(-2 < X < 0)$, (B) Cumulative distribution function, $F(x)$ and (C) $E(X)$ and $Var(X)$. (8)

(ii) Let X be an exponential random variable with $E(X^2) = 1/2$. Obtain (A) $E(X)$ and $Var(X)$, (B) Moment generating function, $M_X(t)$ and (C) $P(X > 3/X > 1)$. (8)

12. (a) (i) The joint p.d.f. of the random variable (X, Y) is given as

$$f(x,y) \begin{cases} ke^{-(x+y)}, & 0 \le y \le x \le \infty \\ 0, & \text{otherwise} \end{cases}.$$

Find (A) the value of k, (B) the marginal p.d.f.s of the random variables X and Y, (C) the conditional p.d.f. $f(x/y)$ of X given $Y = y$. (8)

(ii) The joint p.m.f. of discrete random variable (X, Y) is given as $P(X=-1, Y=0) = 1/8$, $P(X=-1, Y=1) = 2/8$, $P(X=1, Y=0) = 3/8$ and $P(X=1, Y=1) = 2/8$. Compute the correlation coefficient, ρ_{XY} of X and Y. (8)

(OR)

(b) (i) Two randon variables X and Y have joint p.d.f.

$$f(x,y) \begin{cases} \frac{5}{16}x^2y, & 0 < y < x < 2 \\ 0, & \text{otherwise} \end{cases}.$$

(A) Find the marginal p.d.f.s of the random variables X and Y, (B) Obtain the conditional p.d.f. $f(x/y)$, of X given $Y = y$, (C) Are the random variables X and Y independent? Justify. (8)

(ii) The joint p.d.f. of the random variable (X, Y) is given as

$$f(x,y)\begin{cases} e^{-(x+y)}, & x>0, y>0 \\ \\ 0, & \text{otherwise} \end{cases}.$$

Find the p.d.f. of random variables $U = X + Y$ and $V = \dfrac{U}{V}$. (8)

13. (a) (i) Consider a random process $\{X(t); -\infty < t < \infty\}$ defined by $X(t) = U\cos t + V\sin t$, where U and V are independent random variables, each of which assumes the values -2 and 1 with probabilities $1/3$ and $2/3$, respectively. Show that $\{X(t); -\infty < t < \infty\}$ is wide-sense stationary. (8)

(ii) Let $\{X_n : n \geq 0\}$ be a Markov chain having state space $S = \{1, 2, 3\}$ and one-step TPM

$$P = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \\ 0 & \frac{3}{4} & \frac{1}{4} \end{bmatrix}.$$

(A) Draw a transition diagram between for this chain, (B) Is the chain irreducible. Justify your answer, and (C) Is the state-3 ergodic? Explain. (8)

(OR)

(b) (i) State the postulates of the Poisson process and obtain the probability distribution for that. Is the Poisson process stationary? Justify your answer. (8)

(ii) Let $\{X_n; n = 0, 1, 2, 3 \ldots\}$ be a Markov chain having state space $S = \{1, 2, 3\}$ with one-step transition probability matrix

$$P = \begin{bmatrix} 1 & 0 & 0 \\ \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ \\ 0 & 0 & 1 \end{bmatrix}$$

and initial distribution $P(X_0 = i) = \dfrac{1}{3}$, $1 = 1, 2, 3$. Compute

(A) $P(X_3 = 1, X_2 = 1, X_1 = 1, X_0 = 2)$ and

(B) $P(X_2 = 1, X_1 = 1/X_0 = 1)$.

(8)

14. (a) (i) In an $M/M/1/\infty/FCFS$ queueing system, if $\lambda = 10$ and $\mu = 15$, compute (A) L_q, (B) W_s, (C) P_3 and(D) probability that an arriving customer has to wait in the queue. (8)

(ii) For an $M/M/1/\infty$ balking queue derive the steady-state probabilities of the system size by assuming that the service rate as $\mu_n = \mu$, $n = 1, 2, 3, \ldots$, and the arrival rate of the customers as $\lambda_n = \dfrac{\lambda}{n+1}$, $n =, 0, 1, 2, \ldots$, where $n \geq 0$ is the number of customers in the system. (8)

(OR)

(b) (i) A car servicing station has 2 bays where service can be offered simultaneously. Because of space limitation, only 4 cars can wait in the queue. The arrival pattern is Poisson with 12 cars per day. The service time in both the bays is exponentially distributed with $\mu = 8$ cars per day per bay. Find the average number of cars in the service station, the average number of cars waiting for service and the average time a car spends in the system. (8)

(ii) For an $M/M/2/\infty$ FCFS queueing system, derive the system of differential-difference equations for the system size probabilities. Under steady-state condition, obtain the steady-state probabilities of the system size and the mean number of customers in the system. (8)

15. (a) Discuss an $M/G/1/\infty$ FCFS queueing system and derive the P-K mean value formula for the system size. Deduce also the mean number of customers in the system for an $M/M/1/\infty$ FCFS queueing model from the P-K mean value formula. (16)

(OR)

(b) Derive the system of differential-difference equations for the joint probabilities of the system size of two-stations tandem queueing system. Under the steady-state conditions, determine the steady-state probabilities of the system size and hence obtain the expected number of customers in the system and the mean waiting time of a customer in the system. (16)

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